Modeling sawtooth stabilization by energetic ions from neutral beam injection

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Recent advances in modeling the effects of anisotropic energetic ion distributions have enabled the development of a complete coherent physics explanation of sawtooth stabilization in both conventional and spherical tokamaks. As an example, a complete model has been developed to explain the asymmetric stabilization of sawteeth with respect to neutral beam injection direction in the Joint European Torus. This asymmetric sawtooth stabilization [M. Nave *et al.*, Phys. Plasmas **13**, 014503 (2006)] arises because of both the destabilizing contribution from the counterpassing ions and the strong modification of the stabilizing contribution of the nonadiabatic trapped ions due to flow shear. The fast particle effects including pressure anisotropy, sheared flows, and the adiabatic response to the internal kink mode have been modeled in general toroidal geometry for the first time.
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The magnetohydrodynamic (MHD) stability of burning plasmas is a critical issue for operation of the ITER.[1] The reaction $D+T\rightarrow {}^4\mathrm{He}(3.5\text{ MeV})+n(14\text{ MeV})$ produces fusion-born α particles that can affect the stability of the plasma. One key MHD instability which will be affected by these α particles is the sawtooth oscillation—a periodic relaxation of the core plasma density and temperature. Based on experimental evidence, it is thought that the α particles will lead to large amplitude sawteeth,[2,3] which have been shown to result in the triggering of other instabilities called neoclassical tearing modes (NTMs),[4] which can have deleterious ramifications for plasma confinement. As such, recent experiments have identified various methods for the control of sawteeth in order to avoid triggering NTMs whilst retaining the benefits of small, frequent sawtooth crashes, such as the prevention of core impurity accumulation.[5] One such experimental technique is to apply neutral beam injection (NBI) heating in the opposite direction to the plasma current. This has been shown to result in shorter sawtooth periods than those in Ohmically heated plasmas in the Joint European Torus (JET),[6] the Mega Ampère Spherical Tokamak (MAST),[7] and the Tokamak Experiment for Technology Oriented Research (TEXTOR).[8] Furthermore, each experiment exhibits an asymmetry of sawtooth period with respect to NBI direction. This Communication reports on modeling of the stability of the plasma with respect to the $n/m=1/1$ internal kink mode—which is generally accepted to be related to sawtooth oscillations[9]—in the presence of NBI fast ions.

The trigger condition for a sawtooth collapse is believed to be associated with the linear stability threshold for a reconnecting 1/1 mode. Although this differs from the threshold for ideal MHD instability, it nevertheless depends strongly on the magnitude of the ideal potential energy, $\delta W=\delta W_{th}+\delta W_{\mathrm{hot}}$. It has been proposed[2] that, provided the magnetic shear at $q=1$, s_1 exceeds a critical value, reconnection is triggered when $c\rho r_1>\delta W$, where c is a normalization coefficient, r_1 is the radial position at which $q=1$, and ρ is the ion Larmor radius. In Ref. 2, a $1^1/_2$D transport code is used to study the time evolution of $s_1(t)$, $r_1(t)$, and $\delta W(t)$ in order to determine when the trigger inequality is satisfied. Typically, the ideal potential energy $\delta W(t)$ is positive (ideal stable) and reaches a quasistationary value on a relatively short time scale (the reheating time scale for δW_{th}, and the energy deposition time scale for δW_h), while $s_1(t)$ and $r_1(t)$ evolve on a slower time scale, determined by core resistive diffusion. Assuming that the trigger criterion can be represented by a linear time dependence for $r_1(t)\approx\hat{r}_1 t/\tau_\eta$, with τ_η a measure of the resistive diffusion time, and a quasistationary value of $\delta W(t)$, then the sawtooth period, τ_s, can be represented in the form $\tau_s/\tau_\eta\sim(\delta W_{th}+\delta W_h)/c\rho\hat{r}_1$ with longer ramp times predicted when δW_h is large, and shorter ramp times predicted when δW_h is small. In what follows we assume that this applies so that the sawtooth period is roughly proportional to δW.

