Heliumlike Mg XI in the divertor-injected tokamak experiment

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(Received 4 March 1991)

Electron-impact excitation rates for transitions in heliumlike Mg XI, calculated with the R-matrix code, are used to derive the electron-density-sensitive emission line ratio R $(=f/i)$ and temperature-sensitive ratio G $[=(f+i)/r]$, where f is the forbidden $1s^2\,{}^1S-1s2s\,{}^3S$ transition, i the intercombination $1s^2\,{}^1S-1s2p\,{}^3P_{1,2}$ lines, and r the resonance $1s^2\,{}^1S-1s2p\,{}^1P$ transition. A comparison of these with R and G ratios determined from x-ray spectra of the divertor-injected tokamak experiment reveals excellent agreement between theory and observation, with discrepancies of typically 3% and 9% in R and G, respectively. These discrepancies correspond to variations in N_e and T_e of approximately 0.1 and 0.15 dex, respectively, and hence it should be possible to use the theoretical results to derive plasma parameters to this level of accuracy for remote sources for which no independent electron temperature and density estimates exist, such as solar flares.

I. INTRODUCTION

The three principal lines of heliumlike Mg XI, namely, the forbidden (f) $1s^2\,{}^1S-1s2s\,{}^3S$ transition at 9.314 Å, the intercombination (i) $1s^2\,{}^1S-1s2p\,{}^3P_{1,2}$ transitions at 9.232 Å, and the resonance (r) $1s^2\,{}^1S-1s2p\,{}^1P$ transition at 9.169 Å, are frequently observed in the x-ray spectra of laboratory and astrophysical plasmas [1,2]. They may be used to infer the electron density and temperature of the Mg XI emitting region of the plasma through the line ratios R $(=f/i)$ and $G[=(f+i)/r]$, respectively [3–5], and they have often been applied to the analysis of solar flares and active regions [6,7]. However, the theoretical determination of R and G is critically dependent on the atomic data adopted in the calculations, especially for the electron collisonal excitation rates between the ground state and $n=2$ levels [8].

Over the past few years we have been involved in an extensive series of calculations of line ratios for heliumlike ions, and the application of these to solar observations (see Keenan, McCann, and Phillips [9] and references therein). These results are based on electron excitation rates derived with the R-matrix code of Burke and Robb [10], which are probably the most accurate currently available [11].

Recently Keenan *et al.* [12] compared R and G calculations for Si XIII with x-ray spectra from the divertor-injected tokamak experiment (DITE) tokamak, for which the electron density and temperature had been independently determined. Good agreement was found between theory and experiment, which provided observational support for the accuracy of the atomic data adopted in the line ratio calculations. In addition, the results implied that the theoretical R and G ratios could be applied with confidence to the analysis of remote sources for which no independent electron density and temperature estimates exist, such as solar flares.

In this paper we perform a similar analysis for the R and G ratios in Mg XI. Theoretical line strengths, calculated using R-matrix electron excitation rates, are compared with observations from the DITE, so that the suitability of the former as diagnostics for remote sources may be investigated.

II. ATOMIC DATA

The model ion for Mg XI consisted of the 23 $1snl$ states with $n<6$ and $l<3$, making a total of 37 levels when the fine-structure splitting in the triplet terms was included. Energies of all the ionic levels were obtained from Martin and Zalubas [13].

Electron-impact excitation rates among the $1s^2$, $1s2l$, and $1s3l$ states, calculated with the R-matrix code [10], were taken from Tayal [14] and Tayal and Kingston [15,16] for allowed and forbidden, and semi-forbidden (i.e., spin-changing) transitions, respectively. For transitions to higher $1snl$ levels, the above excitation rates were used in conjunction with the n^{-3} scaling law of Gabriel and Heddle [17].

Einstein A coefficients for radiative decays from $1snl$ levels were obtained from Lin, Johnston, and Dalgarno [18] for $n=2$, and Lin, Johnston, and Dalgarno [19] and

Cohen and McEachran [20] for $n > 2$. However, for the intercombination lines $1s^2\,{}^1S - 1s2p\,{}^3P_1$ and $1s^2\,{}^1S - 1s3p\,{}^3P_1$, the results of Laughlin [21] were preferred. The dielectronic and radiative recombination coefficients of Mewe and Schrijver [22] were used in conjunction with the Mg XII–Mg XI ionization balance calculations of Arnaud and Rothenflug [23] to include the effects of these atomic processes on the Mg XI level populations.

