Relaxation revisited

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REVIEW ARTICLES

Relaxation revisited*

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Relaxation is the result of turbulence in a plasma that behaves essentially as an ideal conducting fluid, but has a small resistivity and viscosity. These small effects are locally enhanced by the turbulence and lead to reconnection of magnetic field lines. This destroys an infinity of topological constraints, leaving only the total magnetic helicity as a valid invariant. The plasma therefore rapidly reaches a specific state of minimum energy. This minimum energy ''relaxed state'' can be calculated from first principles and has many striking features. These depend on the topology of the system. They include spontaneous field reversal, symmetry-breaking and current limitation in toroidal pinches, and flux generation and flux amplification in Spheromaks. In addition the relaxed states can be controlled and maintained by injection of helicity from an external circuit. These features, and the profiles of the relaxed states themselves, have been verified in many laboratory experiments. [S1070-664X(00)90505-6]

I. INTRODUCTION

The theory of relaxation[1,2] has been remarkably successful in explaining the behavior of magnetized plasmas in many experiments. In this talk I shall show how relaxation is a consequence of plasma turbulence and field line reconnection, but is controlled by the helicity and topology of the magnetic field. The mathematical (and other) details are available elsewhere[3] so I shall concentrate on the underlying concepts and the results of the theory.

The idea of relaxation arose from the study of the Toroidal Pinch. This is one of the simplest plasma confinement systems. In principle one has just a toroidal solenoid that is also the single-turn secondary of a transformer (Fig. 1). The usual method of operation is first to set up a toroidal field B by energizing the solenoid. Then, after creating an initial plasma, one induces a toroidal plasma current I by a pulse from the transformer. This current heats the plasma and its magnetic field compresses (''pinches'') the plasma towards the axis of the solenoid. (For a review of toroidal pinches, see Ref. 4.)

When these experiments were first carried out they revealed several remarkable common features. Initially the plasma is highly turbulent, as one might expect, but it then settles into a more quiescent state. In this quiescent state the magnetic field configuration is universal, independent of the particular experiment. In fact the quiescent state depends only on a single parameter, the pinch ratio $\theta = 2I/aB$, where a is the minor radius of the torus. Most remarkably of all, if θ is greater than a critical value ~ 1.2, *the toroidal magnetic field is spontaneously reversed* (relative to the initial field) in the outer region of the plasma!

II. THE RELAXED STATE

A. Concept

It is clear from the behavior outlined in Sec. I that during the initial turbulent phase, the plasma seeks out its own preferred configuration—now known as the ''*relaxed state.*''

The idea of a relaxed state can be illustrated by a simple analogy. Suppose a loop of flexible current carrying wire is immersed in a viscous fluid and released. Initially it will move in response to its own magnetic field, but what configuration does it have when it comes to rest?

So long as the wire is moving, energy is being dissipated, so it must come to rest in a configuration of minimum energy—but this will be the minimum energy subject to whatever constraints there are on its motion. Some constraints reflect properties of the wire, but there is also a magnetic constraint; if the wire is highly conducting the magnetic flux through the loop cannot change. This flux is therefore an *invariant* and the relaxed state has minimum energy at fixed magnetic flux. (For a single loop this is also the configuration of maximum inductance.)

Now, the plasma resembles an infinite number of interlinked wire loops and to apply a similar argument we must first identify the plasma constraints. To do this we write the magnetic field $\mathbf{B}$ in terms of the vector potential, $\mathbf{B} = \nabla \times \mathbf{A}$. Then, if the plasma were *perfectly* conducting, as to a good approximation it is, $\mathbf{A}$ must satisfy

$$\frac{\partial \mathbf{A}}{\partial t} = \mathbf{v} \times \mathbf{B} + \nabla \chi. \tag{1}$$

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†Maxwell Prize Speaker.

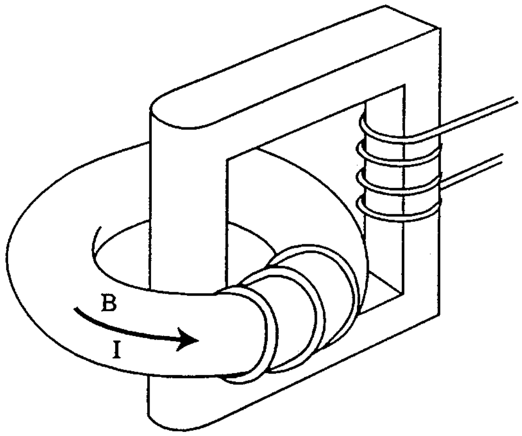

FIG. 1. Basic Toroidal Pinch Experiment.

