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A rotating shell and stabilization of the tokamak resistive wall mode

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The finite resistivity of the wall that surrounds any toroidal plasma confinement device can lead to a branch of instabilities known as the resistive wall mode (RWM). Theory indicates that the RWM is potentially activated whenever the plasma equilibrium is unstable with the wall placed at infinity. In particular, advanced tokamak power plant designs require the plasma β to be above the critical value for this condition to be satisfied. Accordingly, it is important to find a method of stabilizing this mode. In this work we describe a method of stabilizing the tokamak RWM that utilizes a secondary rotating conducting shell surrounding the plasma and first wall. This scheme was first thought of for the reversed-field pinch, but must be reexamined for the tokamak as the mode involved has different characteristics. It is shown that provided the second wall is suitably positioned, RWM stabilization of a tokamak is possible even in the absence of plasma rotation.

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I. INTRODUCTION

It has long been known that if an ideal plasma is unstable in the absence of a wall, then surrounding the plasma with a finitely conducting wall does not alter the equilibrium stability boundaries.¹ The instability that arises has become known as the resistive wall mode (RWM), and is a generic threat to many toroidal plasma confinement devices. Subsequently, much attention has been given to the effect of bulk plasma rotation on the RWM.² Clearly, if the RWM perturbation travels with the plasma then the classical skin effect at the wall would inhibit flux penetration and the wall would appear as highly conducting. On the other hand, if the mode ‘locks’ to the wall despite the plasma rotation then flux penetration would occur and the RWM will continue to grow, albeit initially with a small growth rate. Further, the mode exerts a retarding torque on the plasma, which eventually undergoes a ‘catastrophic’ deceleration while the growth rate suddenly increases.³ This event, which is analogous to the phase changes that occur in a van der Waal’s gas,⁴ has been tentatively observed experimentally.⁵ So, the generation of bulk plasma rotation appears to be an unreliable way of stabilizing the RWM although it does have the effect of suppressing the growth rate.

In the context of the reversed-field pinch (RFP), where the RWM was first identified,^{6,7} it has been proposed that a secondary rotating conducting shell would stabilize the RWM in that device.⁸ (The suggestion was motivated by the TITAN power plant design that proposed using flowing lithium for the blanket⁹—it was later shown that a suitable configuration of external sensors and coils could ‘fake’ the existence of such a shell.¹⁰) The simple idea is that the RWM cannot simultaneously lock to both walls and so its behavior should be strongly affected. In fact, it was shown that pro-

vided the secondary wall was located inside the ideal marginal radius ($r=r_I$, the radial position at which a perfect wall has to be placed to give the ideal mode zero growth rate), the RWM was stabilized for wall rotations of the order of the (longest) inverse wall time. Now, in the RFP, the RWM is generally nonresonant (i.e., nowhere in the plasma does the pitch of the perturbation equal that of the equilibrium field lines). In this paper we revisit this calculation for the tokamak, where the relevant RWM is a different mode—the pressure-driven toroidal external kink. An essential ingredient of this mode is the presence of poloidal harmonics that are resonant in the plasma, so the calculation must take this into account. A cylindrical analog model of this mode was first formulated by Finn,¹¹ and we will use this as a basis. Finn used a cylindrical plasma that was ideally unstable (with no wall) but did possess a resonance. This requires somewhat artificial equilibria but provides a useful qualitative model of the actual Tokamak external kink. Our task is to incorporate a secondary rotating wall into the Finn model.

II. DERIVATION OF THE DISPERSION RELATION

We will use the tokamak ordering,¹² with $m^2 \gg k^2 r^2$, where m , k are the poloidal mode number and toroidal wave number, respectively. Further, we exclude cases with an internal $q=1$ resonance (q being the well-known equilibrium safety factor¹²). If such a resonance existed, then in cylindrical geometry the plasma would be unstable to an ideal internal mode. Hence we are restricted to $m > 1$.

The analysis we present relies on the judicious choice of basis functions that will make up the actual mode eigenfunction. In fact, we choose basis functions that have a direct physical interpretation. We recall that the problem divides naturally into ideal regions and ‘resistive’ layers (namely, the resonance at $r=r_s$ and the static and rotating walls at r_1 and r_2 , respectively). A second-order ordinary differential

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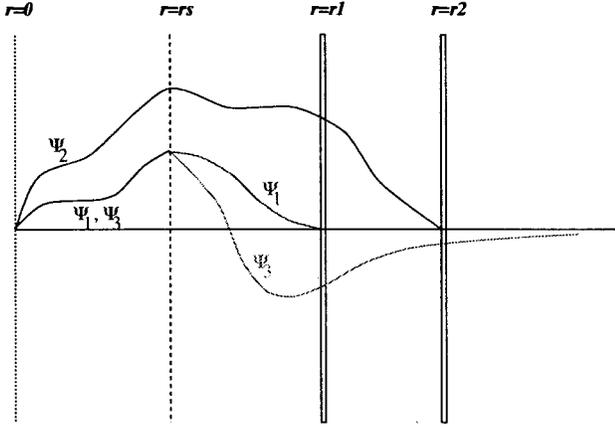