In MAST, the asymmetric stabilization of sawteeth has been explained in terms of the direction of strong toroidal flows, driven by neutral beam injection, relative to the ion diamagnetic drift.[7] Whilst fast particles do have a stabilizing influence upon sawteeth in spherical tokamaks, they cannot explain the experimentally observed minimum in sawtooth period. However, in larger aspect ratio devices, where the toroidal rotation is smaller, the sawtooth behavior can only be explained by the effects of the fast ions. An appropriate tool for studying the effects of anisotropic fast particles on the internal kink mode is the HAGIS code.[10] HAGIS solves the Hamiltonian equations describing the guiding center motion of ions in full toroidal geometry. The code has been extended to calculate the contribution of the fast beam ions to the potential energy of the internal kink mode, δW_h. The code now also includes the effects of the equilibrium flow shear, which modifies the toroidal precession frequency of the particle orbits as well as the electric potential experienced by the beam ions, $\phi_f = rB_0 v_\zeta / q$, where ϕ_f is the electric potential due to the plasma flow, v_ζ is the toroidal rotation velocity, r is the minor radius, and B_0 is the equilibrium magnetic field. The eigenmode structure is computed by the MHD stability code, MISHKA-F,[11] which includes the effects of flows and ion diamagnetic drifts in toroidal geometry. HAGIS subsequently determines the stability of the mode including kinetic effects, where since $\beta_{\text{hot}} \sim 0.2\beta_{\text{bulk}}$ ($\beta = 2\mu_0 p/B^2$) it is appropriate to assume that the kinetic modification of δW does not alter the form of the perturbation, $\vec{\xi}$. HAGIS treats the effects of finite orbits on the adiabatic response to the internal kink mode, as well as the effect of pressure anisotropy on δW_h, which have been neglected in other hybrid kinetic-MHD codes.[12] The kinetic effects of thermal ions at finite rotation have not been considered since this was examined in Ref. 13.

The fast ion distribution function is separated into an equilibrium component, $f_0(\mathcal{E}^0, \mathcal{P}_\zeta^0, \mu)$, and two perturbed components, $\delta f_h = \delta f_{hk} + \delta f_{hf}$, a nonadiabatic (kinetic) and an adiabatic (fluid) part, respectively. Here, the particle energy ($\mathcal{E}^0 = mv^2/2$), the canonical momentum ($\mathcal{P}_\zeta^0 = mB_\zeta v_\parallel / B - e\psi_p$) and the magnetic moment ($\mu = mv_\perp^2 / 2B$) are the unperturbed constants of motion, where m is the particle mass, ζ is the toroidal angle, v is the particle velocity, ψ_p is the poloidal flux, e is the fast ion charge, and "$\parallel$" and "$\perp$" the mean components parallel and perpendicular to the magnetic field, respectively. Analytic theory developed for large aspect ratio circular plasmas[14] implies that these contributions to the perturbed distribution function can be expressed as

$$\delta f_{hk} \sim \sum_{l=-\infty}^{\infty} \frac{\tilde{\omega} - \Delta\Omega - n\omega_{*h}}{\tilde{\omega} - \Delta\Omega - n\langle\dot{\zeta}\rangle + l\omega_b} \frac{\partial f_h}{\partial \mathcal{E}^0} e^{-i(\omega + l\omega_b + n\langle\dot{\zeta}\rangle)t} \quad (1)$$

and $\delta f_{hf} \sim \vec{\xi} \cdot \vec{\nabla} \psi_p \partial f_h / \partial \mathcal{P}_\zeta^0$, where $\omega_{*h} = (\partial f_h / \partial \mathcal{P}_\zeta^0)/(\partial f_h / \partial \mathcal{E}^0)$ is the hot ion diamagnetic frequency, $\Delta\Omega = \Omega_E(r) - \Omega_E(r_1)$ is the sheared toroidal flow, $\tilde{\omega}$ is the Doppler shifted mode frequency, $\omega_b = 2\pi/\tau_b$, and τ_b is the poloidal orbit transit time. In HAGIS the adiabatic[15] and nonadiabatic components of the perturbed distribution function are: $\delta f_{hf} = \alpha g \, \partial f_0 / \partial \mathcal{P}_\zeta^0 + \phi \, \partial f_0 / \partial \mathcal{E}^0$ and $\delta\dot{f}_{hk} = -\dot{\mathcal{P}}_\zeta \partial f_0 / \partial \mathcal{P}_\zeta^0 - \dot{\mathcal{E}} \partial f_0 / \partial \mathcal{E}^0$, respectively, where the covariant ζ component of the magnetic field, $B_\zeta = g(\psi)$, $\mathcal{E}$ is the energy, and the vector potential is given by $\mathbf{A} = \alpha \mathbf{B}_0$. Given δf, the hot particle contribution to the potential energy of the $n=1$ internal kink mode is then calculated as[16–18]

$$\delta W_h = \frac{1}{2} \int \mathrm{d}\Gamma (mv_\parallel^2 + \mu B) \delta f \sum_m \vec{\kappa} \cdot \vec{\xi}^{(m)*}(r,t) e^{-i(n\zeta - m\theta)}, \quad (2)$$

where θ is the poloidal angle, $\vec{\kappa} = \mathbf{b} \cdot \vec{\nabla} \mathbf{b}$ is the magnetic curvature vector, and $\mathbf{b} = \mathbf{B}/B$.