Despite the arbitrary gauge χ, and no matter what the turbulent velocity $\mathbf{v}$, this equation imposes constraints[1,2] on changes in the potential $\mathbf{A}$. A convenient way to express these is that for every infinitesimal flux tube V_i the quantity,

$$K_i = \int_{V_i} \mathbf{A}\cdot\mathbf{B}\,d\tau \tag{2}$$

is an invariant. This infinity of invariants replaces the single invariant of the wire loop. Of course, these invariants are related to the well-known property that the magnetic field is ''frozen in'' to a plasma. But whereas that property implies a constraint on *every* plasma element, the constraints on $\mathbf{A}$ apply only to elements defined by flux tubes, and these are the only constraints.

Unfortunately, when we calculate the state of minimum energy with respect to variations in $\mathbf{A}$ subject to this infinite number of constraints, it bears no resemblance to what is seen in experiments.

B. Implementation

The way out of this difficulty, and the crucial step in the theory,[1,2] is to recognize that the invariants K_i are a mathematical idealization, relevant only if field lines remain intact and can be identified. (So that one knows, e.g., to which field line each invariant belongs!) *This is impossible in a turbulent resistive plasma* because resistivity allows field lines to break and reconnect. Furthermore, they can do so rapidly (compared to the resistive diffusion time) because the effect of resistivity, however small, is enhanced at local concentrations of current created by the turbulence. (Note that *both* turbulence and resistivity are required for this process and that although reconnection has a global effect, by destroying the integrity of extended field lines, it requires only small local changes in the field.)

As a result of the breaking of lines of force, the infinity of invariants K_i become irrelevant. However, the sum of all the K_i, that is the *total* magnetic helicity,

$$\sum K_i \equiv K_0 = \int_{V_0} \mathbf{A}\cdot\mathbf{B}\,d\tau \tag{3}$$

(where V_0 is the total volume of the plasma), does not depend on the integrity of field lines. It therefore remains a valid invariant so long as the resistivity is small. (One might say that the total helicity is continually redistributed among the field lines as they are broken up by the turbulence.) We conclude therefore, that for a turbulent, slightly resistive plasma, there is only a single effective invariant—the total magnetic helicity K_0.

The configuration of lowest energy subject to this single constraint on $\mathbf{A}$ is easily found. It is one in which the current density in the plasma is aligned with the magnetic field and proportional to it, i.e.,

$$\mathbf{J} = \mu\mathbf{B}, \tag{4}$$

where μ is a constant. Thus we see that the relaxed state does indeed depend only on a single parameter μ— just as the observed quiescent state depends only on the single parameter θ. In fact, θ and μ are equivalent, $\theta = \mu a/2$.

Later I will discuss the properties of the relaxed state, but first I would like to say something about the magnetic helicity.

III. MAGNETIC HELICITY AND GAUGE INVARIANCE

The helicity within a flux tube is well defined, *but one cannot say where within that flux tube the helicity is located*! Nor can one say how much helicity is within any region that is not a flux tube. Mathematically this is because $(\mathbf{A}\cdot\mathbf{B})$ is not gauge invariant, although its integral within a flux tube is. (Gauge invariance should not be confused with the ''dynamical'' invariance discussed in Sec. II.) However, a more instructive explanation is that helicity is a *topological* quantity, like the linkage of two hoops. Whether or not two hoops *are* linked is a perfectly valid question, but one cannot ask *where* the linkage is located! In fact, the linkage of hoops is more than just an analogy; helicity *is* a measure of the linkage of magnetic lines of force. One should not, however, assume from this that magnetic helicity is an intangible mathematical notion. In fact it is closely related to the ''volt-seconds'' ($V\,s$) in the discharge—a very solid engineering quantity.

IV. PROPERTIES OF RELAXED STATES

Now I would like to return to the relaxed state itself, which satisfies

$$\nabla\times\mathbf{B} = \mu\mathbf{B} \tag{5}$$

and appropriate boundary conditions. At a conducting boundary the normal component of $\mathbf{B}$ is fixed and for the present we take it to be zero. This ensures that the helicity is well defined (see Sec. III). It also means that the toroidal flux ψ is a constant.