FIG. 1. Basis functions used in the RWM analysis.

equation (ODE) for the perturbed radial magnetic field, the Newcomb equation,¹³ connects the resistive layers and in these layers nonideal effects have to be taken into account. Figure 1 shows our choice of basis functions generated by the Newcomb equation. The basis function Ψ_1 represents the resistive plasma mode when the stationary wall at r_1 is taken to be a perfect conductor [the boundary conditions are then regularity at the origin $r=0$ and $\Psi_1(r_1)=0$]. Here Ψ_2 is, similarly, the resistive plasma mode when the secondary wall is ideal (and the first wall absent). For $r_s < r < \infty$, Ψ_3 is the plasma response when there is no wall present at all, and so has the boundary condition of vanishing as $r \rightarrow \infty$. Finally, for convenience, we choose Ψ_3 to be identical to Ψ_1 in $0 < r < r_1$.

In the following analysis we require to know the signs of $\Psi_{31} \equiv \Psi_3(r_1)$, and $\Psi_{32} \equiv \Psi_3(r_2)$. These can be established by considering the behavior of the small solution (in the Newcomb sense¹³) in (r_s, ∞) , which is clearly given by

$$\Psi_{sm}(r) = \Psi_1(r) - \Psi_3(r). \quad (1)$$

Now we want to ensure that the system displays an ideal RWM. In other words, the ideal mode must be stable if the first wall is perfect but unstable if there are no walls at all. This then implies that Ψ_{sm} vanishes at r_I , and we must have $r_1 < r_I < \infty$. It follows that

$$\Psi_{sm}(r) \propto \left[\left(\frac{r}{r_I} \right)^m - \left(\frac{r}{r_I} \right)^{-m} \right], \quad (2)$$

in the vacuum region, and so

$$\frac{r\Psi'_{sm}}{\Psi_{sm}}(r_1) = -m \frac{[1 + (r_1/r_I)^{2m}]}{[1 - (r_1/r_I)^{2m}]} < -m, \quad (3)$$

where ' denotes d/dr . However, we also have

$$\begin{aligned} \left(\frac{r\Psi'_{sm}}{\Psi_{sm}} \right)(r_1) &\equiv \left(\frac{r\Psi'_1}{-\Psi_3} \right)(r_1) + \left(\frac{r\Psi'_3}{\Psi_3} \right)(r_1) \\ &= \left(\frac{r\Psi'_1}{-\Psi_3} \right)(r_1) - m. \end{aligned} \quad (4)$$

So by comparing Eqs. (3) and (4) and using $\Psi'_1(r_1) < 0$, we deduce that Ψ_{31} is negative, and since $\Psi_3(\propto r^{-m})$ cannot change sign in (r_1, ∞) then Ψ_{32} is also negative. Therefore, Ψ_3 must exhibit the form as shown in Fig. 1.

The use of these ‘‘natural’’ basis functions ensures not only that the subsequent algebra is minimized, but that we can later relate some of the free parameters that arise to those that occurred in the single wall model.³ To start, we write the eigenfunction as a sum of the natural basis functions of Fig. 1:

$$\Psi = \Psi_1 + a\Psi_2 + b\Psi_3, \quad (5)$$

where we can choose the coefficient multiplying Ψ_1 to be unity, and can without loss of generality choose one convenient normalization for each of the $\Psi_{1,2,3}$. In fact, we choose

$$r_1\Psi'_1(r_1) = -1, \quad (6)$$

$$r_2\Psi'_2(r_2) = -1, \quad (7)$$

and

$$\Psi_3(r_s) = \Psi_1(r_s). \quad (8)$$

Because we require Eq. (5) to apply everywhere we must define $\Psi_1(r) \equiv 0, r > r_1$, $\Psi_2(r) \equiv 0, r > r_2$, and we note that Ψ_1, Ψ_2 have discontinuous derivatives at two points (r_s, r_1) and (r_s, r_2) , respectively, while Ψ_3 has a discontinuous derivative only at r_s .

