The stability of ballooning modes in tokamaks with internal transport barriers

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Modern tokamaks can produce transport barriers (TBs)—localized regions with an increased energy confinement. Previous studies have been unable to examine the stability of internal TBs to radially extended short-wavelength magnetohydrodynamic instabilities ("ballooning modes"), for the usual case with a sheared plasma flow and a magnetic shear that passes through zero near the TB. An established technique is adapted to study this situation, finding instability if (1) there is a low-pressure gradient, and if (2) the nearest "resonant surface" at which a Fourier mode is resonant, is sufficiently close. Surprisingly, flow shear is no more stabilizing than for magnetic shears of order one. This is explained. Without a strongly stabilizing mechanism, ballooning modes will fundamentally limit a TB's radial extent, preventing them from extending across the entire plasma radius. [DOI: 10.1063/1.2032742]

INTRODUCTION

Tokamaks[1] used a strong axisymmetric externally applied magnetic field and an axisymmetric driven current to confine a toroidal plasma at temperatures sufficient for the fusion of deuterium and tritium nuclei. The magnetic fields in tokamaks form nested toroidal surfaces upon which the plasma is confined, with the hottest tori contained within cooler tori as one moves radially outwards from the core of the plasma. In general, heat and particles diffuse from one toroidal flux surface to another by turbulent processes. Modern tokamaks, such as the Joint European Torus (JET),[1] achieve high energy-confinement by the production of transport barriers—radially localized regions with a high-energy confinement.[2–4] Transport barriers were first found to form at the plasma's edge, leading to a high-performance *H*-mode confinement, but in recent years it has become possible to improve confinement further by forming an "internal transport barrier" (ITB), deeper inside the plasma. ITBs can appear when the shear in the magnetic field becomes small or zero, are often associated with a strongly sheared plasma flow, and lead to the formation of a high-pressure-gradient region.

High-pressure gradients can drive ballooning modes unstable, hence the need to consider the stability of ballooning modes at an ITB. In addition, ballooning mode studies have much in common with the studies of toroidal electron drift waves and ion temperature gradient modes, so an understanding of the former can lead to a greater understanding of the latter. The following firstly considers the stability of ballooning modes near a minimum in the q profile, where q is the tokamak's "safety factor,"[1] then examines the consequences for the existence and stability of ITBs.

Ballooning modes are well understood,[1,5] as is the stabilizing effect of a sheared plasma flow[6–10] in an otherwise radially uniform plasma. Previous authors often studied the effect of flow shear by an initial value approach,[6,7,10,11] and have attempted to discuss their solutions in terms of the zero flow-shear solutions. This approach has been found to be analytically difficult, partly because a continuum of stable eigenvalues can exist for zero flow shear (for certain values of the parameter k that corresponds to a radial wave number in the ballooning theory[5,11]). Perhaps more importantly, it has been found[12,13] that the most unstable solutions need not be those that correspond to the most unstable zero flow-shear solutions. Instead it has been found that flow shear can cause a previously stable continuum of eigenvalues to generate a discrete set of eigenvalues, and that as the flow shear is increased the largest of these can become larger than their zero flow-shear counterparts. Further evidence for this is found in Ref. 9, and described further below, and the consequences for the calculations of the stability boundaries by Miller *et al.*[7] using an initial value approach, are a subject of our current investigation.

Here we adopt an eigenmode approach, with the growth rate determined as an eigenvalue. The eigenvalue growth rates are identical to the Floquet-mode growth rates that would be obtained from Floquet-mode solutions to the time-dependent initial-value problem (see the Appendix for details). Miller *et al.*[7] confirmed that the Floquet growth rates agreed with those from their initial value calculations, and we also have previously benchmarked our eigenvalue results against Miller.[8,9] In addition Fig. 10 of Ref. 9 appears to display exactly the phenomena described in Bondeson, where increasing the flow shear was found to firstly stabilize the zero flow-shear solution, then to destabilize an apparently different mode at higher flow shears. Hence we are confident that this approach is reliable; it is certainly sufficient to establish the existence of unstable solutions, the only thing that is required for the flow-shear relevant conclusions presented in this paper.

