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Two novel applications of bootstrap currents: Snakes and jitter stabilization

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Both neoclassical theory and certain turbulence theories of particle transport in tokamaks predict the existence of bootstrap (i.e., pressure-driven) currents. Two new applications of this form of noninductive current are considered in this work. In the first, an earlier model of the nonlinearly saturated $m=1$ tearing mode is extended to include the stabilizing effect of a bootstrap current *inside* the island. This is used to explain several observed features of the so-called "snake" reported in the Joint European Torus (JET) [R. D. Gill, A. W. Edwards, D. Pasini, and A. Weller, Nucl. Fusion **32**, 723 (1992)]. The second application involves an alternating current (ac) form of bootstrap current, produced by pressure-gradient fluctuations. It is suggested that a time-dependent (in the plasma frame), radio-frequency (rf) power source can be used to produce localized pressure fluctuations of suitable frequency and amplitude to implement the dynamic stabilization method for suppressing gross modes in tokamaks suggested in a recent paper [A. Thyagaraja, R. D. Hazeltine, and A. Y. Aydemir, Phys. Fluids B **4**, 2733 (1992)]. This method works by "detuning" the resonant layer by rapid current/shear fluctuations. Estimates made for the power source requirements both for small machines such as COMPASS and for larger machines like JET suggest that the method could be practically feasible. This "jitter" (i.e., dynamic) stabilization method could provide a useful form of active instability control to avoid both gross/disruptive and fine-scale/transportive instabilities, which may set severe operating/safety constraints in the reactor regime. The results are also capable, in principle, of throwing considerable light on the local properties of current generation and diffusion in tokamaks, which may be enhanced by turbulence, as has been suggested recently by several researchers.

I. INTRODUCTION

One of the striking predictions of the neoclassical theory[1] that has received some measure of experimental support[2,3] is that of the so-called "bootstrap current." Although it follows generally from Ohm's law that a radial particle flow across flux surfaces could lead to a net toroidal current in addition to the usual Ohmic current, the form and characteristics of this type of current were first given by Bickerton *et al.* in Ref. 1. However, the particle and energy fluxes predicted by neoclassical theory are decidedly not in agreement with tokamak transport measurements. It was, therefore, suggested independently by Yushmanov,[4] Yushmanov and Pereverzev,[5] Connor and Taylor,[6] on the one hand, and by ourselves,[7] on the other, based on quite different physical notions, that there might be a turbulence-enhanced source-driven current associated with turbulent particle and energy losses and possibly also an anomalous resistivity. In a recent paper,[8] we have reviewed various theories of particle and current transport in tokamaks and described their principal features and differences. Whereas Yushmanov and Pereverzev, and Connor and Taylor assume the existence on phenomenological grounds of an anomalously enhanced electron viscosity (equivalent to an anomalous electron–electron collision frequency), our model is related more directly to an anomalous poloidal resistivity and conceivably an anomalous thermal force. These turbulent constitutive properties used in our model are related to the typical ($\mathbf{E}\times\mathbf{B}$) nonlinear drift-wave turbulence measured in tokamaks.[8] Although conceptually quite distinct, all the models referred to lead to turbulent constitutive relations (i.e., fluxes) identical in form and having similar physical consequences, at least as far as particle and current transport are concerned.

The purpose of the present work is to explore some novel consequences of assuming the existence of source-driven currents, of which the neoclassical bootstrap current is the most widely known example and paradigm. Specifically, we consider two applications: one involving direct current (dc) effects in a helically symmetric situation, whereas the second involves alternating current (ac) effects in a poloidally symmetric situation. In the first application, we consider nonlinearly saturated $m=1$ islands[9] maintained by pressure-driven currents with a view to explaining some of the observed features of "snakes".[10] According to Gill *et al.*, the experimental observations of the snake phenomenon reported by them in Ref. 10 do not conform to existing theories of island formation. Some reasons for these apparent discrepancies and the new features of our analysis, including the island pressure-driven currents, as compared, for example, with the earlier work on such currents on island formation[11] of Carrera *et al.*, will be discussed later.

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The second application involves an ac form of bootstrap current. It is suggested that a time-dependent, rf power source can be used to produce localized pressure fluctuations of suitable frequency and amplitude to implement the dynamic stabilization method for suppressing gross modes in tokamaks considered in a recent paper.[12] This method works by "detuning" the resonant layer by rapid current/q profile fluctuations. Estimates are made for the power source requirements both for small machines and large machines. They suggest that the method may be practically feasible and could provide a useful form of active instability control to avoid both gross/disruptive and fine-scale/transportive instabilities, which may set severe operating/safety constraints in the reactor regime. The results are also capable, in principle, of throwing considerable light on the local properties of current generation and diffusion in tokamaks, which may be enhanced by turbulence, as has been suggested recently by several researchers.[13,14]

In view of the paucity of detailed local measurements of various parameters and profiles, the theoretical models are necessarily kept simple. Their purpose is to explain hitherto unexplained facts in a qualitative fashion, within the limitations of purely analytic models.

II. NONLINEARLY SATURATED $m=1$ MODE MAINTAINED BY PRESSURE-DRIVEN CURRENTS

We begin by recalling the salient features of our previously published theory[9] of the nonlinearly saturated $m=1$ tearing mode. The entire discussion is limited to the cylindrical model, though, of course, the bootstrap current is due to toroidal effects. It is important to note that pressure-driven currents (due to $\mathbf{E}\times\mathbf{B}$ turbulence) can occur in cylindrical geometry, as our model[7] and some recent numerical simulations[14] show. The point is that one may discuss the magnetic island structure rather more simply in cylindrical geometry without all the toroidal complications. It may be that the resulting model is oversimplified in some respects and not fully consistent, but the results seem to capture the qualitative features of real experiments.