Now at the second wall we assume a ‘‘thin shell’’ response² so that $\Delta'(p)$, the well-known jump in the logarithmic derivative of the perturbed radial field¹⁴ across the wall, is simply equal to $p\tau_2$, where we have assumed $\exp(pt)$ dependence and τ_2 is the ‘‘long’’ time constant of the second wall (i.e., the characteristic time for a vertical field to penetrate the wall— $\tau_2 = a\delta_w/\eta_w$ with a, δ_w , and η_w being the wall major radius, thickness, and resistivity, respectively). Accordingly, noting that Ψ_3 and Ψ'_3 are continuous through r_2 and using Eq. (7) we have

$$p\tau_2 = -\frac{a}{b|\Psi_{32}|}, \quad (9)$$

where $\Psi_{32} \equiv \Psi_3(r_2)$. Note that when the second wall is rotating with frequency Ω_2 we simply replace p in Eq. (9) by $(p - i\Omega_2)$. Similarly, at r_1 , Ψ_2, Ψ_3 and their derivatives are again continuous, and using Eq. (6) we find

$$p\tau_1 = \frac{1}{a\Psi_{21} - b|\Psi_{31}|}. \quad (10)$$

The last of the jump conditions occurs at the plasma resistive layer at r_s . For the moment we will not specify the plasma response, but symbolize it as $\Delta'_s(p)$. From first definitions we have

$$\Delta'_s(p) = \frac{r_s([\Psi'_1] + a[\Psi'_2] + b[\Psi'_3])}{\Psi_1 + a\Psi_2 + b\Psi_3}, \quad r = r_s. \quad (11)$$

Now recall that we chose $\Psi_{1,2,3}$ to represent the plasma stability properties when, respectively, the first wall is perfect, the second wall is perfect (the first being absent), and when there is no wall at all. So first we simply rewrite Eq. (11) as

$$\Delta'_s(p) = \frac{\Psi_{1s}\Delta'_1 + a\Psi_{2s}\Delta'_2 + b\Psi_{3s}\Delta'_3}{\Psi_{1s} + a\Psi_{2s} + b\Psi_{3s}}, \quad (12)$$

where the $\Delta'_{1,2,3}$ are the stability parameters for the three cases mentioned.

To proceed, we note that for $r \geq r_1$ we have vacuum fields and it is well known that in the tokamak ordering $\Psi \sim r^m, r^{-m}$ in such regions.¹² As we require $\Psi_{22}=0$ and $r_2\Psi'_2(r_2) = -1$ we easily find that

$$\Psi_{21} = \frac{r_2^{2m} - r_1^{2m}}{2mr_1^m r_2^m}. \quad (13)$$

For $r \geq r_1$, Ψ_3 can only be r^{-m} -like, and it follows that

$$|\Psi_{32}| = \left(\frac{r_1}{r_2}\right)^m |\Psi_{31}|. \quad (14)$$

Putting Eqs. (13), (14) into Eqs. (9), (10), we can solve for a and get b in terms of $|\Psi_{31}|$,

$$a = \frac{2mr_1^m r_2^m \tau_2}{[(r_2^{2m} - r_1^{2m})p\tau_2 + 2mr_2^{2m}]\tau_1}, \quad (15)$$

$$b = -\frac{ar_2^m}{p\tau_2 r_1^m |\Psi_{31}|}. \quad (16)$$

By design we have that $\Psi_{3s} = \Psi_{1s}$ and so Eq. (12) contains the unknown fluxes Ψ_{2s} and Ψ_{1s} only in the combination Ψ_{2s}/Ψ_{1s} . To solve for this combination we now use a generic property of any second-order ODE such as the Newcomb equation, namely that it simply converts the values of Ψ and Ψ' at one radial station to their values at another station via a real linear transformation; in particular, we can immediately write

$$\Psi_{1s} = c\Psi_{11} + d\Psi'_{11}, \quad (17)$$

$$\Psi_{2s} = c\Psi_{21} + d\Psi'_{21}, \quad (18)$$

$$\Psi_{3s} = c\Psi_{31} + d\Psi'_{31}, \quad (19)$$

for some real numbers c, d that are functionals of the mode numbers and the equilibrium fields in between r_s and r_1 . The right-hand sides of Eqs. (17) and (19) are $-d/r_1$ and

$-c|\Psi_{31}| + md|\Psi_{31}|/r_1$, respectively, while the right-hand side of Eq. (18) is known because, as remarked above, we can easily solve for Ψ_2 in $r_1 < r < r_2$. So, Eqs. (17)–(19) constitute the required linear algebra problem that will enable us to solve for Ψ_{2s}/Ψ_{1s} . We find

$$\frac{\Psi_{2s}}{\Psi_{1s}} = \frac{1}{\sqrt{Y}} \left(Y - \frac{1}{X|\Psi_{31}|} \right), \quad (20)$$