For simple systems it is not difficult to calculate the relaxed state from Eq. (5). For example, in a large aspect ratio circular cross-section torus, where we can take the cylindrical limit, the appropriate solution is

$$B_r = 0, \quad B_\theta = \alpha J_1(\mu r), \quad B_z = \alpha J_0(\mu r). \tag{6}$$

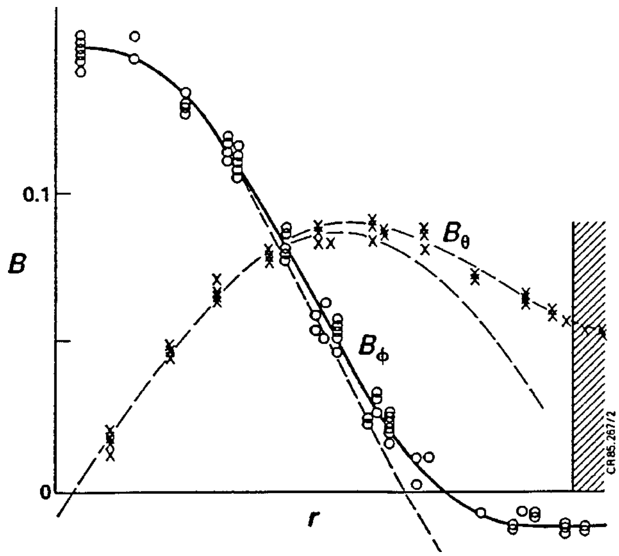

FIG. 2. Experimental and theoretical field profiles in the HBTX-1A (from Ref. 4).

This is the well-known ''Bessel function'' configuration. The ratio K_0/ψ^2 determines the parameter μ and the amplitude α is then determined from either the flux ψ or the helicity K_0. Thus the relaxed state is completely determined by the invariants K_0 and ψ. As we will see, this is a general result; there are no arbitrary or fitted parameters in the calculation of relaxed states.

The calculated field profile agrees well with what is observed experimentally, as shown for example in Fig. 2 from the High Beta Toroidal Experiment (HBTX).[4] Furthermore, we can see from Eq. (6) that spontaneous field reversal should occur at $\mu a = 2.4$, corresponding to a pinch parameter $\theta = 1.2$. This is in remarkably good agreement with the measured value.

So the theory accounts well for the early experiments. However, I want now to turn to some unexpected consequences of the theory.

To explain these I have first to admit that calculation of the relaxed state is considerably more complicated than I have implied. This is because there is actually an infinite number of solutions of Eq. (5) that satisfy the boundary conditions and have prescribed values of the invariants K_0 and ψ. One must select the true minimum energy state from these possibilities!

When this selection has been made it turns out that there are just two possible forms of relaxed state in any torus.[1–3] In the large aspect ratio torus one of these is the ''primitive,'' axisymmetric, Bessel-function solution already mentioned. This correctly describes the relaxed state so long as μa is less than 3.2 (i.e., the pinch ratio $\theta < 1.6$). However at $\mu a = 3.2$, which is an eigenvalue of Eq. (5) associated with zero toroidal flux, this axisymmetric solution ceases to have the lowest energy and the relaxed state is then given by a different solution. This is a superposition of the primitive solution and a *helical* eigenfunction. (Note that this state is not axisymmetric so that the transition from one relaxed state to the other is an example of spontaneous symmetry breaking.) It

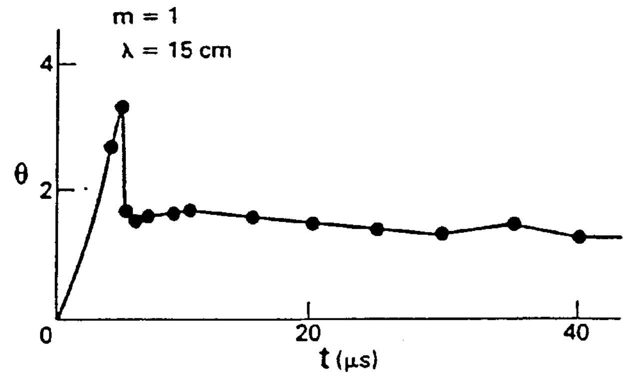

FIG. 3. Current limitation in the HBTX-1 (from Ref. 4).

must be emphasized that, unlike a linear mode, the amplitude of the helical component in the relaxed state is not arbitrary. The helical relaxed state is again completely specified by the invariants K_0 and ψ. (The quantity K_0/ψ^2 now determines the relative amplitude of the helical deformation, μ is fixed by the eigenvalue and K_0 or ψ determines the overall amplitude as before.)