In our earlier paper,[9] we considered the nonlinearly saturated final states, which can be expected to arise in reduced magnetohydrodynamics (RMHD) following an $m=1$ linear tearing instability in tokamaks with $q_0<1$. We found conditions for the existence of a nonlinearly saturated $m=1$ mode, assuming the existence of a purely *inductively* driven current. The solution requires a q profile with a characteristic "shoulder" where $r_i q'(r_i)\equiv s\ll 1$, $q(r_i)=1$. It is believed that experimental observations support this assumption.[15,16] The theory has been extended to the torus,[17] although not in detail. A recent analytic study by Smolyakov[18] implies that the solution found in our steady asymptotic theory is actually the end state of a "Rutherford"-type evolution, starting with the initial tearing instability.

The detailed asymptotic analysis yields a bifurcation relation that expresses the saturated island width w_{island} in terms of the shear parameter s. This relation involves the profile of the current density inside the island. It is an interesting consequence of the analysis that the current profile in the island exterior does not enter the bifurcation relation, which gives the island width. This is different from the $m>1$ problem studied by Carrera *et al.* These authors assumed in their analysis[11] that the pressure gradients within the island are flattened, and therefore not as important as the exterior ones. The experimental observations of snakes[10] show that this may not be the case for $m=1$. We propose to investigate the effect of a pressure-driven current of whatever origin (be it neoclassical or turbulence enhanced) on the structure of the island. The calculations are presented for a simplified model in which only the essential features of the problem are highlighted.

We make the assumption that the "equilibrium" current (i.e., q) profile is specified, as in our earlier paper.[9] This profile implicitly includes, of course, the neoclassical modifications to resistivity. In view of the fact that pressure gradients within the island are crucial to the problem, we also assume that any pressure-driven currents exterior to the island merely modify the geometry of the outer solution slightly, and are not involved in the bifurcation relation. This is also suggested by our asymptotic analysis of the nonlinear equations in the inner region.[9] The problem then reduces to the evaluation of the bifurcation relation in the presence of pressure-driven currents, which are assumed to operate in the interior of the island, in addition to the Ohmic current. The notation and symbols are as in Ref. 9, unless otherwise stated. It should be remembered that the perturbation parameter λ that occurs in the following analysis is a nondimensional measure of the saturated island width.

Using the nondimensional forms established for the helical flux function, $\Psi^*(Y,u,\lambda)$ and the electric potential $\Phi^*(Y,u,\lambda)$, and defining the parameter $\alpha\equiv(2-s)/s, s\equiv r_i q'(r_i)$, the "inner" equations reduce to

$$\frac{\partial^2\Psi^*}{\partial Y^2}=-1-\alpha[1-J^*(\Psi^*)], \tag{1}$$

$$1-H(Y,\lambda)[J^*(\Psi^*)-J^*_{+,p}(\Psi^*)]=\frac{\partial(\Psi^*,\Phi^*)}{\partial(Y,u)}. \tag{2}$$

In these equations, the equivalent radial variable $Y\equiv(r-r_i)/\lambda r_i$ and u is the helical angular variable. All flux functions are nondimensionalized by $\lambda^2 r_i^2 \psi_0''(r_i)$. This is equivalent to nondimensionalizing all current densities by $\psi_0''(r_i)$. Note that $J^*_{+,p}(\Psi^*)$ denotes the pressure-driven current within the island, and $H(Y,\lambda)$ is related to the equilibrium current profile, as in our earlier paper. The neoclassical modifications of resistivity are (implicitly) contained in this profile. The precise form for $J^*_{+,p}(\Psi^*)$ will be discussed after we obtain the new bifurcation relation from Eqs. (1) and (2). The total (nondimensional) current density is, of course, denoted by $J^*(\Psi^*)$.

Thanks to our assumption that the pressure-driven currents are effective only within the island, the formal analysis of Ref. 9 can be repeated virtually verbatim. The principal and crucial difference is that the current density within the island, denoted $J^*_+(\Psi^*)$, must include both the Ohmic and the pressure-driven contributions:

$$J^*_+(\Psi^*) \equiv J^*_{+,\mathrm{Ohmic}} + J^*_{+,p}. \tag{3}$$

The expression for $J^*_{+,\mathrm{Ohmic}}$ has already been obtained. With the present notation, it takes the form

$$J^*_{+,\mathrm{Ohmic}}(\Psi^*) = \frac{2\int_{-v}^{v} du/\sqrt{\sin^2 v - \sin^2 u}}{\int_{-v}^{v} du/\sqrt{\sin^2 v - \sin^2 u}\{H[Y_-(v,u),\lambda] + H[Y_+(v,u),\lambda]\}}. \tag{4}$$

It is evident that within the island we must have $0 < \Psi^* < \frac{1}{2}$, $Y < 0$. The functions $Y_{+,-}$ are given by

$$Y_\pm(\Psi^*,u) = -\cos u \pm \sqrt{\cos^2 u - 2\Psi^*}$$
$$(1 \geqslant \cos^2 u \geqslant 2\Psi^*). \tag{5}$$

In Eq. (4), v is an angle defined by $\sin^2 v \equiv 1 - 2\Psi^*$ and $-v < u < v$.

The bifurcation relation takes exactly the same form as before, and can be written in a form that makes the contribution from pressure-driven currents explicit:

$$2\alpha \int_0^{1/2} [J^*_{+,\mathrm{Ohmic}}(\sigma) - 1] d\sigma = 1 - 2\alpha \int_0^{1/2} J^*_{+,p}(\sigma) d\sigma. \tag{6}$$

Here, the dummy variable σ is a flux-surface label.

