On neutral-beam injection counter to the plasma current

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It is well known that when neutral beams inject ions into trapped orbits in a tokamak, the transfer of momentum between the beam and the plasma occurs through the torque exerted by a radial return current. It is shown that this implies that the angular momentum transferred to the plasma can be larger than the angular momentum of the beam, if the injection is in the opposite direction to the plasma current and the beam ions suffer orbit losses. On the Mega-Ampere Spherical Tokamak (MAST) [R. J. Akers, J. W. Ahn, G. Y. Antar, L. C. Appel, D. Applegate, C. Brickley *et al.*, Plasma Phys. Controlled Fusion **45**, A175 (2003)], this results in up to 30% larger momentum deposition with counterinjection than with co-injection, with substantially increased plasma rotation as a result. It is also shown that heating of the plasma (most probably of the ions) can occur even when the beam ions are lost before they have had time to slow down in the plasma. This is the dominant heating mechanism in the outer 40% of the MAST plasma during counterinjection.
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I. INTRODUCTION

Neutral beams are usually injected in the same toroidal direction as the plasma current in a tokamak. There are at least two reasons for this. First, neutral-beam injection (NBI) drives additional plasma current which eases the requirement on the Ohmic current if the beams are injected in the cocurrent direction. Second, with co-injection the neutral-beam ions move radially inward through the plasma from the point of ionization, so that first-orbit losses are minimized. Especially in small tokamaks, where the injected fast-ion orbits often have a width comparable to the minor radius, such losses can be substantial for countercurrent NBI. Nevertheless, countercurrent NBI can be very useful. On DIII-D (Ref. 1) and the Axially Symmetric Divertor Experiment (ASDEX)-Upgrade[2] it is routinely used to access the so-called quiescent H mode of high confinement without edge-localized modes, and on the Mega-Ampere Spherical Tokamak (MAST) counter-NBI leads to much better confinement than co-NBI.[3] In this paper, we show that the toroidal angular momentum delivered to the plasma can be substantially larger with counterinjection than with co-injection. We also note that NBI can heat the plasma even if the beam ions are lost to the wall before thermalizing and calculate the maximum heating that can be achieved in this way. The heating arising from promptly lost NBI ions appears to be the dominant heating mechanism in the outer region of MAST.

II. TORQUE

The phenomena mentioned above occur because of the way that momentum is transferred from the beams to the plasma, which was the subject of a recent paper by Hinton and Rosenbluth.[4] (The correct answer had actually been found much earlier,[5,6] but Ref. 4 contains the most detailed analysis.) These authors answered the question of how the momentum from the beam can enter the plasma if the beam ions are injected onto trapped orbits. To lowest order in gyroradius such orbits do not carry any angular momentum, so a question arises about what happens to the beam momentum and whether it is taken up by the plasma or by the magnetic-field coils.

To briefly summarize the argument of Hinton and Rosenbluth using a minimum of mathematics, let us consider an injected atom that is ionized on a flux surface ψ. (The poloidal flux ψ is taken to increase with radius.) The electron stays on this flux surface, but the ion moves radially inward or outward depending on whether it was injected parallel or antiparallel to the toroidal plasma current. For the moment, we consider the former case. On a time average (taken on a time scale much longer than the bounce time, but shorter than the slowing-down time), the ion will reside on a different flux surface $\psi+\Delta\psi$ (where $\Delta\psi$ is negative), and in the case of a standard (thin) banana orbit, $\Delta\psi$ is equal to $-RB_p\Delta r_b$, where R is the major radius, B_p the poloidal field strength, and Δr_b the banana width. Because the canonical momentum

$$p_\varphi = m_i R v_\varphi - e\psi$$

is conserved in the absence of collisions, where m_i is the mass of the ion and e its charge, the average angular momentum of the injected ion decreases by the amount

$$\Delta(m_i R v_\varphi) = e\Delta\psi \tag{1}$$

because of its radial displacement. In the case of a banana orbit, this decrease is equal to the initial angular momentum: after moving radially, the ion has lost almost all its initial angular momentum and is traveling along an orbit with no mean toroidal velocity. To understand where this angular momentum has gone, one notes that, because of the charge separation that occurs when the injected electron and ion are deposited on different flux surfaces, a radial current is established in the plasma, which maintains quasineutrality by canceling the fast ion current. This current exerts a $\mathbf{j}\times\mathbf{B}$ torque on the bulk plasma, which exactly accounts for the missing angular momentum. To see this, we consider the momentum equation for the background plasma,

$$\frac{\partial(\rho\mathbf{V})}{\partial t}+\nabla\cdot(\rho\mathbf{V}\mathbf{V}+\pi)=\mathbf{j}\times\mathbf{B}-\nabla p+\mathbf{F}, \tag{2}$$