In the ''primitive'' axisymmetric relaxed state the plasma current increases with the voltage (or more strictly, the V s) applied to the discharge, as one would expect. However, a remarkable feature of the helical relaxed state is that, for any given toroidal flux, *the toroidal plasma current is fixed and independent of the V s applied to the discharge*. (This follows from the fact that μa is a fixed eigenvalue and can be interpreted in the following way: in a higher V s discharge the helical deformation is larger; this increases the circuit inductance and generates a ''back-emf'' that annuls the increased voltage and leaves the plasma current unchanged.)

The existence of this limiting fixed current has been confirmed by experiment. An illustration is shown in Fig. 3, again from the HBTX.[4] This is taken from an experiment in which a very large loop voltage was applied. As one can see, this initially forced the plasma current, and therefore θ, well above the critical value, but the plasma then quickly relaxed and θ fell to near the calculated maximum value, $\theta = 1.6$, and remained there for the rest of the discharge.

V. MULTIPINCH

The phenomena of symmetry breaking and current limitation are even more strikingly demonstrated in another experiment, known as the *Multipinch*.[5] Like the previous experiments this is basically an axisymmetric toroidal solenoid that also forms the secondary of a pulse transformer, and it is operated in exactly the same way. Its novel feature is that the minor cross section of the solenoid is strongly noncircular. Instead it has a ''figure-eight'' form with a narrow waist (Fig. 4).

The relaxed state for such a torus can be calculated[5] from Eq. (5) just as for the circular cross-section device. As in that case, we find that below a critical value of $\mu \sim 2.21$ the relaxed state of the Multipinch is axisymmetric, and the plasma current increases with V s. However, when μa

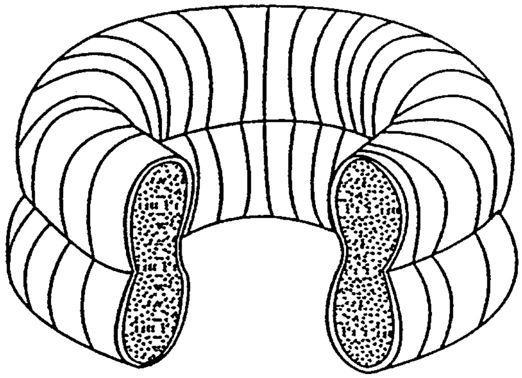

FIG. 4. Multipinch (general atomics).

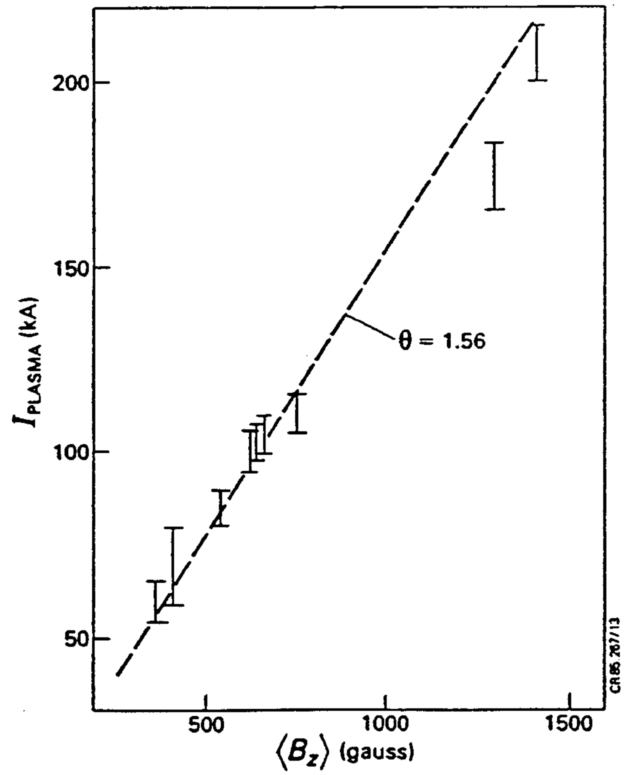

FIG. 6. Variation of the limiting current with toroidal flux in Multipinch (from Ref. 5).

reaches 2.21, which is a zero-flux eigenvalue for the Multipinch, there is a symmetry-breaking transition to a state in which the toroidal current no longer increases with voltage. But in the Multipinch the symmetry that is broken is not the continuous symmetry around the torus; it is the discrete ''up-down'' symmetry between the upper and lower lobes of the ''figure-eight'' cross section! Below the transition the current in each of the two lobes is the same and increases with voltage. Above the transition an increase in voltage produces more current in one lobe but less in the other, and leaves the total toroidal current unchanged.