Using the above atomic data and the statistical equilibrium code of Dufton [24], relative emission line strengths were calculated for a range of electron densities and temperatures. The procedure was similar to that used by Dufton *et al.* [25], where details of the approximations involved may be found.

III. EXPERIMENTAL DATA

The experimental results were obtained from the DITE tokamak at the UKAEA Culham Laboratory [26]. This tokamak has major and minor radii of 1.2 and 0.24 m, respectively, a maximum toroidal field of 2.7 T, and maximum toroidal current of 300 kA. The present experimental setup was similar to that adopted in the case of Si XIII by Keenan *et al.* [12], where further details of the procedures involved may be found. Briefly, the central electron density was varied between approximately 1 and 6×10^{13} cm^{-3}, while the central electron temperature was typically 700 eV, these plasma parameters being measured with a microwave interferometer [26] and Thomson scattering of a 2-*J* ruby laser [27], respectively. The estimated uncertainties in these measurements is approximately $\pm10\%$ in both N_e and T_e.

All of the spectra were obtained with a Bragg rotor spectrometer [28], examination of the radial Mg XI emission profile showing that almost all of the emission was from the central region of the plasma, where N_e is almost constant and T_e does not vary significantly, as was previously found for Si XIII (see Fig. 1 of Keenan *et al.* [12]). This localization of the emission allows the R and G ratios to be measured from the central-chord integrated signal alone, without the need for radial profiles at every plasma density. A comparison of the relative intensities of the Mg XII Ly-α and Mg XI resonance lines showed that there were approximate coronal equilibrium in the central region of the plasma.

In deriving the R and G ratios, account was taken of the change in spectrometer sensitivity through the spectrum. An increase in sensitivity of about 5% is calculated between the resonance and forbidden lines, mainly due to the change in the Bragg reflection integral of the ammonium dihydrogen phosphate crystal [29]. The contribution of unresolved dielectric satellites to the f, i, and r lines was calculated to be negligible [30] at the electron temperature under consideration, and hence the observations were not corrected for the presence of these.

In Fig. 1 we show our Mg XI spectra at two electron densities spanning the N_e coverage of our observations, to illustrate the high quality of the observational data, as well as the variation of the R ratio with density.

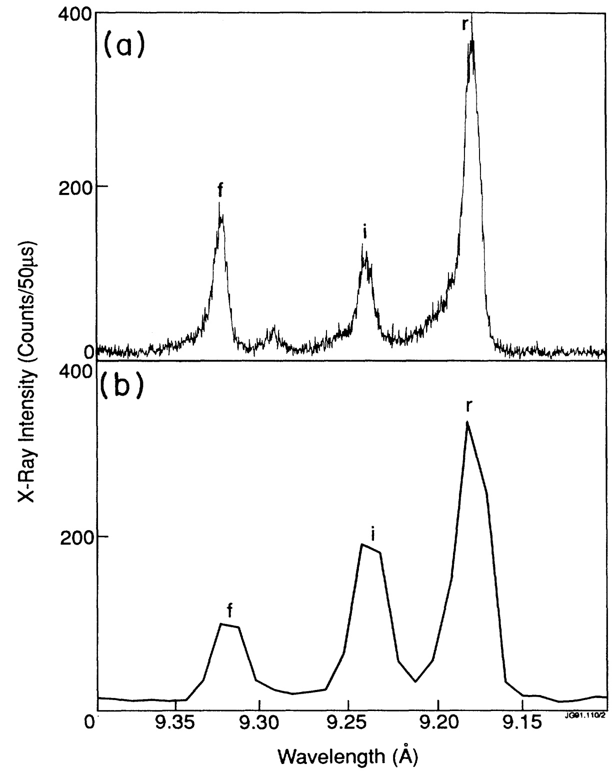

FIG. 1. (a) and (b) Spectra of Mg XI from the DITE tokamak, showing the forbidden (f) $1s^2\,{}^1S-1s2s\,{}^3S$ transition at 9.314 Å, the intercombination (i) $1s^2\,{}^1S-1s2p\,{}^3P_{1,2}$ transitions at 9.232 Å, and the resonance (r) $1s^2\,{}^1S-1s2p\,{}^1P$ transition at 9.169 Å. (a) is for discharge no. 33849 with $N_e=1.1\times10^{13}$ cm^{-3} and crystal scan speed 1200° sec^{-1}, while (b) is for discharge no. 28289 with $N_e=5.5\times10^{13}$ cm^{-3} and crystal scan speed 40° sec^{-1}. The variation of $R=f/i$ with N_e may be clearly seen by comparing (a) and (b).