where $p=p_e+p_i$ is the plasma pressure, $\mathbf{V}=\mathbf{V}_i$ the ion velocity, $\rho=m_i n$ the density, $\pi=\pi_i+\pi_e$ the viscosity, and $\mathbf{F}$ the beam-plasma friction force. Taking the $R\hat{\varphi}$ projection, forming a flux-surface average,

$$\frac{\partial\langle\rho RV_\varphi\rangle}{\partial t}=\langle\mathbf{j}\cdot\nabla\psi-R\hat{\varphi}\cdot\nabla\cdot(\rho\mathbf{V}\mathbf{V}+\pi)+RF_\varphi\rangle, \tag{3}$$

and integrating over the plasma volume gives

$$\frac{d}{dt}\int\rho RV_\varphi dV=\int\mathbf{j}\cdot\nabla\psi dV+\int RF_\varphi dV - \text{transport losses}. \tag{4}$$

The first term on the right is equal to

$$\int\mathbf{j}\cdot\nabla\psi dV=\dot{N}e\Delta\psi,$$

where $\dot{N}$ is the number of injected ions in unit time. It now follows from (1) and (4) that the rate of change of the plasma angular momentum is equal to the injected angular momentum per unit time minus viscous losses.

Now consider what happens if the beam is directed in the opposite direction to the plasma current, so that the injected ions move outward from the point of ionization and are lost from the plasma by hitting the first wall or some other in-vessel component. The same argument as above then applies if the flux surface where the loss occurred is denoted by $\psi+\Delta\psi$. Because of p_φ conservation we have

$$e\Delta\psi=m_i(R_0v_{\varphi0}-R_1v_{\varphi1}), \tag{5}$$

where $m_iR_0v_{\varphi0}$ is the injected angular momentum and $m_iR_1v_{\varphi1}$ the lost momentum. Again, a return current is established between ψ and $\psi+\Delta\psi$, and this current exerts a torque equal to (5) multiplied by the number of injected ions in unit time,

$$\text{Total torque}=\dot{N}m_i(R_0v_{\varphi0}-R_1v_{\varphi1}). \tag{6}$$

Not suprisingly, the delivered torque is equal to the injected momentum minus the lost momentum. However, there are two points worth noting. First, if the ions are traveling along the outboard leg of banana orbits when being lost, then $v_{\varphi1}$ is positive while $v_{\varphi0}$ is negative and the total delivered torque is larger in absolute terms than the injected angular momentum. The injection of countercurrent momentum is supplemented by the loss of cocurrent momentum, leading to a larger effect than if there were no first-orbit losses. Second, if the loss occurs some distance away from the separatrix (or last closed flux surface), then all the $\mathbf{j}\times\mathbf{B}$ torque is not exerted on the plasma. If the plasma is surrounded by a vacuum region, then the return current outside the separatrix (which will be denoted by ψ_s) flows in the metal structure supporting whatever in-vessel component that the ions hit. Only the current in the region $\psi_0<\psi<\psi_s$ exerts a torque on the plasma. This means that instead of (6), the torque on the plasma is equal to

$$\text{Torque on plasma}=\dot{N}m_i(R_0v_{\varphi0}-R_sv_{\varphi s}),$$

where $m_iR_sv_{\varphi s}$ is the angular momentum at the point where the injected ions cross the separatrix. In other words, for purposes of angular momentum bookkeeping, the ions are effectively lost when crossing the separatrix. If they do so with positive toroidal velocity, the torque delivered to the plasma is larger than the angular momentum from the beam. This effect is quite significant in MAST.

III. HEATING

A. Fluid picture

It is instructive also to consider the energy balance of the plasma. The work done by the torque must of course show up in the energy equation.[6] To see how this occurs, we take the scalar product of the plasma fluid velocity $\mathbf{V}$ with the momentum equation (2), where, for clarity, we neglect the collisional interaction between the plasma and the beam ions, assuming that the latter are all lost before they have had time to slow down. This gives an equation for the kinetic-energy balance of the plasma,

$$\frac{\partial}{\partial t}\left(\frac{\rho V^2}{2}\right)+\nabla\cdot\left(\frac{\rho V^2\mathbf{V}}{2}\right)=\mathbf{V}\cdot(\mathbf{j}\times\mathbf{B}-\nabla p-\nabla\cdot\pi), \tag{7}$$

where the first term on the right is the work that the $\mathbf{j}\times\mathbf{B}$ force performs on the moving plasma. The total energy of the plasma thus increases, as is apparent from the sum of the ion and electron energy equations,[7]

$$\frac{\partial}{\partial t}\left(\frac{\rho V^2}{2}+\frac{3p}{2}\right)+\nabla\cdot\left(\frac{\rho V^2}{2}\mathbf{V}+\frac{5p\mathbf{V}}{2}+\pi\cdot\mathbf{V}+\mathbf{q}\right)=\mathbf{j}\cdot\mathbf{E}, \tag{8}$$

where $\mathbf{q}=\mathbf{q}_i+\mathbf{q}_e+(5p_e/2)\mathbf{u}+\pi_e\cdot\mathbf{u}$ is the heat flux and $\mathbf{u}=\mathbf{V}_e-\mathbf{V}=-\mathbf{j}/ne$.[7,8] Here, the energy input from the radial current shows up as "radial Ohmic heating" on the right-hand side.