Previous work has not considered how ballooning mode stability may be affected when there is: a radially dependent

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equilibrium, a small (or zero) magnetic shear, and a sheared toroidal plasma flow. Earlier theoretical work with a small magnetic shear,[14,15] was strictly valid in the limit with the toroidal mode number $n \to \infty$. This corresponds to a mode width in real space that tends to zero, and takes the magnetic shear and pressure gradient to be constant, independent of the radius. Such studies discount the possibility of a mode being unstable at a nearby radial position, and extending into the otherwise stable region. In addition previous work did not allow the effect of flow shear to be studied in a rigorous way, as was done for moderate values of magnetic shear in Refs. 7–10, in which it was suggested[8,9] that flow shear will be increasingly important at the small values of magnetic shear found near a minimum in q (where the magnetic shear is zero). Hence the following work considers ballooning mode stability in a region with a minimum in the q profile, an axisymmetric toroidal plasma flow that is sheared with respect to the minor radius,[1] and that allows an enhanced pressure gradient. For simplicity we consider the usual s-α model with circular flux surfaces, neglect the flow's centrifugal effects, and take a low-β ordering (so that compressibility can be neglected). This model is described in Ref. 9. For the above model with a monotonic q profile, a linear stability analysis in the limit of the toroidal mode number $n \to \infty$ leads at lowest order in $1/n$ to a single equation,[8,9]

$$s^2\frac{d}{dx}\left[[(x-M)^2+\Gamma^2]\frac{du_m}{dx}\right]-[(x-M)^2+\Gamma^2]u_m+\alpha\left\{-\frac{s}{2}\frac{d}{dx}\{[(x-M)^2+\Gamma^2](u_{m+1}-u_{m-1})\}-\frac{s}{2}[(x-M)^2+\Gamma^2+1]\frac{d}{dx}[u_{m+1}-u_{m-1}]-s(x-M)\frac{d}{dx}[u_{m+1}+u_{m-1}]+\frac{1}{2}[u_{m+1}+u_{m-1}]\right\}-\frac{\alpha^2}{2}\left\{[(x-M)^2+1+\Gamma^2]\left[u_m-\frac{1}{2}[u_{m+2}+u_{m-2}]\right]-(x-M)\times[u_{m+2}-u_{m-2}]\right\}=0, \tag{1}$$

for which the growth rate γ is obtained as an eigenvalue (where the plasma displacement $\vec{\xi} \sim e^{\gamma t}$), and the full eigenmode depends solely upon $\nabla\psi \cdot \vec{\xi}/RB_p = \Sigma_m u_m(x) e^{im\theta}$, with θ the usual poloidal straight field-line coordinate. In the Appendix it is demonstrated that the growth rate γ is identical to the Floquet-mode "growth rate" obtained using a ballooning expansion and a time-dependent eikonal. The other terms in the equation are the pressure gradient parameter $\alpha = -(2r^2/RB_p^2)(dP/dr)$, the magnetic shear $s=(r/q)(dq/dr)$, and $\Gamma^2(x) = [(\gamma + ixd\Omega/dq)^2 R^2 q^2/c_s^2](p/B^2)$, which is the only term through which the growth rate γ and the flow's angular velocity enter the equations. Here the radial coordinate $x = n(q_m - q)$ is used, with q_m an arbitrary reference value of q. If the magnetic shear is constant, this coordinate has equally spaced rational surfaces that occur at integer values of x.

To study a minimum in the q profile, a weak radial dependence of the equilibrium parameters s and α must be reintroduced. Although not rigorous, previous calculations[8,9,16] support the belief that the main role of reintroducing weak equilibrium variations is to break the equivalence between rational surfaces, so as to produce a mode that remains of finite width as $d\Omega/dr \to 0$, and hence may be calculated numerically.