This current-limiting behavior can be clearly seen in the Multipinch experiments.[5] Figure 5 shows the plasma current measured as a function of the transformer voltage V_{CB} (roughly equivalent to the V s in the discharge). At low voltages the plasma current, which is equally shared between the two lobes, increases with V_{CB}. At higher voltage the current ceases to increase with voltage and is no longer equal in the two lobes. (One might ask why the current begins to rise again at the high voltage end of the plateau. This occurs when the imbalance between the two lobes has become so large that essentially *all* the current is in one lobe and none in the other. Beyond this point the plasma is contained entirely in one lobe of the cross section.)

It can also be seen from Fig. 5 that the limiting current depends on the average toroidal field, i.e., on the flux ψ, as it should according to the theory. The variation of the limiting current with the toroidal flux is shown in Fig. 6. The line $\theta = 1.56$ corresponds to $\mu a = 2.42$, in good agreement with the predicted value $\mu a = 2.21$.

Thus we see that both the conventional circular cross-section torus and the Multipinch show a symmetry breaking transition to a current limited state, but in the former it is axisymmetry that is broken whereas in the latter it is the ''up–down'' symmetry. The switch from one form of symmetry breaking to the other depends on the exact cross-section, particularly the width of the ''waist.''[6]

One might now ask what would be the behavior of a system that has no symmetry? In a general toroidal system the theory[3,7] also predicts a transition from a relaxed state in which the current increases with V s to one in which the current is fixed. However, the transition from one state to the other becomes more gradual as the system departs further from axisymmetry and, of course, it can no longer be identified with spontaneous symmetry breaking.

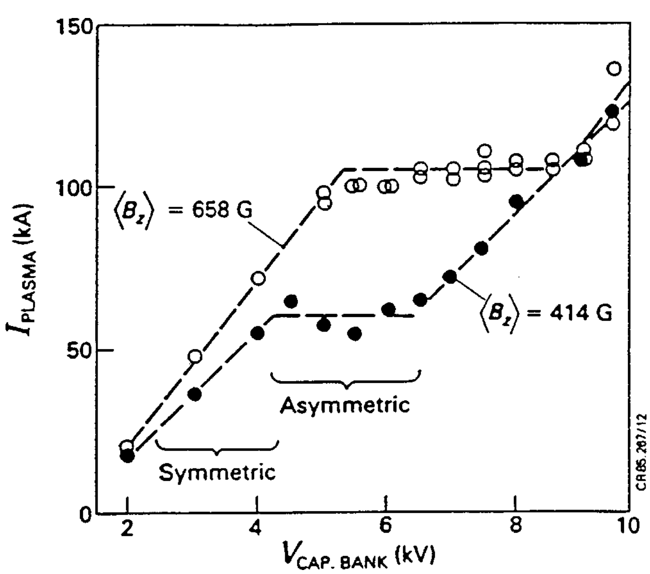

FIG. 5. Current limitation in multipinch. (from Ref. 5).

VI. SPHERICAL DEVICES

So far I have considered only toroidal systems. However, the properties of relaxed states depend strongly on the topology of the experiment.[8] This becomes clear when we consider relaxation in a spherical container. Such experiments are known as ''*Spheromaks*''[9] and are illustrated in

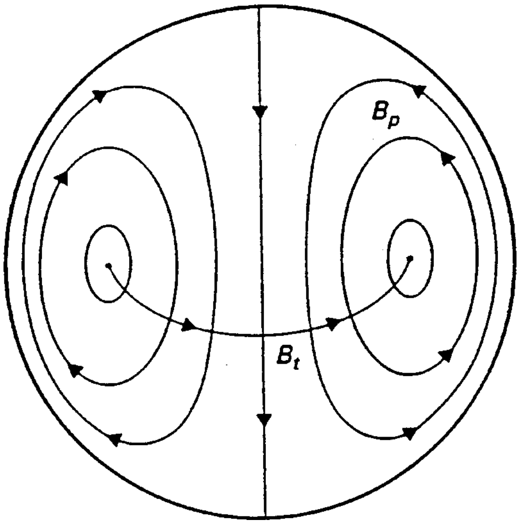

FIG. 7. Spheromak, schematic.

Fig. 7. The quiescent magnetic field has the usual toroidal surfaces but the containing vessel has no central aperture; i.e., there is no ''hole in the doughnut'' and consequently no solenoid or pulse transformer. Topologically speaking, a Spheromak is a simply connected region, as distinct from the multiply connected region of the toroidal systems. It is this feature that distinguishes the two classes of experiment, not any particular shape of the bounding vessel. For a review of Spheromak research, see Ref. 10.