Before we proceed to extract the physical implications of this equation, it is necessary to take into account the explicit form of the pressure-driven current. Neoclassical theory[1] gives the following well-known expression for the bootstrap current density (in Gaussian units), namely,

$$j_\parallel^{\mathrm{BS}} = -\kappa \left(\frac{r}{R}\right)^{1/2} \frac{c}{B_{\mathrm{pol}}} \frac{dp}{dr}, \tag{7}$$

where κ depends, among other things, on collisionality, ratios of density, and temperature gradients, etc. In certain cases,[19] $\kappa \simeq 4.88(d \log n / d \log p) + 0.27 d \log T / d \log p$. As discussed in detail in our recent paper,[8] both our model of turbulence-enhanced poloidal resistivity and the theories of Yushmanov and Pereverzev[4,5] and Connor and Taylor[6] lead to pressure-driven currents of the form

$$j_p \simeq -\frac{c}{B_{\mathrm{pol}}} \frac{dp}{dr}. \tag{8}$$

We can treat all the cases in a unified way if we make the following simple observations regarding the currents within the island. Let us set $\Delta p \equiv p_{\max} - p_{\mathrm{separatrix}}$. A typical poloidal magnetic field is given by $B_{\mathrm{pol}} = (r_i/R) B_z$. We consider an arbitrary pressure profile, $F(\sigma)$ within the island, vanishing on the separatrix and equal to unity at the O point of the island. Noting that $\lambda r_i \simeq w_{\mathrm{island}}$, and taking account of the current density normalizations of Ref. 9 [i.e., $s = R\psi_0''(r_i)/B_z$], we obtain the following simple expression for the pressure-driven current, $J^*_{+,p}(\sigma)$:

$$J^*_{+,p}(\sigma) = \left(\frac{\delta\beta^*_p}{\lambda}\right) \frac{dF}{d\sigma}. \tag{9}$$

The nondimensional constant $\delta\beta^*_p$ is simply related (as might be expected on elementary physical grounds) to the increase in pressure in the island interior. Thus, we find

$$\delta\beta^*_p \equiv \kappa . \frac{\Delta p}{s B^2_{\mathrm{pol}}(r_i)} \left(\frac{r_i}{R}\right)^{1/2}. \tag{10}$$

The constant κ is of order unity in neoclassical theory, but can be larger [i.e., typically of order $(r_i/R)^{-1/2}$] in the turbulent pressure-drive models. It should also be remembered that this constant includes all possible geometrical factors arising in averaging the bootstrap current over the island flux surfaces. The fraction $\delta\beta^*_p/\lambda$ is a measure of the pressure gradient within the island. Carrera *et al.* derive a very similar expression with specific forms for κ for $m > 1$ islands in their paper.[11] The details of the island geometry enter only through the constant κ, and are expected to be unimportant for the purposes of the present investigation.

Substituting these expressions in the general bifurcation relation, Eq. (6), we get, upon making use of $\int_0^{1/2} (dF/d\sigma) d\sigma \equiv 1$ [this simply arises from the definition of the pressure profile function, $F(\sigma)$], the new bifurcation relation,

$$2\alpha \int_0^{1/2} [J^*_{+,\mathrm{Ohmic}}(\sigma) - 1] d\sigma = 1 - 2\alpha \delta\beta^*_p/\lambda. \tag{11}$$

This is to be regarded as an equation for the as yet undetermined amplitude parameter λ, in terms of the given profile properties such as α and the island pressure parameter, $\delta\beta^*_p$.

It must be emphasized that Δp (and hence $\delta\beta^*_p$) may, in general, depend upon λ. It can be treated as an autonomous parameter to be estimated from experiment and used as such to obtain λ by solving Eq. (11) or, alternatively, it may be calculated as a function (among other things) of λ and appropriate energy and particle sources by solving the relevant transport equations within the island interior. In the spirit of the present analysis, we shall follow the first route here. It should be noted that experimental evidence suggests that the pressure gradients within the island do not bear any simple relation to the exterior values, indicating that the island confinement (of particle density; heat transport is more like the outer region[10]) is better than that of the exterior region. At present, there is no theoretical explanation of this fact. It is, however, consistent with the idea that island states such as the ones we consider are more nearly neoclassical relative to the tokamak as a whole, especially as regards particle confinement.

A more complete analysis, which lies outside the scope of the present paper, can be expected to include the energy and particle balance equations within the island; the bifurcation relation would then be able to predict the actual value of Δp as a function of λ and relevant transport coefficients. Such an analysis is, of course, much more complicated than the present one, not only from a mathematical point of view (it is doubtful if a purely analytic treatment is at all possible for this more general problem) but also from a purely physical view, in view of the uncertainties surrounding tokamak transport. For this reason, we have used experiment to guide us with regard to the island pressure confinement and use our bifurcation relation to elucidate the magnetic structure of the snake without seeking to explain the "good" particle (and pressure) confinement of the resultant nonlinearly saturated island/snake.

Note that the actual pressure *profile* within the island [i.e., $F(\sigma)$] is irrelevant to the determination of λ, and hence the island width w_{island}.

Returning to the bifurcation relation, the integral on the left of Eq. (11) is already known from Ref. 9. Writing, $\alpha \simeq 2/s$, we obtain

$$\lambda = \lambda_0 \left(1 - \frac{4\delta\beta_p^*}{s\lambda} \right), \tag{12}$$

where $\lambda_0 \equiv \frac{1}{2}\alpha^*[d \log q(r_i)/d \log \eta(r_i)]$ is the solution in the absence of pressure. The numerical constant $\alpha^* \equiv [\int_0^1 \pi\, dx/2K(x)]^{-1}$, where $K(x)$ is the elliptic integral, $K(x) = \int_0^{\pi/2} dw/\sqrt{1-x\sin^2 w}$. It is known that $1 < \alpha^* < \frac{3}{2}$. The island width is then given by $w_{\text{island}} \equiv 2\lambda r_i$.

It is instructive to note that the bifurcation relation [Eq. (12)] derived from our steady-state asymptotic theory can also be obtained as a result of Smolyakov's[18] Rutherford-type equation, upon taking proper account of the bootstrap current in the *interior* of the $m=1$ island. This alternative derivation also makes useful contact with general formulas due to Carrera *et al.* for the island bootstrap, with the important difference that the pressure gradient is evaluated within the island rather than with reference to the exterior distribution.

Without going into extensive detail, Smolyakov's Rutherford equation modified by the island bootstrap current (but neglecting the Pfirsch–Schlüter contribution, which makes only a minor contribution to the island width normalization) takes the form[18]

$$\frac{\partial w}{\partial t} = \mu_* D_R \left(\frac{1}{w} - \frac{1}{w_0} - \frac{8 r_i \delta\beta_p^*}{s w^2} \right). \tag{13}$$

We use the notations $w = 2\lambda r_i$ and $w_0 = 2\lambda_0 r_i$. It is easily seen that the expression for the island bootstrap is consistent with that of Carrera *et al.*, when it is realized that the pressure gradient to be used refers to the island interior and is proportional to $\Delta p/w$. The steady-state solution of this Rutherford–Smolyakov equation is exactly that given by the bifurcation relation, Eq. (12). Furthermore, it is clearly the case that our steady solutions (with and without the island bootstrap current) are the end states of the corresponding Rutherford-type evolution described by Smolyakov's nonlinear $m=1$ model.