At a minimum in q, two problems become apparent. Firstly, the coordinate $x = n(q_m - q)$ will become double valued in either side of the minimum at $q = q_m$. In addition, as $s \to 0$ the second-order radial derivatives must tend to infinity if the mode is to have finite width (a finite mode width envelope requires nonzero second-order derivatives or higher). The problem arises through the use of the rapid radial coordinate x, and could have been avoided by using the minor radius as a coordinate, leading to terms d/dx becoming $nq_m d/dr$. Here instead the coordinate $X = n(q_m - q)/s(q)$ is used, removing the prefactors of the magnetic shear s from the second-order derivatives, while also being a single-valued coordinate. Such a coordinate emerged naturally from an analytic calculation of ballooning-mode stability at low magnetic shear,[14] motivating its use in our study. In this coordinate system we obtain

$$\frac{d}{dX}\left[[s^2(X-\bar{M})^2+\Gamma^2]\frac{du_m}{dX}\right]-[s^2(X-\bar{M})^2+\Gamma^2]u_m+\alpha\left\{-\frac{1}{2}\frac{d}{dX}\{[s^2(x-\bar{M})^2+\Gamma^2](u_{m+1}-u_{m-1})\}-\frac{1}{2}[s^2(X-\bar{M})^2+\Gamma^2+1]\frac{d}{dX}[u_{m+1}-u_{m-1}]-s(X-\bar{M})\frac{d}{dX}[u_{m+1}+u_{m-1}]+\frac{1}{2}[u_{m+1}+u_{m-1}]\right\}-\frac{\alpha^2}{2}\left\{[s^2(X-\bar{M})^2+1+\Gamma^2]\left[u_m-\frac{1}{2}[u_{m+2}+u_{m-2}]\right]-s(X-\bar{M})[u_{m+2}-u_{m-2}]\right\}=0, \tag{2}$$

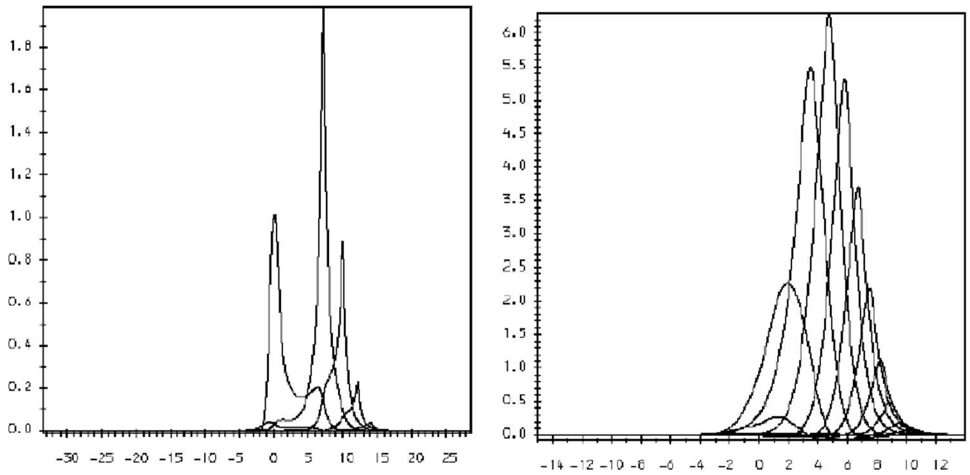

FIG. 1. Mode structures for $\Delta X=6.8$ (left) and 3.4 (right). The minimum in q is at $X=0$, negative shear is for $X<0$, and each line is the amplitude of a Fourier mode. For $\Delta X=6.8$ the mode is a maximum at the first rational surface, for which $s\simeq 0.056$, whereas for $\Delta X=3.4$ the maximum is near $X\simeq 5$ for which $s\simeq 0.43$. Hence the mode structure for $\Delta X=3.4$ is much closer to conventional ballooning mode structures (with $s\sim 1$), e.g., see Ref. 8.