The work done by the $\mathbf{j}\times\mathbf{B}$ force acts not only to spin up the plasma, but also to heat it since rotational energy is dissipated as heat by viscosity. This heating goes into whatever species contributes most to the viscosity, i.e., most probably the ions rather than the electrons since it is the ions that carry the momentum. To see this mathematically, consider the situation where NBI deposits electrons in the plasma while the injected ions are lost to the edge. When a steady state has been reached, there is a radial electron flux but no ion particle transport (other than that balancing the ionization of edge neutrals in the plasma, which we ignore). The thermal energy balance equation for the ions is

$$\frac{\partial}{\partial t}\left(\frac{3p_i}{2}\right)+\nabla\cdot\left(\frac{5p_i\mathbf{V}}{2}+\mathbf{q}_i\right)-Q_i=\mathbf{V}\cdot\nabla p_i-\pi_i:\nabla\mathbf{V}, \quad (9)$$

where the second term on the right represents viscous heating and Q_i is the collisional energy exchange with the electrons. In steady state, the integral of the right-hand side over the plasma volume is

$$\int(\mathbf{V}\cdot\nabla p_i-\pi_i:\nabla\mathbf{V})dV=\int\mathbf{V}\cdot(\nabla p_i+\nabla\cdot\pi)dV$$

$$=\int\mathbf{V}\cdot(\mathbf{j}\times\mathbf{B}-\nabla p_e)]dV,$$

where we have neglected electron viscosity, used Eq. (7), and assumed that the flow velocity is small at the boundary. The term containing p_e vanishes since the flow velocity is generally of the form $\mathbf{V}=\omega(\psi)R\hat{\varphi}+u(\psi)\mathbf{B}/n$, where $\omega(\psi)$ and $u(\psi)$ are flux functions, in which case

$$\int\mathbf{V}\cdot\nabla p_e dV=\int\frac{u\mathbf{B}\cdot\nabla(nT_e)}{n}dV,$$

which vanishes if the electron temperature is a flux function. (The density n need not be a flux function; it varies poloidally if the plasma rotates quickly enough.) The $\mathbf{j}\times\mathbf{B}$ work thus finally appears as ion viscous heating (which is true also for the ordinary collisional torque[6]), and the volume integral of Eq. (9) becomes

$$P_i=\int\left(\frac{5p_i\mathbf{V}}{2}+\mathbf{q}_i\right)\cdot d\mathbf{S}=\int[\mathbf{V}\cdot(\mathbf{j}\times\mathbf{B})+Q_i]dV,$$

in a steady state where this heating is balanced by transport losses across the plasma edge. If, in this steady state, the viscosity and friction force on the ions are smaller than the pressure gradient (as is practically always the case), so that the ion momentum equation reduces to

$$\rho\mathbf{V}\cdot\nabla\mathbf{V}=ne(\mathbf{E}+\mathbf{V}\times\mathbf{B})-\nabla p_i,$$

then the ion heating becomes

$$P_i=\int\left[\mathbf{j}\cdot\left(\mathbf{E}-\frac{1}{ne}\nabla p_i-\frac{m_e}{e}\mathbf{V}\cdot\nabla\mathbf{V}\right)+Q_i\right]dV.$$

On the other hand, the total energy delivered to the plasma by the beams is given by the integral of the right-hand side of Eq. (8) over the plasma volume,

$$P_i+P_e=\int\mathbf{j}\cdot\mathbf{E}dV. \quad (10)$$

If the plasma rotates rapidly, with a velocity exceeding the diamagnetic velocity, then most of the heating goes to the ions, $P_e+P_i\simeq P_i$. We now proceed to estimate the maximum energy that can be delivered to the plasma in this way. This is most easily done from a particle picture.

B. Particle picture

An injected ion that hits the wall first loses some kinetic energy by traveling radially outward against the electric field. It is this energy that is deposited in the plasma [as follows from Eq. (10)], and it is generally less than the injection energy. The energy lost by the particle is equal to the depth of the electrostatic potential well, which is smaller than the injection energy, so the particle necessarily has some remaining kinetic energy when crossing the separatrix. In order to calculate this energy analytically, we make the simplifying assumptions that the radial electric field is approximately constant across the width of the poloidal ion orbit, and that the latter may be approximated by a standard (thin) banana orbit. We also assume that the plasma flow velocity is mainly in the toroidal direction, $\mathbf{V}=\omega(\psi)R\hat{\varphi}$, as predicted by neoclassical theory as a result of poloidal flow damping.[10] For simplicity we only consider horizontal injection in the midplane, so that the injection speed is

$$\mathbf{v}_0=v_{oR}\hat{\mathbf{R}}+v_{0\varphi}\hat{\varphi},$$

where $\hat{\mathbf{R}}=\nabla R$ is the unit vector in the direction of the major radius. However, no assumption is made regarding the aspect ratio of the torus, which is thus arbitrary.