We now discuss the implications of the preceding theory. It is obvious that the results of Ref. 9 on the saturation of the $m=1$ tearing mode are recovered if $\delta\beta_p^* \equiv 0$. The bifurcation relation in this case relates λ and the island width to the reduced shear parameter, $d \log q(r_i)/d \log \eta(r_i)$. When $\delta\beta_p^*$ is nonzero, the relation becomes formally a quadratic in λ. Setting $x \equiv \lambda/\lambda_0$, for sufficiently small pressure within the island, we obtain

$$x_{\pm} = \frac{1 \pm \sqrt{(1 - 16\delta\beta_p^*/s\lambda_0)}}{2}. \tag{14}$$

Thus, there are either two physically meaningful (i.e., real, positive) solutions or no solutions. For given s and λ_0, if the pressure is sufficiently high (i.e., when $4\delta\beta_p^*/s\lambda_0 > 1$), there are no real solutions. As the pressure within the island rises, the two solutions approach each other and disappear when the critical value is exceeded. This result for x_- may indicate a "β crash" of the saturated $m=1$ island at sufficiently high internal pressure, and could be conceivably involved in triggering sawteeth crashes. When the equilibrium fails, it is most likely the crash would occur on a rapid, Alfvénic time scale.

When the pressure-driven current parameter, $16\delta\beta_p^*/s\lambda_0$, is less than unity, there are two real solutions. Both these solutions are smaller than the pressure-free solution, λ_0. Note that as the parameter increases, the two solutions approach each other and "coalesce" at the value $\lambda = \lambda_0/2$. The smaller of the two solutions (i.e., $\lambda_0 x_-$) is interesting, in that at a sufficiently small value of the pressure parameter, it leads to $\lambda = 4\delta\beta_p^*/s$. This is manifestly independent of the Ohmic current within the island. Indeed, it leads to small saturated self-consistent islands, even when the shear parameter s is *not* small! These "self-sustained" islands held together by pressure-driven currents are quite different in character from the previously determined "small shear" islands.

We can demonstrate generally that in order to fulfill the condition of q' having a zero close to but inboard of the resonant point r_i, it is necessary for the equilibrium toroidal current $j_{0z}(r)$ to have a relative maximum at some radius $0 < r < r_i$. This follows from $1/q(r) \equiv RB_\theta(r)/rB_z$ and Ampère's law using the identity

$$\frac{rq'}{q^2} = \frac{4\pi R}{cB_z} \left(-\frac{1}{r^2} \int_0^r j_z'(u) u^2\, du \right). \tag{15}$$

If $j_z' < 0$, rq' cannot vanish close to r_i. Hence, in order that the $s \simeq 0$ inboard of the $q=1$ radius, the current density must have a sharp local maximum there (and, of course, a nearby local minimum as well, provided the current density is peaked at the magnetic axis).

This proves that for the purely Ohmic case of the $m=1$ island, the equilibrium resistivity must have a local minimum slightly inboard of the resonant point r_i. In the presence of an increased pressure within such an island, an extra pressure-driven current effectively reduces the need for this Ohmic effect. Indeed, in this case, it is no longer necessary for s to be small (although it may be for other reasons unconnected with the island saturation). Thus we see that the existence of a pressure-driven current obviates the need to have a minimum in the equilibrium resistivity profile or a zero of q'.

It follows from the preceding considerations, that our model predicts two distinct types of islands: in the purely Ohmic case, the requirement of a current density *maximum* inboard of the $q=1$ radius implies a minimum of resistivity (under steady conditions), which, in turn, requires a temperature maximum at the O point of the island. This might be termed the "hot" island scenario. However, as we have seen, if there are pressure-driven currents present, all that is needed to satisfy the bifurcation relation is a *pressure* gradient within the island. The temperature in this case can even be lower within the island, provided the density is sufficiently high. This can be called the "cool island" scenario.

III. APPLICATION TO SNAKES IN JET

In a recent paper,[10] Gill *et al.* have documented observations of large, very localized pressure perturbations observed in JET due to the formation of a small region on the $q=1$ surface, which persists for a surprisingly long time, called the "snake." Several types of snakes are discussed. The most common type are "pellet-induced" snakes. More rarely, "spontaneous" snakes associated with increased resistivity changes and impurity accumulation prior to the onset of sawteeth are seen, as are more exotic "double" and "negative" snakes. We shall apply the ideas developed in the previous section to gain a qualitative description of these phenomena.

We begin by considering a typical example of a pellet-induced snake. Thus consider case B (Table 1 in Ref. 10), which is a pellet-produced snake with a lifetime of nearly 2 secs. According to Gill *et al.*, the temperature hardly changes within the island ($<10\%$), whereas the density increase is about 16%. We assume that $\Delta p \simeq \delta n_e . T_e \simeq 2.0 \times 10^4$ erg/cm^3. We also take $r_s = r_i = 50$ cm; $R=300$ cm; $B_z \simeq 3$ T. The island width is given by experiment to be $l_r \simeq 20$ cm. The shear parameter s is not known in this case. A reasonable estimate is $s \simeq 10^{-1}$. We now estimate $\delta\beta_p^* \simeq \kappa 4 \times 10^{-3}$.

If we assume the constant $\kappa=1$ and consider the case when the pressure-driven current maintains the island entirely on its own, we get the estimate, $\lambda \simeq 4\delta\beta_p^*/s \simeq 0.16$, leading to $w_{\text{island}} \simeq 16$ cm, which is in moderate agreement with experiment, given the roughness of the estimates. Thus, it seems that the model can qualitatively account for the observations in this case.