where $\bar{M}=(m-nq_m)/s(q)$, and which we solve numerically in the usual manner.[9] When Taylor expanding $\Gamma(X)$, we avoid the factor of $1/s(q)$ that would arise by simply substituting $x(q)=Xs(q)$, by noting that $(r-r_m)/r_m\sim X/nq_m$, and Taylor expanding in the real-space coordinate. The reason this is possible is explained further below, and it leads to $\Gamma(X)=\gamma+i(d\Omega/dr)X+O(1/n)$, where $d\Omega/dr$ is written in place of $(2r_m/q_m)(d\Omega/dr)$. Note that in the $X=n(q_m-q)/s(q)$ coordinate, the rational surfaces are no longer at evenly spaced integer values.

Analytic calculations at low values of magnetic shear s have shown that ballooning modes are likely to be unstable *only* if the pressure gradient parameter α is also small. Such calculations take $s\sim\alpha^2$ and calculate the growth rate to be[17] $\gamma^2\simeq -s^2/2+3s\alpha^2/4$, a result that also predicts stability for negative magnetic shear. Writing $s=a\alpha^2$ and maximizing γ^2 with respect to a then gives the most unstable $s(\alpha)$ as $s\simeq(3/4)\alpha^2$. This guides our choice $\alpha(s)=\sqrt{4/3}\times\sqrt{|s|}\exp(-X^2/n^2q_m^2W^2)$, which we expect to be close to the most unstable equilibrium profile for small s and X, with the Gaussian envelope of α limiting the mode's radial extent. W is a measure of the Gaussian's width in real space, taken here as approximately 3% of the minor radius ($W=0.013$). The profile is chosen to produce a robust instability, allowing us to explore stability properties for a range of parameter values, and, in particular, to assess the effect of flow shear. Future work will explore the stability of arbitrary aspect ratio configurations with experimental profiles, using a modified version of ELITE.[16] The resulting equations are solved numerically, as described in Ref. 9.

Throughout we have taken a quadratic q profile,

$$q=q_m+\left.\frac{1}{2}\frac{d^2q}{dr^2}\right|_{r_m}(r-r_m)^2, \tag{3}$$

so our free parameters are a normalized q'' that replaces $(r_m^2/2q_m)(d^2q/dr^2)$, the product nq_m, and the α profile's width W. For distances $r-r_m$ that are much smaller than the minor radius, $X\simeq nq_m(q_m-q)/r_m(dq/dr)\simeq -(nq_m/2)((r-r_m)/r_m)$, and $s(X)\simeq(r_m/q_m)(dq/dr)\simeq(-X)/\Delta X^2$. To a good approximation, when $n\gg 1$ the quantities nq_m and q'' only appear in the combination nq_m/q'', which is given in terms of $\Delta X\simeq\frac{1}{2}\sqrt{nq_m/q''}$, the distance from q_m to the closest radial position at which a Fourier mode is resonant [with $n(q_m-q)=1$].[18] Note that in all of the following results, nq_m is an integer.

RESULTS

We find that ballooning instabilities can extend across the minimum in the q profile (in addition to being unstable near the minimum in q, as predicted in Ref. 14). Plots of the mode structures for a zero flow shear are given in Fig. 1. The plots are of the Fourier modes' amplitudes versus radial coordinate, for different values of n that correspond to a distance ΔX to the first rational surface of 3.8 and 6.8, respectively. The modes are clearly positioned in the regions with higher magnetic shear, but extend all the way to q_m where the perturbation is applied, and a little way into the negative magnetic shear region by a distance of order a rational surface.