It is useful to write the electrostatic potential as $\Phi=\bar{\Phi}(\psi)+\tilde{\Phi}(\psi,\theta)$, where $\bar{\Phi}=\langle\Phi\rangle$ is the flux-surface average of Φ and the poloidally varying part is given by[10]

$$\frac{e\tilde{\Phi}}{T_e}=\frac{m_i\omega^2(R^2-\langle R^2\rangle)}{2(T_e+T_i)}. \quad (11)$$

The poloidal electric field arises because the centrifugal force pushes ions but not electrons toward the outboard side of each flux surface. The total energy of an injected ion is

$$H=\frac{m_iv^2}{2}+e\Phi\simeq\frac{m_iv^2}{2}-\omega e\psi+e\tilde{\Phi}+\text{const},$$

where $\omega=-d\bar{\Phi}/d\psi$,[10] and it follows that the energy given up by the particle to the plasma is

$$\Delta W=\frac{m_i(v_0^2-v_s^2)}{2}=\omega e(\psi_0-\psi_s)+e(\tilde{\Phi}_s-\tilde{\Phi}_0),$$

where the subscripts 0 and s refer to the position of ionization and separatrix crossing, respectively. It is clear that this energy is maximized if the ion is lost in the midplane of the outer leg on a trapped banana orbit. It is also clear that the particle cannot in general deliver *all* its injection energy to the plasma in this way since, in order for trapping to occur, the magnetic moment must be nonzero. (An exception can occur for low-energy particles that become electrostatically trapped, see below.) The magnetic moment is an adiabatic invariant, and it follows that the particle necessarily has some perpendicular kinetic energy when hitting the wall—energy that is not delivered to the plasma.

The particle orbits are most easily studied in a frame rotating toroidally with the angular frequency ω, where the average radial electric field vanishes. The particle velocity measured in this frame is denoted by $\mathbf{u}=\mathbf{v}-\omega R\hat{\varphi}$, and we note that for a particle that is lost in the outer midplane

$$\Delta W=\frac{m_i\omega RB_\varphi u_{0\|}}{B}=2\left(\frac{B_\varphi}{B}\right)^2 m_i\omega R(v_{0\varphi}-\omega R).$$

The fractional energy lost is thus

$$\frac{\Delta W}{W} = \frac{4\omega R u_{0\parallel}}{v_0^2}\frac{B_\varphi}{B} = 4\left(\frac{B_\varphi}{B}\right)^2 \frac{x-1}{x^2+(v_{0R}/\omega R)^2}, \tag{12}$$

where $x=v_{0\varphi}/\omega R$ and all quantities are evaluated in the outer midplane. This expression is valid under the condition that the radial injection speed v_{0R} is such that the particle is marginally trapped. To formulate this condition mathematically, we examine constants of the motion. The energy can be written as

$$H = \frac{m_i u^2}{2} - \frac{m_i \omega^2 R^2}{2} + e\tilde{\Phi} + \omega p_\varphi + \text{const},$$

where we can choose the constant so that the last two terms cancel,

$$H = \frac{m_i u^2}{2} + e\tilde{\Phi} - \frac{m_i \omega^2 R^2}{2}. \tag{13}$$

A second constant of motion is furnished by the magnetic moment measured in the rotating frame,

$$\mu = \frac{m_i u_\perp^2}{2B}. \tag{14}$$

We allow the rotation speed to be comparable to the injection speed, which implies that the electrostatic potential varies relatively rapidly in the radial direction (about $m_i v^2/e$ over a gyroradius). In this situation the magnetic moment measured in the laboratory frame, $m_i v_\perp^2/2B$, is not conserved, but its counterpart (14) in the rotating frame is an adiabatic invariant provided the rotation frequency ω varies slowly on the gyroradius length scale.[10] The parallel kinetic energy is obtained from Eqs. (11) and (13) and is equal to

$$\frac{m_i u_\parallel^2}{2} = H - \mu B(\theta) + V(\theta),$$

where

$$V(\theta) = \frac{m_i \omega^2 [T_i R^2(\theta) + T_e \langle R^2\rangle]}{2(T_e+T_i)}.$$