The case of spontaneous snakes involves impurity accumulation and consequently increased resistivity. They are fairly rare and are not strictly steady-state phenomena. It seems that pressure-driven currents within the island offer the most plausible explanation for them. In the absence of pressure-driven currents, the theory requires a minimum in rq' inboard of the island, and this does not appear to be consistent with an enhanced resistivity. The same would appear to be true of the "negative" snakes, which seem to have depressed temperatures within the island.

We estimate the parameter $4\delta\beta_p^*/s$ for case C in Gill *et al.* Taking the numbers given by the authors and $\kappa=1$, $s=0.1$, we find that $\delta\beta_p^* \simeq 5 \times 10^{-3}$, $w_{\text{island}} \simeq 20$ cm, nearly twice the value quoted by the experimenters. It should, however, be remembered that in this case the collisionality effects can be expected to lower the constant κ and also the uncertainty in the parameter s can easily reconcile the theory and experiment. Indeed, even the sudden appearance of the snake may be due to the fact that as the profile evolves, the parameter $16\delta\beta_p^*/s\lambda_0$ evolves from a large positive value to below unity, establishing conditions for the appearance of a snake, although this scenario is at present tentative. Thus, within the limits of theoretical and experimental uncertainty, we conclude that the model can describe spontaneous snakes.

There are no detailed measurements available for the more exotic "double" and the "negative" snakes. We should point out that our earlier paper[9] actually predicted the possibility of "double" snakes, provided the q profile had two $q=1$ resonances. There is therefore little difficulty in explaining double snakes in the presence of pressure-driven currents. Negative snakes can also be qualitatively explained. The model makes a clear cut prediction: even if the temperature within the island is lower than the surroundings, the *pressure* within the island must be higher.

We can now discuss some of the main differences between our model and earlier approaches. Gill *et al.* do not consider the effects of pressure-driven currents in their brief discussion of island models to account for their observations. The data presented by them indicate that the pressure within the island is substantial. Our previous model of the saturated $m=1$ supplemented by Smolyakov's analysis[18] shows that Ohmic currents alone are inadequate to explain snakes that correspond more closely to the "cool" island scenario than to the Ohmic, "hot" island one. The work of Carrera *et al.*[11] is not immediately applicable to the $m=1$ case. More importantly, they find that the effect of island bootstrap in their problem is principally in the exterior region, where there are considerable pressure gradients. This exterior contribution in the more symmetrical $m>1$ islands tends to *destabilize* the tearing mode, at least initially. In our case, the island pressure-driven currents in the interior actually tend to maintain the neighboring equilibrium and "sustain" the island against the $m=1$ tearing instability. Such pressures are also indicative of good particle confinement within the island, as shown by the accumulation of impurities.

At present, both the model and the data are in too primitive a state to conclude definitely that the "anomalous" pressure-driven currents, as opposed to the neoclassical bootstrap currents, are actually needed to explain the experimental observations. The effects of transport and toroidal geometry on the islands remain to be elucidated. These are expected to be crucial to explaining the fact that as the discharge evolves is time s is thought to increase experimentally (although it is not clear whether this refers to the shear at $q=1$), even though the island width appears not to decrease.

Note that our model is not yet capable of making definite predictions about how pellet-induced snakes actually form and why such structures are experimentally found to be remarkably stable to large perturbations such as sawteeth crashes. These problems are not only inherently transient on rather rapid time scales (considerably less than a millisecond), they also involve the solution of energy and other transport equations. Thus the magnetic structure cannot be discussed in isolation from transport in the consideration of such rapid transients. It seems that this limitation of the model cannot be readily overcome by purely analytic means, given the large uncertainties in turbulent transport under the physical conditions in question. Thus these problems are clearly outside the scope of this paper.

In any event, the results presented suggest that the present model would appear to provide a reasonable starting point for understanding the snake phenomenon associated with the $m=1$ mode.

IV. APPLICATION OF FLUCTUATING PRESSURE-DRIVEN CURRENTS TO JITTER STABILIZATION

Active control of system stability is a standard tool in many areas of engineering. Recently, it has even been suggested that it might be particularly applicable to "chaotic" systems, which are known to be especially sensitive to small perturbations. The concept and ideas relating to dynamic stabilization are well known in plasma physics.[20,21] Active control of plasma instabilities, both of the dynamic and the feedback variety, can be expected to play an increasingly important engineering role in the reactor regime, where severe constraints set by both gross/disruptive and fine-scale/transportive instabilities must be avoided for both economic and safety/operational grounds.

The aim of this work is to briefly describe the possibilities of a type of active control of gross plasma instabilities suggested by the recent paper of Thyagaraja, Hazeltine, and Aydemir.[12] In view of the fact that gross modes could place severe constraints on reactor operations, it seems worthwhile to explore possibilities suggested by "jitter" stabilization of resonant tearing modes, which work by effectively "detuning" the resonance. This is complementary to the dc control of tearing-like modes, which involves modification of the equilibrium j (and, consequently, q) profiles by suitable heating/current-drive schemes, being actively pursued across the world in many laboratories. If it is possible to use an ac current-drive source of noninductive origin, it might be feasible to to superpose a time-periodic oscillatory component to the equilibrium j. It was shown by Thyagaraja *et al.*[12] that such oscillations of suitable magnitude and frequency relative to the $m=1$ (linearly unstable) tearing mode could lower or even completely eliminate the growth of the mode. More recent work[22] suggests that a similar theoretical conclusion might apply to higher m tearing modes, and possibly also to fine-scale modes involved in drift turbulence.

In all cases, the key experimental issue is this: how can the tokamak current be "jittered" at the right frequency and amplitude so that a resonant mode can be "detuned?" If the resistivity is sufficiently high, it can easily be seen that resistive diffusion of current is consistent with localized perturbations of q of the size and type desired. At low collisionality, however, it is not easy to see how the jitter "penetrates" to the resonance zone of the mode.