Because there *is* no solenoid or pulse transformer, it is difficult to create high temperature plasmas in Spheromaks. However, one successful method is to inject the plasma from a coaxial plasma ''gun''[10,11] as shown in Fig. 8. Magnetic forces eject a plasma from the gun into a ''flux conserver'' where reconnection occurs and it relaxes to the Spheromak configuration.

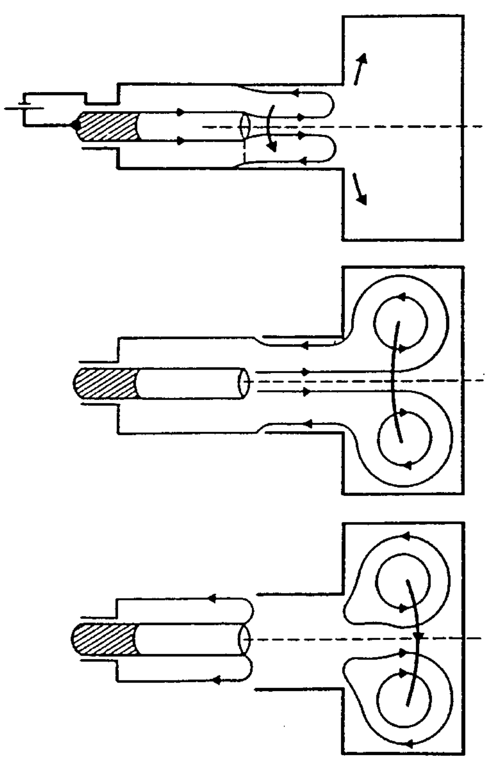

FIG. 8. Spheromak formation by the coaxial gun (from Ref. 11).

As in the toroidal case, helicity is conserved in a Spheromak and the relaxed state satisfies Eq. (5). However the change to a simply connected domain completely alters the calculation. In a torus there are two invariants, K_0 and ψ, and these specify the relaxed state. But toroidal flux is not conserved in a Spheromak; it can be annihilated or created at the axis of symmetry. Consequently there is only one invariant. On the other hand, because the plasma domain is simply connected *there is now only one solution of Eq. (5) that can represent a relaxed state*. This corresponds to the lowest eigenvalue μ_{cs} of Eq. (5) for the given flux-conserver. Consequently, the profile of the magnetic field in a Spheromak is determined entirely by its flux conserver; the invariant K_0 serves only to determine the magnitude of the field.

The magnetic fields observed in Spheromaks agree well with those calculated for the relaxed state. However, the feature of these experiments that I want to emphasize is that the field is actually generated during relaxation. This is illustrated in Fig. 9, from the S1 experiment,[12,13] which shows the evolution of the poloidal and toroidal fields. It can be seen that during the relaxation phase, poloidal flux is destroyed [Fig. 9(a)], and toroidal flux is created [Fig. 9(b)], until their ratio, represented by the pitch q of field lines near the magnetic axis, reaches the calculated value 0.65 [Fig. 9(c)]. (The generation of magnetic fields by turbulence, as demonstrated in these experiments, is sometimes called the *dynamo effect* and has a long history in connection with research on the earth's magnetic field.)

VII. HELICITY INJECTION

So far we have considered behavior resulting from the invariance, i.e., *conservation*, of helicity. But, of course, before helicity can be conserved it must be created. In the experiments described so far this occurs spontaneously, in a rather uncontrolled way, as the plasma is formed. However, helicity can also be created in a controlled fashion. This is often known as ''*helicity injection*.'' The basic principle is that if a voltage V is applied between electrodes that are linked by a common magnetic flux ψ, then helicity is created[14] between the electrodes at a rate

$$\frac{dK}{dt}=2V\psi. \tag{7}$$

A conceptually simple form of helicity injection is obtained by modifying a Spheromak to have a core of flux passing along its axis from the North to the South pole. This creates the Flux-core Spheromak,[15] Fig. 10. Then a voltage applied between electrodes at the poles can ''inject'' helicity into the system according to Eq. (7).