A particle is thus trapped on the outboard side of the flux surface if

$$\mu > \mu_c = \frac{H+V_{\text{in}}}{B_{\text{in}}},$$

where $V_{\text{in}}=V(\pi)$ and $B_{\text{in}}=B(\pi)$ are the values of V and B on the inboard side of the flux surface (assuming that this is where B is largest). Note that, unlike the situation in a nonrotating plasma, the trapped-passing boundary here depends on the particle energy. For a marginally trapped particle in the outer midplane ($\theta=0$, subscript "out"), the parallel velocity is thus given by

$$\frac{m_i u_{0\parallel}^2}{2} = H - \mu_c B_{\text{out}} + V_{\text{out}} = \left(1-\frac{B_{\text{out}}}{B_{\text{in}}}\right)H + V_{\text{out}} - \frac{B_{\text{out}}V_{\text{in}}}{B_{\text{in}}}.$$

On the other hand,

$$u_{0\parallel} = \frac{B_\varphi}{B}(v_{0\varphi}-\omega R)$$

and

$$H = \frac{m_i}{2}[(v_{0\varphi}-\omega R)^2 + v_{0R}^2] - V(0),$$

so it follows that for trapped particles

$$\left(\frac{v_{0R}}{\omega R_{\text{out}}}\right)^2 > a(x-1)^2 - b, \tag{15}$$

where

$$a = \frac{B_{\varphi\text{out}}^2/B_{\text{out}}^2}{1-B_{\text{out}}/B_{\text{in}}} - 1,$$

$$b = \frac{1-R_{\text{in}}^2/R_{\text{out}}^2}{(B_{\text{in}}/B_{\text{out}}-1)(1+T_e/T_i)}.$$

The largest fractional energy loss (12) is obtained if the radial injection speed is made as small as possible subject to the constraint that the particle should be trapped, and is thus given by

$$\frac{\Delta W}{W} = 4\left(\frac{B_\varphi}{B}\right)^2_{\text{out}} \frac{x-1}{x^2+\max[a(x-1)^2-b,0]}. \tag{16}$$

This expression depends on the magnetic geometry through the parameters a and b, and on the ratio $x=v_{0\varphi}/\omega R$. The latter is determined by the injection rate and the momentum confinement in the region of interest. In general we expect $x \gg 1$ as the injection speed is much greater than the rotation speed. The energy transfer is then relatively small, $\Delta W/W = O(1/x)$, as one would expect. In a standard circular equilibrium with small inverse aspect ratio, $\epsilon \ll 1$, we have

$$a \simeq \frac{1}{2\epsilon},$$

$$b \simeq \frac{2}{1+T_e/T_i},$$

and Eq. (16) reduces to

$$\frac{\Delta W}{W} = \frac{8\epsilon}{x-1}.$$

If the plasma rotates more quickly, it is interesting to see how (surprisingly) large the fractional energy transfer (16) can be made. We thus proceed to maximize this expression with respect to x. First of all, if $a<0$, then Eq. (15) indicates that the injected ion must be trapped regardless of the normalized toroidal injection speed x. In this case, toroidally tangential injection only gives rise to trapped orbits and the energy loss (16) becomes

$$\frac{\Delta W}{W} = 4\left(\frac{B_\varphi}{B}\right)^2_{\text{out}} \frac{x-1}{x^2}. \tag{17}$$

The maximum over x of this expression occurs for $x=2$ and is equal to

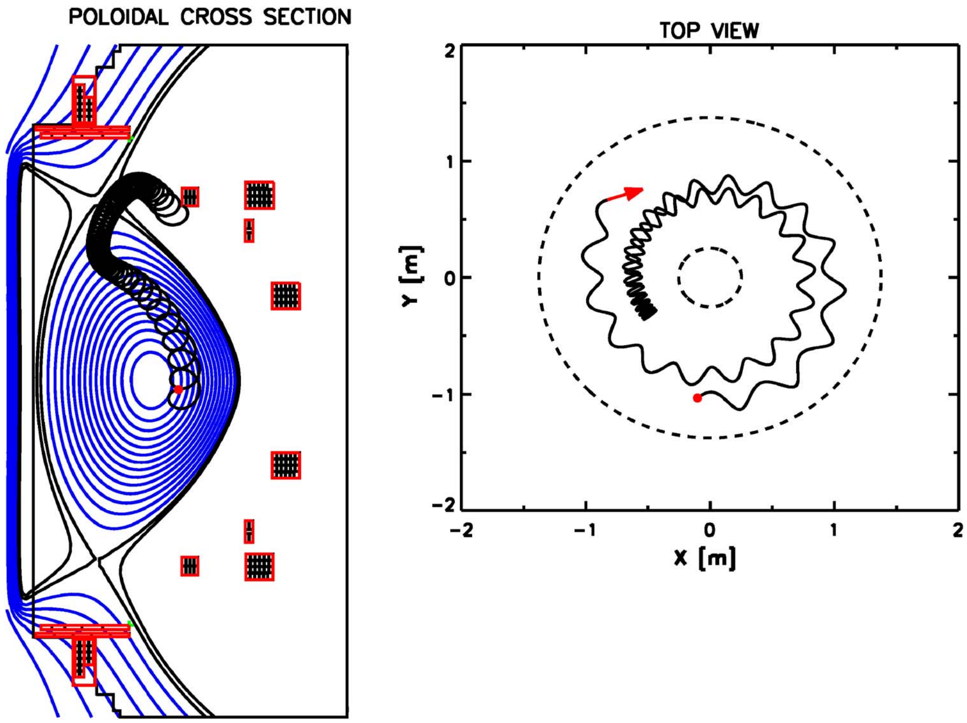

FIG. 1. (Color online). A typical loss orbit of a counterinjected beam ion in MAST. The ion is injected with negative (counterclockwise) momentum but lost with positive momentum as it hits one of the poloidal field coils. These coils are located inside the vacuum vessel on MAST.