If $A_{\parallel}(r,t)$ is the parallel vector potential, Ohm's law states that $A_{\parallel}$ must evolve according to

$$E_{\text{ext},\parallel}-\frac{1}{c}\frac{\partial A_{\parallel}}{\partial t}=\eta_{\parallel}\left[-\frac{c}{4\pi}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial A_{\parallel}}{\partial r}\right)-j_{\parallel}^{c.d.}\right],\quad(16)$$

where we have used Ampère's law, and $j_{\parallel}^{c.d.}$ is the "noninductively" driven current source (of whatever origin). It follows that if $j_{\parallel}^{c.d.}=(2/\pi)(I^*/2\pi r_s)[\Delta\cos\omega t/\Delta^2+(r-r_s)^2]$, where Δ is the radial localization width of the driven current, I^* is the total maximum noninductive current, and r_s is the resonance radius of the mode required to be jitter stabilized, the jitter amplitude of q is obtained as[12]

$$\frac{\delta q}{q}\simeq\left(\frac{c^2\eta_{\parallel}}{4\pi\,\Delta^2\omega}\right)\left(\frac{I^*}{I_{\text{plasma}}(r_s)}\right).\quad(17)$$

For stabilization of the $m=1$ mode, we require, $\delta q V_{\text{Alfvén}}/R\omega\gg1$. This formula is approximate, but is is expected on very general grounds to give the correct order of magnitudes. It shows immediately that low frequency, strong localization, large amplitude, and high resistivity are "good" for jitter stabilization. Now the frequency ω is required to be higher than the growth rate of the "unjittered" mode (γ_c) for the resonance to be properly "detuned." Fortunately, γ_c only grows like a weak power ($\frac{1}{3}$–$\frac{3}{5}$) of $\eta_{\parallel}$. The localization width Δ is set by the method used to drive the noninductive current. Obviously, it will be small for rf (ECRH, for instance) rather than beams. The amplitude is also a function of the power available and the efficiency of current drive [as measured by the ratio $I^*/I(r_s)$].

The general formula shows that as the collisionality decreases, η decreases rather sharply with temperature (if assumed Spitzer or neoclassical), and the requirements for penetration seem rather stringent. The recent work[8] of Thyagaraja and Haas suggests that anomalous bootstrap currents and associated enhanced resistivity may provide the solution to this problem. Studies by Shaing[13] and Dawson[14] also suggest that the neoclassical bootstrap current may be considerably enhanced by turbulence effects, reinforcing earlier phenomenological theories,[6–8] which lead to similar conclusions. If this is indeed the case, it might be possible to get out of the "resistivity straitjacket" implied by the preceding relations.

We now consider a novel application of pressure-driven currents (otherwise known as "neoclassical" or "anomalous" bootstrap currents), made to oscillate by suitable energy sources at any desired location within the plasma. It is convenient to proceed quite generally at first. Consider Ohm's law for the equilibrium (i.e., $m=0$) parallel current. According to Rosenbluth *et al.*,[19] a detailed calculation of neoclassical transport in the banana regime leads to

$$E_{\parallel} = \frac{\eta_{\parallel}^{\text{Spitzer}}}{1-1.95(r/R)^{1/2}}(j_{\text{tor}} - j_{\parallel}^{\text{BS}}), \tag{18}$$

where $j_{\parallel}^{\text{BS}}$ is given by Eq. (7). If conditions are steady, $E_{\parallel} \simeq E_{\text{ext},z}$, the latter being the transformer field applied to drive the Ohmic current in the plasma. If we seek to "jitter" the system by introducing, say, a localized (in r), time-periodic perturbation in the pressure denoted by $\delta p(r,t)$, Eq. (18) governs the plasma response. In particular, we may estimate the fluctuations in B_θ, or equivalently in $q(r,t)$. It should be noted that Eq. (18) is actually nonlinear, due to the form of the bootstrap current. It can, therefore, be solved, in general, as a boundary value problem for given δp and $\delta\eta_{\parallel}$ only numerically. Fortunately, in the application we are interested in, it is entirely appropriate to assume that δp is localized in radius, periodic in time, and also that it is small in the sense that the perturbed currents are small relative to the equilibrium current. In view of this, the equation for the perturbed parallel vector potential $\delta A_{\parallel}$ (r,t) can be derived from Eq. (18) by linearization. This equation takes the following form:

$$-\frac{1}{c}\frac{\partial \delta A_{\parallel}}{\partial t} = \eta\left[-\frac{c}{4\pi}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \delta A_{\parallel}}{\partial r}\right) - \delta j_{\parallel}^{\text{BS}}\right] + \frac{\delta\eta}{\eta}E_{\text{ext},z}. \tag{19}$$

In this equation, η refers to the equilibrium parallel resistivity and all quantities without the "δ" are the unjittered quantities. The equation may be rewritten in the compact form

$$\frac{\partial \delta A_{\parallel}}{\partial t} = \frac{c^2\eta}{4\pi}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \delta A_{\parallel}}{\partial r}\right) + \delta S(r,t). \tag{20}$$

The source term $\delta S(r,t)$ comprises three distinct contributions. The fluctuations in the pressure are generally associated with temperature fluctuations leading to resistivity fluctuations. Pressure fluctuations δp drive $\delta j_{\parallel}^{\text{BS}}$. Finally, the $\delta A_{\parallel}$ themselves lead to fluctuations in B_θ, which also contribute to the fluctuations of the bootstrap current. These observations are embodied in the following expression for δS,

$$\delta S(r,t) \equiv \frac{c\delta\eta}{\eta}E_{\text{ext},z} + \left(\frac{c}{B_\theta}\frac{\partial \delta A_{\parallel}}{\partial r} - \frac{L_p}{p}\frac{\partial \delta p}{\partial r}\right)c\eta j_{\parallel}^{\text{BS}}, \tag{21}$$

where $L_p \equiv p/dp/dr$. It is easily seen that the first and third terms are "forcing" while the second is a "response" term. We shall analyze this expression under the assumptions (1) ($\delta p \ll p$, (2) The typical radial localization length Δ of the pressure perturbation satisfies $\Delta \ll L_p \simeq r_i$, where r_i denotes the the resonance radius, and (3) the frequency of the applied pressure perturbation satisfies, $\omega \gg c^2\eta/4\pi r_i^2$. It is now plain that we may order the first term in δS out in comparison with the third. Similarly, the second, response term is negligible in comparison with $\omega\delta A_{\parallel}$ on the left-hand side of Eq. (20). Furthermore, normalizing $(r-r_i)/\Delta \equiv x, \omega t \equiv \tau$, Eqs. (20) and (21) may be simplified to

$$\frac{\partial \delta A_{\parallel}}{\partial \tau} = \frac{c^2\eta}{4\pi\,\Delta^2\omega}\cdot\frac{\partial^2 \delta A_{\parallel}}{\partial x^2} - \frac{L_p}{\omega\Delta}\frac{\partial}{\partial x}\left(\frac{\delta p}{p}\right)c\eta j_{\parallel}^{\text{BS}}. \tag{22}$$