As the boundary of a Flux-core Spheromak is not a flux surface, the helicity is not immediately well-defined (see Sec. III). One way to rectify this[16] is to imagine that the flux leaving and entering the boundary is extended throughout the exterior as a vacuum field. Then the total helicity $\int \mathbf{A}\cdot\mathbf{B}$,

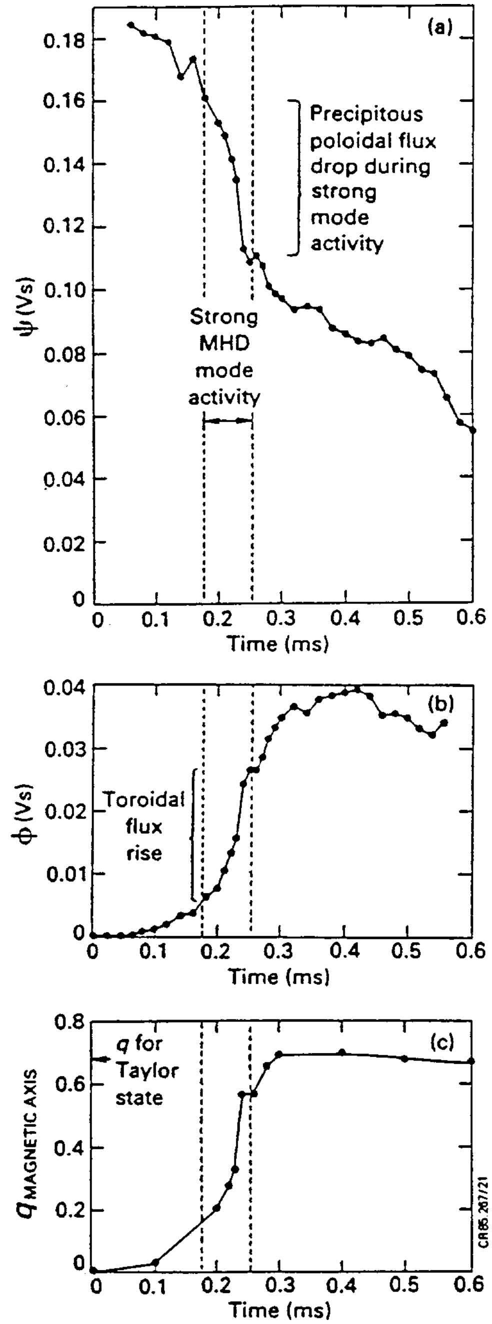

FIG. 9. Evolution of the flux in the S-1 Spheromak (from Refs. 12 and 13). (a) Evolution of poloidal flux. (b) Evolution of toroidal flux. (c) Ratio of toroidal/poloidal flux.

inside *and outside*, the boundary *is* well-defined. If the boundary is a conductor, changes in the interior field do not affect the exterior. Then the difference in helicity between two configurations that differ inside the Spheromak but have identical normal components at the surface (and hence identical hypothetical extensions outside) may be taken as their relative helicity. This is well-defined and gauge invariant and plays the same dynamical role as K_0.

The need to include the contribution to $\int \mathbf{A}\cdot\mathbf{B}$ from the exterior field in the relative helicity, even though the exterior field does not change, reflects the fact that helicity is not a local quantity. Other definitions of relative helicity have been proposed[10,14,17] that involve only fields in the interior, but these do not bring out the connection with the nonlocal nature of helicity.

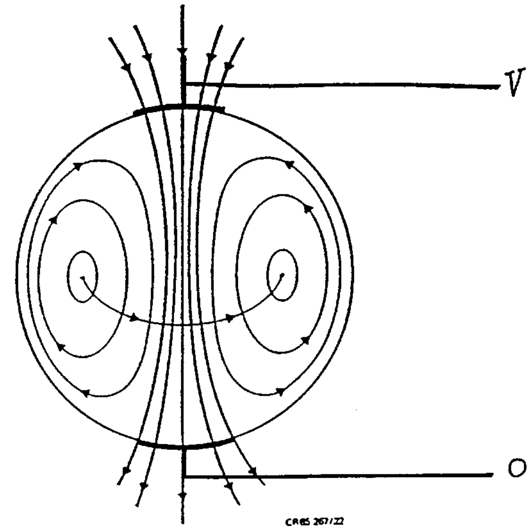

FIG. 10. Flux-core spheromak, schematic.

In a Flux core Spheromak the relaxed state again satisfies Eq. (5) but, unlike the simple Spheromak, the relaxed state is no longer unique. It is determined in different ways depending on the mode of operation.[3] In the helicity injection mode the current I_z and flux ψ_z through the polar caps are maintained by external circuits. Then μ is given by I_z/ψ_z and the relaxed state can be controlled, and, in principle, maintained indefinitely against resistive decay. Of course, it is not surprising that a plasma can be maintained by voltage and current between electrodes! The remarkable feature of helicity injection is that an *axial* voltage maintains an *equatorial* current. This is possible only because of turbulent relaxation. Another surprising feature is that if the external circuits are adjusted so that I_z/ψ_z approaches the eigenvalue μ_{cs} for the given flux conserver, then the ratio of flux in the plasma to the flux ψ_z through the electrodes, increases indefinitely. In practice a flux amplification of ~ 5 has been observed and higher values have been inferred.[18]

The principle of helicity injection illustrated in Fig. 10 is not limited to Spheromak-like devices; it can equally be applied to the toroidal pinch or the Tokamak.[15,19,20] In this case too the discharge can be controlled and maintained without toroidal voltage (i.e., ''noninductively''), and the flux again increases indefinitely as μ approaches a critical value.