$$\max \frac{\Delta W}{W} = \left(\frac{B_\varphi}{B}\right)^2_{\mathrm{out}}$$
$$(a < 0, \text{ or } 2ab + b - a > 0 \text{ and } 0 < a < b). \tag{18}$$

Thus, if $a<0$ and the momentum confinement is so good that $x=2$ where the injected atoms are ionized (i.e., the edge plasma rotates at half the injection speed), then the lost ions deliver a fraction $(B_\varphi/B)^2$ (evaluated in the outer midplane) to the plasma before hitting the wall. This theoretical maximum will of course not be achieved in practice, but it is notable that it is so large. In MAST, where a typically is very slightly negative, $(B_\varphi/B)^2_{\mathrm{out}} \simeq 0.7$.

If $a>0$, then

$$\frac{d}{dx}\left(\frac{\Delta W}{W}\right) = -4\left(\frac{B_\varphi}{B}\right)^2_{\mathrm{out}} \times \begin{cases} \dfrac{x-2}{x^3}, & x < 1 + \sqrt{b/a}, \\ \dfrac{(1+a)(x-1)^2 + (b-1)}{[x^2 + a(x-1)^2 - b]^2}, & x > 1 + \sqrt{b/a}, \end{cases}$$

and the maximum depends on the location of the zeros of these derivatives. The second one vanishes at

$$x = x_* = 1 + \sqrt{\frac{1-b}{1+a}},$$

and we must distinguish between the cases $x_* > 1 + \sqrt{b/a}$ and $x_* < 1 + \sqrt{b/a}$. In the first case the the maximum occurs at $x = x_*$ and is equal to

$$\max \frac{\Delta W}{W} = \frac{2}{1 + \sqrt{(1+a)(1-b)}}\left(\frac{B_\varphi}{B}\right)^2_{\mathrm{out}}$$
$$(2ab + b - a < 0 < a).$$

In the latter case, $d(\Delta W/W)/dx < 0$ in the region $x < 1 + \sqrt{b/a}$, so the maximum energy transfer occurs for $x < 1 + \sqrt{b/a}$. If $b > a$, this happens at $x=2$ and the maximum energy loss is again equal to Eq. (18). If instead $b < a$ then the maximum is found at $x = 1 + \sqrt{b/a}$ and is equal to

$$\max \frac{\Delta W}{W} = 4\left(\frac{B_\varphi}{B}\right)^2_{\mathrm{out}} \frac{\sqrt{b/a}}{(1 + \sqrt{b/a})^2}$$
$$(b < a, a > 0 \text{ and } 2ab + b - a > 0).$$

This is the case that applies to the standard large-aspect-ratio equilibrium (unless $T_e > 3T_i$), in which case the maximum energy transfer becomes

$$\max \frac{\Delta W}{W} \simeq 8\sqrt{\frac{\epsilon}{1 + T_e/T_i}},$$

which is formally a small number but in practice not negligible.

IV. THE CASE OF MAST

First-orbit losses are substantial during counterinjection in MAST. To assess the situation quantitatively, we have used the LOCUST full-orbit Monte Carlo code[3] to analyze the statistics of slowing down beam ions in a number of MAST discharges. The TRANSP code,[6] which tracks guiding centers but does not directly follow gyromotion, was also used for comparison. The magnetic equilibribrium is obtained in slightly different ways by these codes. LOCUST uses an EFIT

TABLE I. Heating power in different channels for a typical pair of discharges with co-NBI (no. 8505) and counter-NBI (no. 8322) in MAST. Most of the beam power is absorbed by the plasma in the case of co-NBI, while most of it is lost to the wall in case of counter-NBI. Most of the aborbed power is delivered to the plasma by ordinary collisional friction in both cases, but about 1/6 of the absorbed power in the counter-NBI discharge is delivered in the way described by Eq. (10).