Bearing in mind the relations $\delta B_\theta = -(1/\Delta)(\partial \delta A_{\parallel}/\partial x), \delta q/q = -\delta B_\theta/B_\theta$, we get the following equation for $\delta q/q \equiv Q$ from Eq. (21) by differentiating with respect to x and using the following relations: $\Delta \ll L_p; f^{\text{BS}} \equiv j_{\parallel}^{\text{BS}}/j_{\parallel}^{\text{Tot}}; j_{\parallel}^{\text{Tot}} \simeq (c/4\pi)(1/r)(\partial/\partial r) \times (rB_\theta), q = rB_z/RB_\theta$:

$$\frac{\partial Q(x,\tau)}{\partial \tau} = \frac{c^2\eta}{4\pi\,\Delta^2\omega}\left[\frac{\partial^2 Q}{\partial x^2} - \frac{\partial^2}{\partial x^2}\left(\frac{\delta p}{p}\right)\frac{2L_p f^{\text{BS}}(1-s)}{r_i}\right], \tag{23}$$

where $s \equiv r_i q'(r_i)/q(r_i)$ and f^{BS} is the "bootstrap" fraction [i.e., the ratio of the bootstrap to the total current density at the resonance radius; we are presently concerned mainly with $q(r_i)=1$]. For the present purposes we may assume that all the equilibrium quantities in the above equation are evaluated at $r=r_i$ and take $\delta p/p = \epsilon\exp(-x^2/2)\cos\tau$. In practice, we are interested in situations when the parameter $c^2\eta/4\pi\,\Delta^2\omega$ is small. The approximate solution of Eq. (23) is then readily estimated by

$$\begin{aligned} Q(x,\tau) &= \frac{\delta q}{q} \\ &\simeq \epsilon\frac{c^2\eta}{4\pi\,\Delta^2\omega}\left(\frac{2L_p f^{\text{BS}}(1-s)}{r_i}\right)(1-x^2) \\ &\quad\times\exp\left(-\frac{x^2}{2}\right)\sin\tau. \end{aligned} \tag{24}$$

In general, the diffusive response should also be included. This leads to a phase shift between Q and the pressure perturbations, as might be anticipated. Clearly, for given ϵ, ω, which will presumably be fixed by technical requirements, the response δq is large when the resistivity is as large as possible and the localization length Δ as small as possible. We also require the ratio $(1-s)f^{\text{BS}}L_p/r_i$ as large as possible.

The work of Thyagaraja *et al.*[12] suggests that the $m=1$ tearing mode can be "jitter stabilized" by resonance detuning if $\delta q_{\max}(V_A/\omega R) \geqslant 1$, where, $V_A \equiv B_z/(4\pi\rho)^{1/2}$. We can now estimate the size of the pressure perturbations needed to meet this requirement:

$$\left(\frac{\delta p}{p}\right)_{\text{Max}} \geqslant \left(\frac{\omega R}{V_A}\right)\left(\frac{4\pi\,\Delta^2\omega}{c^2\eta}\right)\left(\frac{r_i}{2L_p(1-s)f^{\text{BS}}}\right). \tag{25}$$

We first consider the COMPASS example given in Thyagaraja *et al.*[12] We take $\omega \leqslant 3\times10^4\ \text{sec}^{-1}$ ($\simeq 5$ kHz), $V_A/R = 10^7\ \text{sec}^{-1}$, $\Delta = 2$ cm, $r_i/L_p = 0.3$, $s = 0.1$, $f^{\text{BS}} = 0.5$, $n_e \simeq 2\times10^{13}/\text{cm}^3$, $T_e \simeq 1$ keV. We find that $\epsilon = \delta p/p \geqslant 2\%$ for stabilization to occur.

It is plainly the case that as the temperature gets higher, η decreases fairly sharply. It would, therefore, be interesting to consider the results for a model in which one makes use of turbulence-enhanced resistivity and pressure-driven currents. The theories of Connor and Taylor[6] and Haas and Thyagaraja[7,8] offer the interesting possibility of producing jitter of the required characteristics, even though the neoclassical resistivity is too small to obtain a useful result. The first point to note is that using these theories amounts to merely altering the value to be used for η in the preceding formulas together with a concomitant change of f^{BS} to f^{p}, which measures the ratio of the pressure-driven current from both neoclassical and turbulent origin to the total current. For example, in the Haas Thyagaraja model, $\eta=\eta_{\parallel}^{\mathrm{Neo}}+\eta_{\theta}(r_i/R)^2$, where η_θ could, in general, be very much larger (due to $\mathbf{E}\times\mathbf{B}$ turbulent friction forces in the poloidal directions), irrespective of toroidal and collisional effects, than the neoclassical term. Although this means that γ_c, the linear tearing growth rate will also be higher (jitter stabilization requires $\omega\gg\gamma_c$), this increases as $\eta^{1/3}$, whereas the parameter $c^2\eta/4\pi\,\Delta^2\omega$ will, in general, increase like $\eta^{2/3}$ while maintaining consistency with stabilization theory.