Figure 2 shows the dependence of the growth rate on the distance to the first rational surface. As ΔX becomes large, the fourier modes can no longer extend from q_m to the more unstable values of s and α that are driving the instability, stabilizing the mode at q_m. So in addition to a carefully cho-

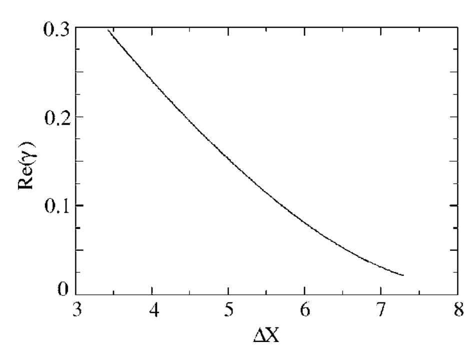

FIG. 2. The growth rate γ is plotted against the distance to the first rational surface ΔX, by holding nq_m constant and decreasing q''. Note that γ is real for zero flow shear.

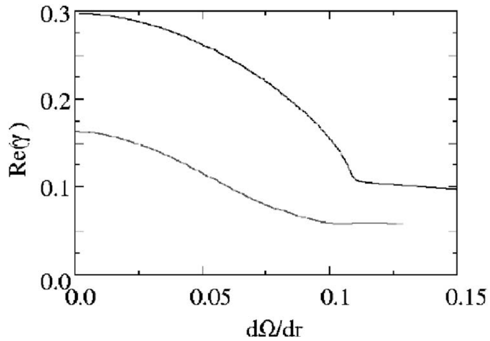

FIG. 3. The real part of the growth rate γ of the two most unstable solution branches, is plotted against the flow shear $d\Omega/dr$. Here $\Delta X=3.4$.

sen pressure profile, instability at q_m requires the first rational surface to be sufficiently close to the minimum in q.

TOROIDAL FLOW SHEAR

Previous work found ballooning modes to be stabilized by a flow shear $d\Omega/dq\sim 1$, so because $d\Omega/dq\sim(d\Omega/dr)\times(1/s)$, as $s\rightarrow 0$ we might expect the effects of flow shear to be especially strong. Moreover, at the first rational surface $n(q-q_m)=1$, so if there is a linear q profile with $q=q_m+(dq/dr)dr$, then the distance to the first rational surface is $dr\sim 1/nq'$, whereas a quadratic q profile $q=q_m+(d^2q/dr^2)\times(dr^2/2)$ gives a larger $dr\sim 1/\sqrt{nq''}$. Therefore, as the inertial terms $\vec{v}_\phi\cdot\vec{\nabla}\vec{\xi}\sim n\Omega(r)\sim n(d\Omega/dr)$, the change in flow speed as we go to the first rational surface is of order $(d\Omega/dr)(1/q')$, for a linear q profile, but of order $d\Omega/dr\sqrt{n/q''}$ for a quadratic q profile. Thus at high n we would expect ballooning mode stability to be very sensitive to small flow shears. However, this is not found to be the case (see Fig. 3), because $\sqrt{n/q''}\sim\Delta X$, and unless ΔX is small then the ballooning modes are stable (without any flow shear).[19]

DISCUSSION

Before concluding it is worth considering the consequences of these results. Firstly, we note that for small or negative magnetic shear, the TB's high-pressure gradient region is likely to be in the second-stable region (see Fig. 4), and hence is not limited by the stability of ballooning modes. However, consideration of the path taken in the plane of s and α, moving radially from the center of the plasma to the plasma's edge, indicates that a conventional s-α equilibrium must have a radial position at which ballooning modes are unstable, where the path in s-α space cuts across the unstable region (see Fig. 4). Because we find flow shear unlikely to have a strong stabilizing affect, the existence of an ITB requires a region where ballooning modes are stabilized by some other property of the equilibrium (such as shaping of the plasma). More importantly, as emphasized in Fig. 4, even with sufficient stabilization of ballooning modes to allow a TB to exist, they (and related instabilities) will constrain the radial extent of the transport barrier.