Power (MW)	Co-NBI	Ctr-NBI
Total beam power	1.8	2.6
Absorbed power	1.66	0.91
Slowing down on electrons	0.78	0.30
Slowing down on ions	0.87	0.45
$\mathbf{j}\times\mathbf{B}$ heating	0.01	0.16

magnetic reconstruction constrained to match the D_α emission at the plasma boundary, while TRANSP in addition solves the current diffusion equation to determine more accurately the magnetic field in the interior of the plasma. Figure 1 shows a typical example of a loss orbit in MAST during counter-NBI calculated by LOCUST. After reversing its toroidal velocity, the injected ion first crosses the separatrix and then hits a poloidal field coil. These coils are situated inside the vacuum vessel on MAST.

In Table I and Fig. 2 a typical co-NBI and counter-NBI discharge are compared. Table I gives the total power from the beams and going into the plasma through various channels, and Fig. 2 shows the radial profiles of the power densities. In the co-NBI plasma, nearly all the power is deposited in the plasma through ordinary collisional friction, with approximately equal powers going into the ions and electrons. The $\mathbf{j}\times\mathbf{B}$ work on the plasma is negligible. In contrast, in the case of counter-NBI less than half the beam power is absorbed by the plasma as most of the power is deposited on in-vessel components and, by charge-exchange reactions, in the gas blanket surrounding the plasma. Approximately half of the absorbed power goes into frictional heating of the ions and one third into electron friction. The remaining 1/6 of the absorbed power is delivered to the