Previous modeling has concentrated primarily on the effects of the trapped fast particles.[12] However, the neutral beam heating in JET gives rise to a predominantly passing population. Recent analytic theory[19,20] has suggested that the copassing energetic particles can stabilize the 1/1 internal kink mode, whereas counterpassing fast ions can have a destabilizing influence. These effects are modeled in general tokamak geometry for the first time, and are found to be important for analyzing the asymmetric dependence of sawtooth stability with respect to the direction of the NBI. Wang *et al.*[20] proposed that the nonadiabatic passing ion effects arose due to the gradient $\vec{\nabla} f_h$ integrated over the $q=1$ radius. In contrast, Graves[19] suggested that the nonadiabatic passing particle effects are counteracted by an adiabatic contribution, but that an additional adiabatic contribution survives from the fast ions which intersect the $q=1$ flux surface. This latter mechanism depends on $\partial f_h / \partial P_\zeta$ at $q=1$ only, and is more sensitive to localized heating. Figure 1 shows the passing particle contribution to δW_h in a typical JET discharge for a nonsymmetric fast ion distribution, which is Maxwellian with respect to energy and Gaussian with respect to pitch angle. The distribution function is artificially taken to be zero outside a finite radius, indicated on the x axis. It is evident that when no gradient exists in a region around $q=1$ ($s = \sqrt{\psi} \sim 0.3$) bounded by the orbit width, the passing ions do not contribute to the kink mode stability. The fact that unbalanced passing ions contribute only via a radial gradient in f_h close to $q=1$ has important implications for sawtooth control in ITER using negative-ion NBI heating at varying deposition radii.[19] The strong contribution of the passing particles comes from ions close to the trapped-passing boundary where their orbit widths, Δ_b, are large, $\delta W_h \sim \Delta_b$.

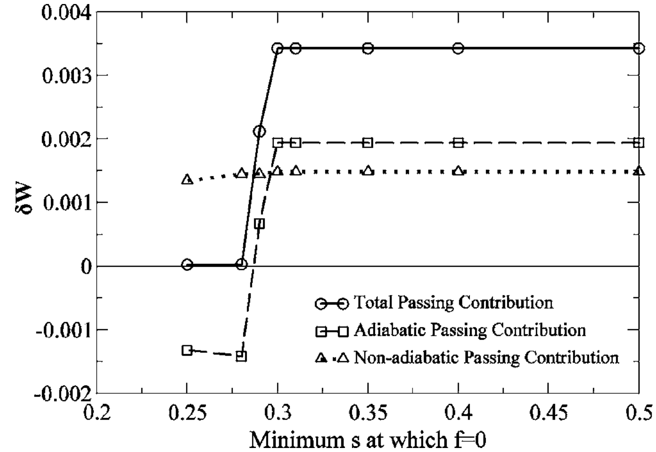

FIG. 1. The contribution to δW_h from passing ions for a hot ion distribution function nullified outside $s = \sqrt{\psi}$.

In order to analyze the JET experimental results[6] concerning sawtooth stability, HAGIS has been used to calculate δW_h. Together with the contribution from MHD effects including toroidal flow, calculated using MISHKA-F, the stability of the kink mode for a range of beam powers can be evaluated. The JET discharge which has been analyzed is shot 60998 (coinjected beams of 4.1 MW, $I_p = 2.3$ MA, $B_T = 2.5$ T, $\bar{n}_e = 3.2 \times 10^{19}$ m^{-3}, and $v_\zeta \sim 45$ km/s). The neutral beam current drive has been calculated[6] and found to be broadly deposited and less than 10% of the Ohmic current,