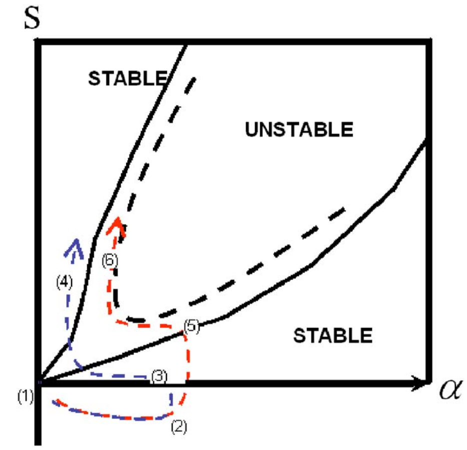

FIG. 4. For a large aspect ratio s-α equilibrium, ballooning mode stability is summarized in the above stability diagram. The dashed arrows indicate typical paths described by an equilibrium with an ITB, starting from the center of the plasma (1) and moving out past the ITB (2)–(3) and (2)–(5), to the plasma edge (4) and (5). Shaping (for example) can help stabilize ballooning modes, reducing the size of the unstable region (dashed curve), and allowing the possibility for an ITB to exist. However, the unstable region will limit the maximum radial extent of an ITB's highpressure-gradient (high α) region, e.g., (2)–(5).

CONCLUSIONS

We have considered ballooning mode stability for an s-α-type model of the equilibrium, with a circular cross-section equilibrium, a radially localized region in which the pressure gradient is taken to be balanced by the second derivative of the Shafranov shift, and a quadratic q profile (as opposed to the linear q profile considered in previous calculations[9]). The quadratic q profile results in a magnetic shear that is both positive and negative, with a zero at the minimum in q. (Note that this study has been restricted to high-n ballooning modes and has not considered the stability of low-n "infernal" modes.[20,21])

We have considered the stability of ballooning modes at the minimum in q (where the magnetic shear is zero), taking an optimally unstable equilibrium profile for illustration. We then find that a ballooning instability can extend past the minimum in q, and extend a distance of order a rational surface into the region with negative magnetic shear. However, instability requires a *small* pressure gradient with a profile that is carefully chosen to maximize the likelihood of instability, and also requires the first rational surface to be sufficiently close to the minimum in q.

A priori, flow shear would be expected to have a strong effect on the ballooning mode stability. This is because as $n\rightarrow\infty$ the separation between rational surfaces (in X space) tends to infinity, and hence the differences in flow speeds at the Fourier modes' resonant positions also tend to infinity (strongly shearing, and hence stabilizing the instability). However, if the distance from the minimum in q to the first rational surface ΔX is large, then ballooning modes become stable at q_m. Hence either ballooning modes are already stable near q_m, or the separation between rational surfaces is sufficiently small that the effect of flow shear is similar to magnetic shears of order one. Normalizing the flow speed ΩR by the sound speed c_s, we find that flow shears

$r_m[d(\Omega R/c_s)/dr] \sim \sqrt{B^2/p}$ are required to noticeably stabilize ballooning modes.[22] This and the results from Refs. 7–9 and 14 suggest that toroidal flow shear is unlikely to have a *significant* direct stabilizing influence on ballooning modes near ITBs, or indeed anywhere in the core of the plasma.

We have considered the path in the s-α plane that would be described by moving from the plasma's center to the plasma's edge (for the equilibrium considered here). A conclusion is that for ITBs to exist, then ballooning modes must be stabilized in at least one radial location, either by appropriate shaping of the plasma or some other property of the equilibrium. In addition, it indicates that in the absence of a new unrecognized stabilizing mechanism, ballooning modes (and related instabilities), place a constraint on a TB's radial extent, fundamentally preventing them from extending from the core to the plasma's edge.

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APPENDIX: TRANSFORMATION TO THE TIME-DEPENDENT EIKONAL REPRESENTATION

By providing a transformation from the Fourier-mode representation to the time-dependent eikonal representation, it is shown below that the growth rate calculated via the eigenmode approach, corresponds to the "growth rate" of the Floquet solutions to the time-dependent equation obtained using a "ballooning transformation" and the time-dependent eikonal.