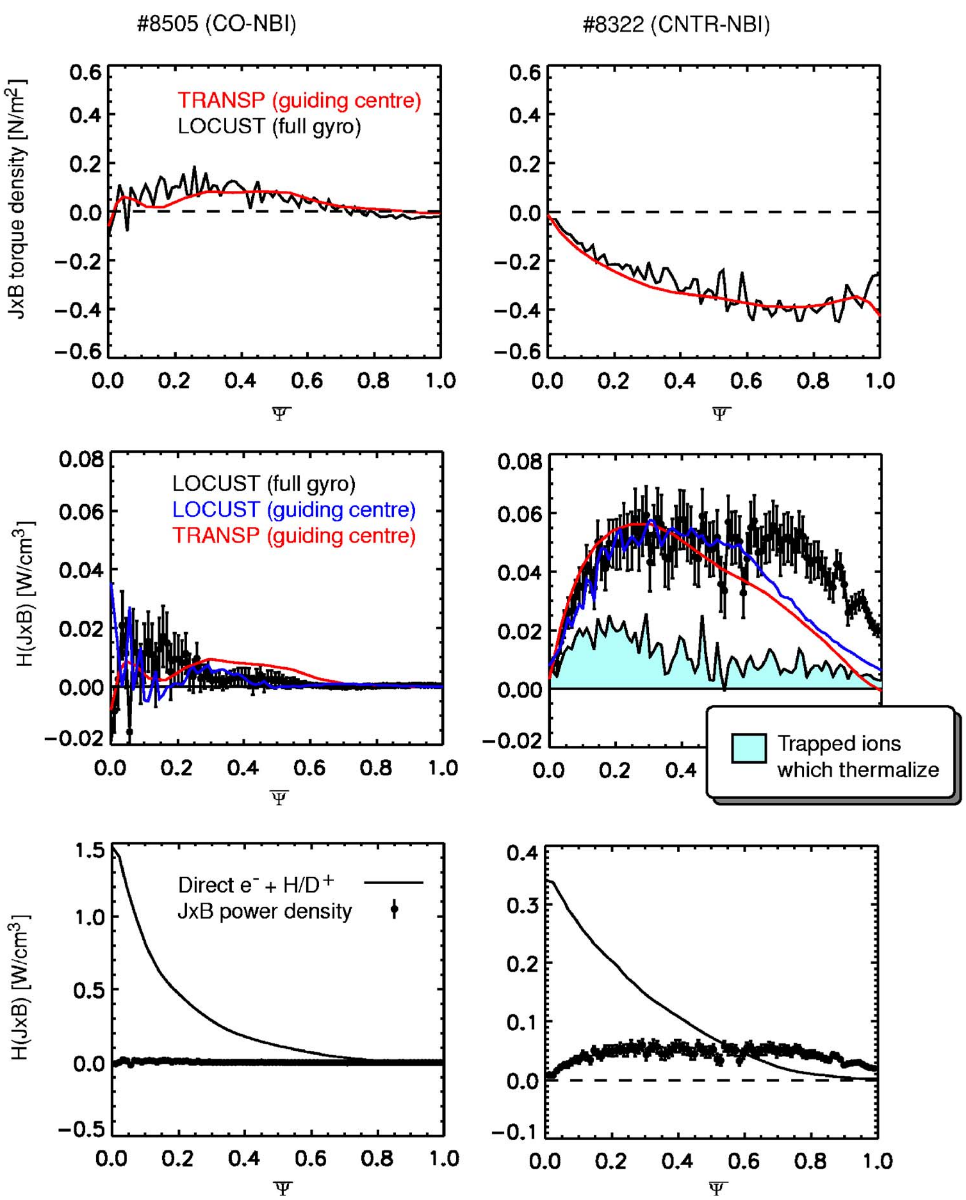

FIG. 2. (Color online). Comparison between a typical co-NBI (no. 8505) and counter-NBI (no. 8322) discharge in MAST. The panels in the first row show the $\mathbf{j}\times\mathbf{B}$ torque density as a function of normalized poloidal flux. This torque is larger in the counterinjection case because of the loss of comoving NBI ions. The next row shows the "$\mathbf{j}\times\mathbf{B}$ heating" power density, i.e., the heating arising from the radial movement of NBI ions according to Eq. (10). In the case of counter-NBI, most of this heating is due to promptly lost ions, but some (blue area) comes from banana orbits that stay in the plasma. The heating is calculated by tracking the full gyromotion orbits in the black curve, and using only guiding-center orbits in the blue and red curves. Including gyromotion increases the heating signficantly in the edge, as it allows ions to be lost without their guiding center crossing the separatrix. The final pair of panels show the total and $\mathbf{j}\times\mathbf{B}$ heating power densities. Note that the latter dominates in the outer 40% of the counter-NBI plasma.

plasma by the radial movement of NBI ions, by the mechanism discussed in Secs. II and III. Some of this power originates from banana orbits that stay (and are thermalized) in the plasma, but most of it comes from promptly lost NBI ions. This is the dominant heating mechanism in the outer 40% of the plasma. It is important to include the full gyromotion in these calculations, rather than just tracking guiding centers, since gyromotion allows particles to be lost from the plasma although their guiding center does not cross the separatrix. A correction to this effect is currently being applied to the NUBEAM Monte Carlo tracker in TRANSP.

The numerical result that about 1/6 of the beam power is delivered to the plasma by promptly lost ions is in approximate agreement with the estimate (17), which is the case that is applicable to MAST since the parameter a is slightly negative. For a typical rotation speed of 100 km/s (representative of the rotation at the banana center of the NBI ions) and injection energy of 30 keV, this expression gives $\Delta W/W \sim 0.16$.

V. THE CASE OF NO RETURN CURRENT

Finally, it is perhaps of interest to examine where the injected momentum goes if the radial return current is somehow prevented from flowing in the plasma (for instance, if the plasma is replaced by an insulator). Again, we consider the situation where in unit time $\dot{N}$ electrons are deposited on the flux surface ψ and an equal number of ions on $\psi+\Delta\psi$. If there is no return current, the resulting charge separation leads to a rapid buildup of an electric field, which causes *the electromagnetic field* to acquire angular momentum. Since the momentum density in an electromagnetic field is

$$\mathbf{P} = \epsilon_0 \mathbf{E} \times \mathbf{B},$$

its total angular momentum is equal to

$$L = \int R\hat{\varphi} \cdot \mathbf{P} dV = \epsilon_0 \int \mathbf{E} \cdot \nabla \psi dV,$$

where the integral is taken over all of space. The field angular momentum thus increases at the rate

$$\frac{dL}{dt} = \epsilon_0 \frac{d}{dt} \int [\nabla \cdot (\psi \mathbf{E}) - \psi \nabla \cdot \mathbf{E}] dV = -\dot{N} e \Delta\psi,$$

where the first term has been converted to a surface integral at infinity and Poisson's equation has been used in the second term. According to Eq. (1), the field thus gains angular momentum at the same rate as it is lost by the injected ions.

VI. SUMMARY

If neutral-beam ions are injected counter to the plasma current and are therefore lost to the wall on their first orbit, they still transfer some of their momentum and energy to the plasma. Indeed, if the injected ions are trapped and leave the plasma on the outer leg of their banana orbits, then *more* momentum is delivered to the plasma than if the ions did not suffer orbit losses. For conventional (thin) banana orbits lost in the outboard midplane, the resulting torque on the plasma is twice as large as if the ions had stayed in the plasma. In practice, the torque is enhanced by up to about 30% in the case of MAST, which should help to explain why the plasma is observed to rotate significantly faster during counter-NBI.

The work done by the torque increases the rotational energy of the plasma, which in turn is dissipated as heat by viscosity. As it is the ions rather than the electrons that carry angular momentum, it is likely that the ion viscosity (whether anomalous or neoclassical) dominates over electron viscosity. Most of the heating is thus likely to occur in the ion channel. The heating power achieved in this way is of course always smaller than the NBI power, in practice by a wide margin. However, the theoretical maximum (provided the edge plasma rotates quickly enough) is substantial, about 70% in the case of MAST. The actual power is less than this theoretical maximum because (i) the injected ions do not all follow orbits that are optimal (i.e., barely trapped) for this purpose and (ii) the edge rotation velocity is much lower than that giving the largest energy loss for the injected particles. Nevertheless, Monte Carlo simulations suggest that this form of heating dominates in the outer region of counter-NBI discharges and thus plays an important role for the energy balance of the plasma edge.

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