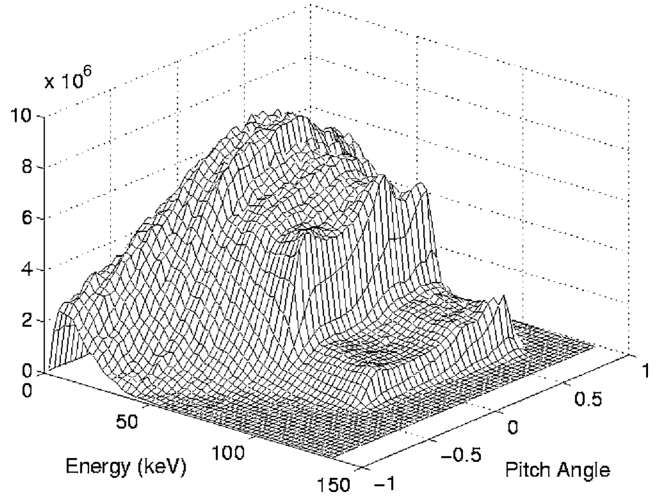

FIG. 2. The fast particle distribution function as a function of particle energy and pitch angle, $\lambda = v_{\parallel}/v$, at $r/a = 0.35$, $\theta = 0$. The distribution function is approximately Gaussian with respect to λ for high-energy particles, but approaches isotropy for low-energy particles. The distribution function dependence upon λ is also biased in terms of radial location.

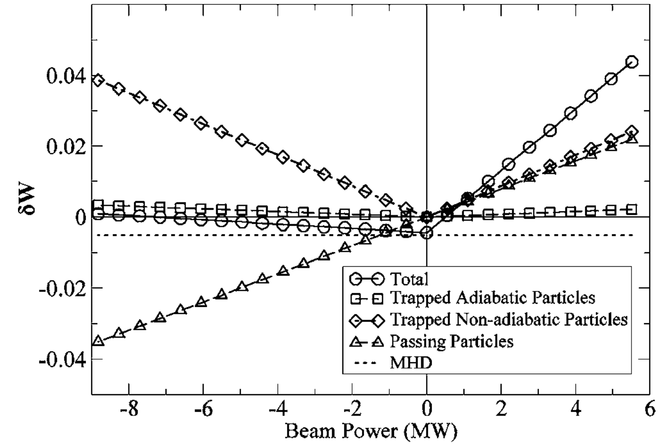

FIG. 3. The contribution to δW from each energetic particle species with respect to injected beam power for a JET equilibrium (discharge 60998) unstable to a 1/1 kink mode.

meaning it is relatively insignificant in these discharges. The equilibria are reconstructed using the HELENA code,[21] which takes as input the plasma shapes and q profiles from the EFIT equilibrium code[22] and the pressure profile obtained from the TRANSP transport code.[23] The position of the $q=1$ surface is constrained by the inversion radius found from the soft x-ray diagnostic. The pressure profile includes a contribution from the neutral beam fast particles, which are treated as isotropic at this stage. The equilibrium does not include toroidal flow or pressure anisotropy.

The fast particle distribution function was obtained from TRANSP. The exact distribution function is retained, though it can be described approximately as a slowing down distribution with respect to energy and a Gaussian distribution with respect to pitch angle, centered around $\lambda = v_{\parallel}/v = 0.5$. The dependence of the distribution function upon energy and pitch angle is illustrated in Fig. 2. By retaining the complete distribution function, the complicated dependence of the pitch angle distribution width upon the normalized poloidal flux and the particle energy is treated accurately. This is important since the degree of anisotropy of the fast particle distribution can significantly change the contribution of the trapped particles to δW_h.

The contribution to the stability of the $n=1$ internal kink mode from each class of particles for a static plasma is shown in Fig. 3. The potential energy is normalized in the same way as Ref. 14, such that $\delta \hat{W}_h = \delta W_h \mu_0 / (6\pi^2 R_0 \vec{\xi}_0^2 \epsilon_1^4 B_0^2)$, where $\epsilon_1 = r_1/R_0$ and $\vec{\xi}_0$ is the displacement at the magnetic axis. It can be seen that for the realistic beam distribution function employed in these simulations, the passing particles, which are often ignored in studies concerning energetic particles, are as important as the trapped particles. In accordance with analytic theory,[19] the copassing particles are strongly stabilizing, whereas the counterpassing particles give a destabilizing contribution which nearly balances the strong stabilization from the trapped population. Whilst only ideal stability is considered here, it has been shown[24] that including resistivity only slightly modifies the stability boundary and the instability drive is still from the asymmetric passing ions. The nonadiabatic trapped particles are always stabilizing,[25,26] and for the level of parallel anisotropy present in this distribution function, the adiabatic terms are only weakly stabilizing. The contribution from the passing particles helps to explain the different sawtooth behavior exhibited in JET when injecting the beam in different directions relative to the plasma current, since the $\vec{\nabla} B$ drifts of the energetic particles determine whether they are stabilizing or not. However, as shown below, only the inclusion of toroidal flow shear enables an understanding of the minimum in sawtooth period.