We restrict our attention to the large aspect ratio model with circular flux surfaces, for which a linear stability analysis using a ballooning transformation and a time-dependent eikonal leads to[7,11]

$$\frac{\partial}{\partial\eta}(1+h^2)\frac{\partial X}{\partial\eta} + \Gamma X = \frac{1}{\gamma_A}\frac{\partial}{\partial t}(1+h^2)\frac{\partial X}{\partial t}, \tag{A1}$$

where $h=s[(\eta+\eta_0)-s_v t-\alpha\sin\eta]$, $\Gamma=\alpha(\cos\eta+h\sin\eta)$, $\gamma_A = B/\rho^{1/2}Rq$, and $s_v = sd\Omega/dq$. Normalizing such that $s_v \to s\Omega'/\gamma_A$, with $\Omega' = d\Omega/dq$, $t \to \Omega'\gamma_A t$, then $h \to s(\eta+\eta_0) - st - \alpha\sin\eta$, and we have

$$\frac{\partial}{\partial\eta}(1+h^2)\frac{\partial X}{\partial\eta} + \Gamma X = \Omega'^2\frac{\partial}{\partial t}(1+h^2)\frac{\partial X}{\partial t}. \tag{A2}$$

Changing coordinates to $\theta=\eta+\eta_0-t$, then at constant θ the equation is periodic in t, and will have Floquet solutions $X(\eta,t)=F(\theta,t)e^{\mu t}$, with μ being the "Floquet growth rate" and F being periodic in t. This gives

$$\frac{\partial}{\partial\eta}(1+h^2)\frac{\partial F}{\partial\eta} + \Gamma F = \left(\mu+\Omega'\frac{\partial}{\partial t}\right)(1+h^2)\left(\mu+\Omega'\frac{\partial}{\partial t}\right)F. \tag{A3}$$

Now we return to the eigenmode problem in the Fourier mode representation [Eq. (1)], and write

$$u_m(x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(x-M)(\eta+\eta_0)-ikx} y(\eta+\eta_0,k)\,d(\eta+\eta_0)\,dk. \tag{A4}$$

Now note that $e^{i(x-M)(\eta+\eta_0)-ikx} = e^{ix(\eta-k)-iM\eta}e^{i(x-M)\eta_0}$, and take $\eta\to\eta+2\pi$, $k\to k+2\pi$, for which since M is an integer $u_m(x)$ becomes

$$u_m(x) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(x-M)(\eta+\eta_0)-ikx} y(\eta+\eta_0+2\pi,k+2\pi)\,d(\eta+\eta_0)\,dk, \tag{A5}$$

and because $u_m(x)$ is independent of η and k, it implies that $y(\eta+\eta_0,k)=y(\eta+\eta_0+2\pi,k+2\pi)$. So writing $\theta=\eta+\eta_0-k$ then at constant $\theta, y(\theta,k)$ is periodic in k.

Substituting (A4) into Eq. (1) gives (after some algebra and integration by parts),

$$\frac{\partial}{\partial\eta}(1+h^2)\frac{\partial y}{\partial\eta} + \Gamma y = \left(\gamma+\Omega'\frac{\partial}{\partial k}\right)(1+h^2)\left(\gamma+\Omega'\frac{\partial}{\partial k}\right)y, \tag{A6}$$

where $h=s(\eta+\eta_0)-sk$. This equation has the same coefficients and boundary conditions as (A3), and hence the growth rate γ of the eigenmode problem is identical to the Floquet growth rate of the time-dependent initial value problem [Eq. (A3)].

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[17] See Ref. 14 and references within for details.
[18] The distance to the first rational surface in real space is $dr=(r_m/nq_m)\Delta X$.
[19] Note that at flow shears higher than those shown in Fig. 3 the root finding method ZERINT (Ref. 9) is no longer able to follow the solution, due to the solution's radius of convergence tending to zero. This may indicate that at higher flow shears the growth rate is no longer one of a discrete set of eigenvalues, but that a continuous spectrum of growth rates exists instead.
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