As a tentative application of these ideas we consider parameters typical of JET. We take $T_{e,i}\simeq 5$ keV, $n\simeq 10^{14}$ cm^{-3}, $B_z=3$ T, $r_i/L_p=0.3$, $f^p=0.8$, $R=300$ cm, $Z_{\mathrm{eff}}\simeq 2,\omega=10^4$ sec^{-1}, and $\Delta=3$ cm. These parameters lead to $V_{\mathrm{A}}\simeq 10^9$ cm sec^{-1}, $\eta\simeq 2\times 10^{-18}$ cgs, $c^2\eta/4\pi\,\Delta^2\omega=2\times 10^{-3}$, and $\epsilon\geqslant 25\%$. Clearly, this is a rather large perturbation (of the same order as a typical sawtooth crash). If the "anomalous" resistivity is five times larger than the neoclassical value, γ_c and ω will be essentially unchanged, whereas ϵ will be brought down to nearly 5%, a much more practical proposition.

The frequency ω in the plasma frame may be achieved in various ways, for example, by switching a source on and off at the required frequency (in the lab frame) or similar amplitude modulation through plasma rotation. The power requirements can be estimated as follows. We suppose that the total power input to the equilibrium plasma is P_{tot} and the associated energy confinement time is τ_E; taking a to be the minor radius and P_{jitter} to be the total power required to jitter the plasma pressure by the relative amplitude $\delta p/p$ at frequency ω and radial localization length Δ, we have

$$\frac{P_{\mathrm{jitter}}}{P_{\mathrm{tot}}}\simeq(\omega\tau_E)\left(\frac{\delta p}{p}\right)\left(\frac{r_i\Delta}{a^2}\right). \tag{26}$$

Using the preceding theory, we obtain the requirement

$$\frac{P_{\mathrm{jitter}}}{P_{\mathrm{tot}}}\geqslant(\omega\tau_E)\left(\frac{r_i\Delta}{a^2}\right)\left(\frac{\omega R}{V_{\mathrm{A}}}\right)\left(\frac{4\pi\,\Delta^2\omega}{c^2\eta}\right)\left(\frac{r_i}{2L_p(1-s)f^{\mathrm{BS}}}\right). \tag{27}$$

For COMPASS-type numbers, taking $\tau_E\simeq 30$ msec, we find P_{jitter} to be between 10%–20% of the total power. In larger machines such as JET, although the resistivity is smaller (if entirely collisional) and the confinement time larger, there are compensating effects due to larger size and lower tearing growth rates (implying lower effective values for ω). The transient power needed is somewhat larger, of the order of 40%, for $\tau_E\simeq 1$ sec. As noted previously, in larger machines "jittered bootstrap" at the required frequencies does not seem practicable unless the effective local resistivity is larger than neoclassical, due to turbulence effects (or, possibly Landau-type collisionless kinetic effects). Note, however, that this power is really applied in "bursts" at relatively high frequency in a radially localized region and does not significantly add to the plasma β (i.e., $\delta p/p\ll 1$); thus the *energy* added to the plasma by the ac power source is always a small fraction of the steady-state, global energy input. The question of whether this ac bootstrap is more or less efficient than an ac form of ECRH current drive is not easily answered, since the latter, if localized and at the required frequency, necessarily also generates its own "bootstrap" effect, in addition to the fluctuating current driven directly. To some extent the effects are mixed together. Thus the calculation of the relative efficiency of bootstrap versus direct ac drive is complicated by the specifics of the power source and absorption efficiency and lies outside the scope of the present paper.

The point about the present scheme is that it relies solely on heating the plasma rather than attempting to produce $\delta j_{\parallel}^{c.d.}$ directly. As such, it is not affected by the efficiency of alternating current drive. It also suggests a possible mechanism for the turbulence in the plasma itself to produce a fluctuating bootstrap current that could modify the stability properties of gross modes. In this scenario, gross tokamak modes could be stabilized or "healed" by the plasma itself due to the "intrinsic" jitter. Further experimental and theoretical work is obviously required to study these questions in detail.

An interesting feature of the "anomalous" pressure-driven current models is that although the *effective* resistivity is increased, this does *not* mean that for a given total current the loop voltage or $E_{\mathrm{ext},\parallel}$ is necessarily increased. This is because we may write $E_{\mathrm{ext},\parallel}=\eta j^{\mathrm{Tot}}(1-f^p)$. This means that even if η were to be considerably enhanced, if f^p is also enhanced, the measured resistance will appear proportional to $\eta(1-f^p)$ and could be consistent with $\eta^{\mathrm{neoclassical}}$, or even η^{Spitzer}! Recent measurements[2,3] would appear to indicate that experiments actually operate in a neoclassical regime as far as resistivity and bootstrap are concerned. However, in view of the possibility of cancellation effects we have noted, equilibrium measurements could also be consistent with anomalous effects. Thus, in order to unambiguously verify neoclassical bootstrap and resistivity, it is necessary to carry out perturbative experiments, such as the one suggested here by the "jitter" theory. Such experiments may throw new light on turbulence-enhanced resistivity and associated pressure-driven currents. This is exactly analogous to the well-known situation in the case of the particle flux, where the outward diffusive and the inward pinch can "conspire" to produce an apparently small *effective* diffusivity; the "true" D,U are only determinable by local perturbative meaasurements, supplementing the equilibrium particle balance. When they are determined in this way, they usually turn out to be far larger than neoclassical values and seem to have a turbulent origin.

V. CONCLUSIONS

In this paper we have considered some novel effects associated with pressure-driven currents. Such currents arise at sufficiently high poloidal beta in tokamaks, either due to neoclassical effects or to turbulence effects. The first application deals with a relatively steady situation, where the pressure-driven current within an island is used to explain various "snake" phenomena noted in JET in terms of an extended version of a previously developed nonlinear $m=1$ theory.

We also propose an ac version of bootstrap currents driven by pressure fluctuations created by a suitable rf or other source to implement a method of dynamic stabilization of the $m=1$ linear tearing mode (in the first instance). Estimates suggest that the idea may be feasible on existing machines.

In particle, energy, and impurity transport, equilibrium transport analysis and perturbative techniques are complementary, and *together* provide fundamental information about both "diagonal" and "off diagonal" transport matrix elements. It is reasonable to suppose that investigations of both dc and ac bootstrap current generation and diffusion would similarly provide a more detailed understanding of the combined effects of neoclassical theory and plasma turbulence.

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