IV. RESULTS AND DISCUSSION

The theoretical R ratio is plotted as a function of electron density at electron temperatures of $T_e=2\times10^6$, 4×10^6, and 1×10^7 K in Fig. 2, while in Fig. 3 the sensitivity of the ratio to recombination processes is illustrated at the temperature of maximum Mg XI emissivity, $T_m=7\times10^6$ K [31]. An inspection of the figures shows that R does not vary strongly with T_e, with a factor of 5 change in the latter corresponding to only about a 30% variation in the former. However, over the electron density range typical of tokamak plasmas ($N_e\simeq10^{13}-10^{14}$ cm^{-3}), the ratio varies by approximately a factor of 6. This is significantly larger than the factor (2.1) calculated by Keenan *et al.* [12] for R in Si XIII over the same density interval, so that tokamak observations of Mg XI should allow a much better experimental verification of the density sensitivity of this ratio predicted by theory.

In Fig. 4 the theoretical G ratio is plotted as a function of electron temperature at an electron density of $N_e=10^{13}$ cm^{-3}, although we note that changing the

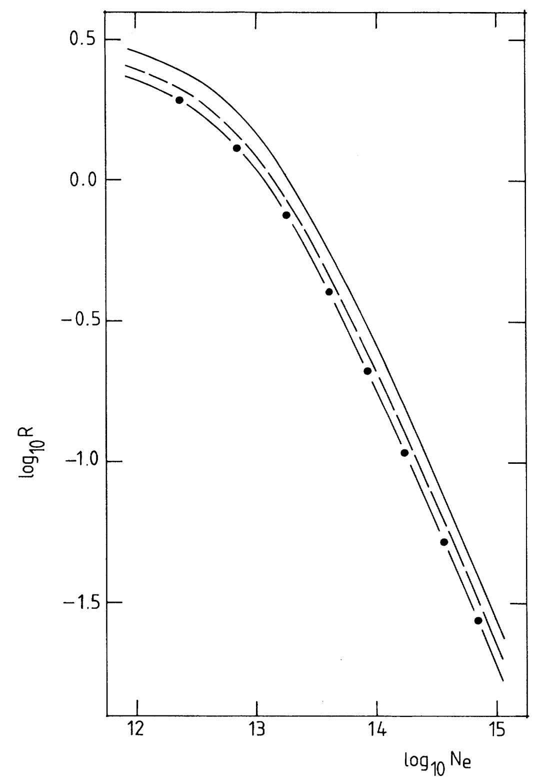

FIG. 2. The theoretical emission line ratio R (line intensities in photons) plotted as a function of electron density at electron temperatures of $T_e=2\times10^6$ K (dash-dotted line), 4×10^6 K (dashed line), and 1×10^7 K (solid line), with dielectronic and radiative recombination included in the calculations.

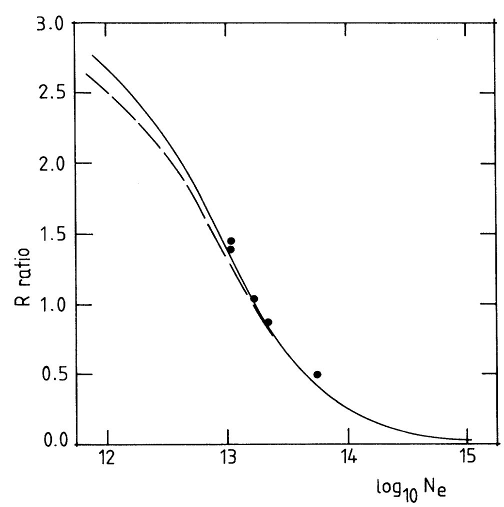

FIG. 3. The theoretical emission line ratio R (line intensities in photons) plotted as a function of electron density at the temperature of maximum Mg XI emissivity, $T_m=7\times10^6$ K [31], with dielectronic and radiative recombination either included in (solid line) or excluded from (dashed line) the calculations. Typical errors in the experimental data (solid points) are $\pm5\%$ in R_{obs} and ±0.04 in $\log_{10}N_e$.