The effect of toroidal flow shear is modeled by prescribing the experimental toroidal rotation profile measured by the charge exchange diagnostic. The co- and counter-NBI profiles are very similar and are approximately linearly sheared with respect to poloidal flux, ψ. Whilst the absolute values of the flows are only relatively small, there are strong toroidal flow *shears* present in JET, which can modify the stabilization of the energetic ions.[27] The effect of sheared rotation on both $\Re(\delta W_h)$, which quantifies the stabilizing effect of the fast ions, and $\Im(\delta W_h)$, agrees well with Fig. 6(b) of Ref. 14.

Conservation of the third adiabatic invariant, Φ, which produces strong stabilization from trapped fast particles,[17] is only obtained[27] when $\langle \omega_d \rangle + \Delta\Omega - \tilde{\omega} \gg 0$, where $\langle \omega_d \rangle$ is the bounce-averaged hot particle toroidal drift precession frequency. Since this condition is more readily satisfied for $\Delta\Omega > 0$, corotating plasmas with velocity shear support more effective stabilization of the kink mode. Conversely, the stabilizing effect is diminished in counter-rotating plasmas ($\Delta\Omega < 0$), since Φ conservation is inhibited and the stabilizing contribution can only come from the fewer higher energy ions.

When the nonadiabatic effect of the trapped particles is modified by the sheared flow, the contribution to the stability of the internal kink mode changes significantly, as illustrated in Fig. 4. The modeled toroidal flow profile is given by the

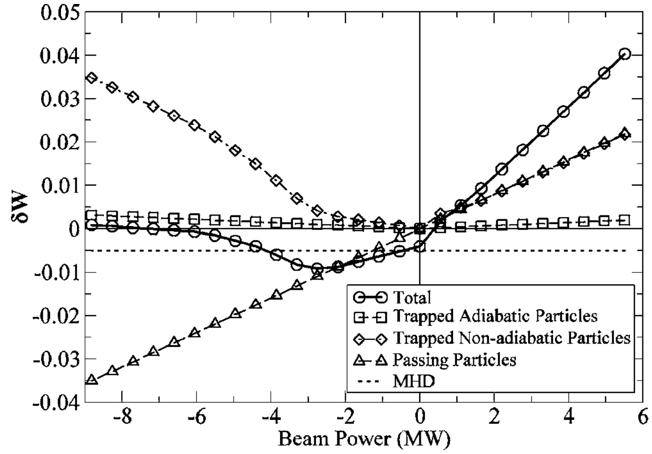

FIG. 4. The contribution to δW from each energetic particle species with respect to injected beam power for a JET equilibrium (discharge 60998) including flow shear effects.

charge exchange diagnostic and is scaled linearly with beam power. The Ohmic mode frequency, $f=\omega_{*i}\sim 0.5$ kHz, is of the same order as the toroidal plasma rotation frequency. The minimum in mode stability occurs at approximately 3 MW of counter-NBI power, in excellent agreement with the minimum in sawtooth period exhibited experimentally.[6] This minimum arises because (i) the flow shear in JET reduces the stabilizing effect of the trapped ions injected countercurrent and (ii) the counterpassing ions provide a strongly destabilizing contribution. In corotating plasmas the sheared flows amplify the stabilizing contribution from the trapped ions and the copassing ions are also strongly stabilizing.

We present thorough modeling of JET discharges, which exhibit an asymmetric dependence of 1/1 mode stability upon the direction of neutral beam heating. The effects of anisotropy and flow shear have been included for both adiabatic and nonadiabatic trapped and passing fast particles. It is found that the passing ions, which represent the majority of energetic particles injected into JET through NBI heating, are strongly stabilizing when injected in the same direction as the plasma current, but destabilizing for counter-NBI. For co-NBI, the stabilizing role of the nonadiabatic trapped ions is increased when flow shear is included. Conversely, for counter-NBI, the trapped particles are less stabilizing. This means that the plasma is most unstable to $n=1$ kink modes when heated with countercurrent neutral beams, in excellent accordance with the experimental data. By considering the effects of toroidal flow shear and pressure anisotropy, a complete model now exists, which can accurately simulate $n=m=1$ mode stability including kinetic effects in general toroidal geometry.

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