However, a different, and more remarkable, form of helicity injection has been demonstrated in a Tokamak. This involves mixing of a spherical and a toroidal plasma. In an experiment[21] a small Spheromak is created in a coaxial gun and injected into a Tokamak discharge—much as one is injected into a flux-conserver in Fig. (8). This Spheromak plasma then merges into the larger Tokamak plasma and the effect on the Tokamak current is observed.

To understand the results of the experiment we must recall that helicity is a pseudoscalar, i.e., it has a right- or left- ''handedness.'' (For example, the helicity in the Tokamak plasma is reversed if the relative directions of its initial

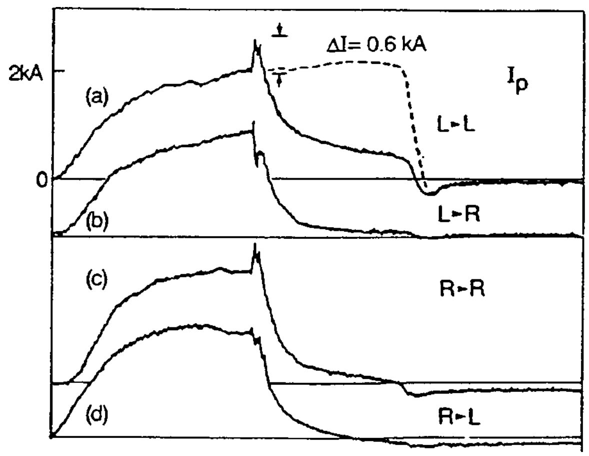

FIG. 11. Effect of spheromak injection on the Tokamak current (from Ref. 21). (a) Left-handed spheromak into the left-handed Tokamak. (b) Left-handed into right-handed. (c) Right-handed into right-handed. (d) Right-handed into left-handed.

magnetic field and current are reversed.) Thus there are four permutations of the experiment; a left- or right-handed Spheromak injected into a left- or right-handed Tokamak.

The effect of injection on the Tokamak current in the four cases is shown in Fig. 11. (The arrow shows the time of injection and the broken line is the behavior if there is no injection.) It can be seen that when injection is ''favorable,'' i.e., when the Spheromak and Tokamak have the same handedness, as in cases (a) and (c), it produces a small rise in the toroidal current. Conversely, when the injection is ''unfavorable,'' as in cases (b) and (d), it produces a drop in the toroidal current. (In all cases the initial change is followed by collapse of the current due to the influx of cold plasma and gas from the gun.) This is a striking demonstration of the important role of helicity in magnetized plasmas.

VIII. SUMMARY AND CONCLUSIONS

In a turbulent plasma, field line reconnection occurs rapidly because the effect of resistivity, however small, is enhanced at local concentrations of current. This destroys an infinity of topological constraints that exist in an ideal plasma, leaving only a single robust invariant, the magnetic helicity. As a result a turbulent plasma rapidly reaches a unique lowest energy configuration, the relaxed state.

Note that the theory of relaxation is not a variational principle such as those in classical mechanics. In mechanics the variations one considers are *virtual*; but in relaxation they are real and due to turbulence. Consequently, if a plasma is not turbulent it need not relax.

The relaxed state itself exhibits many remarkable features, which depend on the topology of the system. In a toroidal system these include spontaneous field reversal, symmetry breaking and current limitation—and the relaxed state can be maintained without a toroidal voltage. In a spherical system they include the generation and amplification of flux and the formation of a unique profile controlled solely by the shape of a flux conserver.

All the features predicted for the relaxed state agree remarkably well, both qualitatively and quantitatively with what is seen in experiments. But of course the agreement is not perfect! Some discrepancies are due to the fact that relaxation is incomplete because of excessive dissipation in cold plasma near boundaries. In this respect it is interesting that plasma fluctuations are substantially reduced if an external current drive is used to bring the current profile closer to the fully relaxed one.[22]

Finally, I should like to emphasize that the relaxed state in any system is fully determined and can be calculated completely from first principles, i.e., without empirical or fitted parameters. This is, of course, an unusual achievement in the field of turbulence!

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