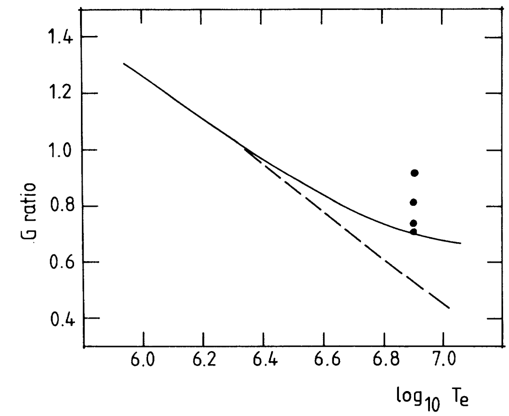

FIG. 4. The theoretical emission line ratio G (line intensities in photons) plotted as a function of electron temperature T_e at an electron density of $\log_{10}N_e=13.0$, with dielectronic and radiative recombination either included in (solid line) or excluded from (dashed line) the calculations. Typical errors in the experimental data (solid points) are $\pm9\%$ in G_{obs} and ±0.06 in $\log_{10}T_e$.

adopted N_e by an order of magnitude results in less than a 1% variation in G. As may be seen from Figs. 3 and 4, the effects of recombination on G are much larger than on R, increasing the former by 26% at $T_e=T_m$, but the latter by less than 7%. Unfortunately, recombination leads to the temperature sensitivity of G being greatly reduced at high values of T_e. For example, between $\log_{10}T_e=6.6$ and 7.0 G varies by a factor of 1.8 if recombination is excluded from the line ratio calculations, but inclusion of this process reduces this factor to 1.2.

In Table I we list the observed and theoretical R and G ratios. Unfortunately, G could not be measured in discharge no. 33904 as the resonance line in this spectrum was saturated. The experimental values of G are plotted on the theoretical G versus $\log_{10}T_e$ curve in Fig. 4, while the R observations are shown on the theoretical R versus $\log_{10}N_e$ curve in Fig. 3, which is for $\log_{10}T_e=6.85$. We may justify comparing the theoretical and observed R ratios on one graph by noting that the electron temperatures of *all* the DITE discharges are similar ($\log_{10}T_e\simeq6.91$) and that R is only weakly dependent on T_e (Fig. 2). An inspection of Table I and Figs. 3 and 4 shows that the agreement between theory and observation is generally good, with the typical discrepancy being 3% in R and 9% in G, which provides experimental support for the reliability of the R-matrix electron-impact excitation rates adopted in the present paper.

The good agreement found between theory and experiment for the Mg XI line ratios implies that the theoretical results may be applied with confidence to the analysis of remote sources for which no independent estimates of N_e and T_e exist, such as solar flares. Under flare conditions, when the electron density of the Mg XI emitting region is $N_e\simeq10^{12}$ cm^{-3} and the temperature $T_e\simeq T_m$ [6], the 3% and 9% discrepancies in R and G correspond to variations in N_e and T_e of approximately 0.1 and 0.15 dex, respectively. Hence in principle it should be possible to derive flare parameters to this level of accuracy using the

TABLE I. Comparison of observed and theoretical Mg XI R and G ratios. As the electron temperatures of all the discharges were similar ($T_e = 700 \pm 100$ eV), only one theoretical G ratio (for $T_e = 700$ eV) is quoted.

$N_e/10^{13}$ cm^{-3}	Discharge no.	R_{obs}	R_{theory}	G_{obs}	G_{theory}
5.5	28289	0.51	0.46	0.92	0.71
1.1	33849	1.45	1.40	0.73	0.71
1.1	33850	1.40	1.40	0.71	0.71
2.2	33904	0.87	0.87		0.71
1.7	33914	1.04	1.07	0.81	0.71

results in Fig. 2 and 4.

Finally, we note that the present line ratio calculations may also be useful in deriving highly accurate electron densities in those tokamaks whose N_e has not been independently determined, as at tokamak densities a 3% change in R corresponds to only a 5% variation in N_e. However, we should point out that the results are applicable only when the atomic processes in the plasma are appropriate to the core region where ion diffusive time scales are long compared to the time to reach ionization equilibrium.

ACKNOWLEDGMENTS

We would like to thank Dr. R. W. P. McWhirter for his continued interest in the work. S. M. M. is grateful to the SERC for financial support. The authors are indebted to the Culham Laboratory for financial assistance (in the cases of R. B. and J. D.), and for the enthusiastic support from the DITE tokamak group during the experimental program. We are also grateful to I. Coffey for his help in preparing the figures. This work was supported by the Nuffield Foundation.

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