where

$$X = \frac{2m}{1-Y}, \quad Y = \left(\frac{r_1}{r_2}\right)^{2m}.$$

Note that using Eqs. (15), (16), and (20), and the fact that $\Psi_{1s} = \Psi_{3s}$, the dispersion relation, Eq. (12), now contains only one unknown flux, namely $|\Psi_{31}|$. This final unknown can be solved for when we realize that Ψ_1, Ψ_2 , and Ψ_3 are not independent in the plasma region (again because the governing Newcomb equation is of second order). Accordingly we can write

$$\Psi_2 = e\Psi_1 + f\Psi_3, \quad (21)$$

for some real constants e and f . Applying this relationship at $r = r_s$ gives

$$\Psi_{2s} = (e+f)\Psi_{1s}, \quad (22)$$

while differentiating Eq. (21) and evaluating at either side of $r = r_s$ gives

$$\Delta'_2 = \frac{e\Delta'_1 + f\Delta'_3}{(e+f)}. \quad (23)$$

Applying Eq. (21) directly at $r = r_1$ gives

$$\Psi_{21} = -f|\Psi_{31}|. \quad (24)$$

Now Eqs. (13), (22), (23), and (24) constitute another linear algebra problem from which we can deduce $|\Psi_{31}|$. In fact,

$$|\Psi_{31}| = \frac{1}{XY} \frac{(\Delta'_2 - \Delta'_3)}{(\Delta'_2 - \Delta'_1)}. \quad (25)$$

So now, using Eqs. (15), (16), (20), and (25) together with $\Psi_{1s} = \Psi_{3s}$, we find the basic dispersion relation,

$$\Delta'_s(p) = \frac{(\Delta'_2 - \Delta'_1)\Delta'_3 X^2 Y + (\Delta'_3 - \Delta'_1)\Delta'_2 X Y p \tau_2 + (\Delta'_3 - \Delta'_2)\Delta'_1 (X + p \tau_2) p \tau_1}{(\Delta'_2 - \Delta'_1) X^2 Y + (\Delta'_3 - \Delta'_1) X Y p \tau_2 + (\Delta'_3 - \Delta'_2) (X + p \tau_2) p \tau_1}. \quad (26)$$

To ensure that we are investigating a RWM, we must have a plasma equilibrium that is ideal-magnetohydrodynamic unstable in the absence of walls.^{1,2} This means that the Δ' of the ideal eigenfunction with no walls must display an ideal, inertial response at the resonance r_s , i.e., $\Delta' = -1/(p\tau_A)$ with τ_A the inertial layer Alfvén time.¹⁵ Now the ideal mode eigenfunction is given by $\Psi_3, r_s < r < \infty$, and $-\Psi_3, 0 < r < r_s$ (i.e., Ψ_3 passes through zero within the inertial layer at r_s , as an ideal mode cannot reconnect flux there). However,

note that the value of Δ' is unchanged by this sign reversal of Ψ_3 in $(0, r_s)$. So if we write $\Delta'_3 = -1/\epsilon$ then positive ϵ gives the (τ_A) normalized growth rate of the ideal mode in the absence of a wall. (The parameter ϵ appeared, with the same interpretation, in the earlier work on the single wall model.³) We also follow the notation of Ref. 3 by writing $\Delta'_1 = -\delta$, so for conventional tearing modes, say, positive δ would imply stability.¹⁴ Using this notation the dispersion relation (26) can be rewritten as

$$\Delta'_s(p) = \frac{X^2 Y (\delta + \Delta'_2) - \delta (1 + \epsilon \Delta'_2) (X + p \tau_2) p \tau_1 + X Y \Delta'_2 (1 - \epsilon \delta) p \tau_2}{-\epsilon X^2 Y (\delta + \Delta'_2) + (1 + \epsilon \Delta'_2) (X + p \tau_2) p \tau_1 + X Y (1 - \epsilon \delta) p \tau_2} \quad (27)$$

III. ANALYSIS OF THE DISPERSION RELATION

We choose the actual layer response to be ‘‘viscoresistive,’’¹⁶

$$\Delta'_s = p \tau_{VR}, \quad (28)$$

where $\tau_{VR} \sim \tau_A^{1/3} \tau_R^{5/6} / \tau_V^{1/6}$ and τ_R, τ_V are, respectively, characteristic layer resistive and viscous times. This response is appropriate to most tokamak plasmas.¹⁶ We recall that this choice of layer response is ‘‘pessimistic’’ in that in the case of the single wall problem, a viscoresistive layer response eradicated all RWM stability windows.³ With the choice of Eq. (28) as the layer response the dispersion relation Eq. (27) is a complex cubic [bearing in mind that we Doppler shift $p \tau_{VR} \rightarrow (p - i\Omega_{pl}) \tau_{VR}$ and $p \tau_2 \rightarrow (p - i\Omega_2) \tau_2$ to simulate plasma and second wall rotation]. We shall mainly investigate Eq. (27) numerically, but before that there is one analytic observation we can make.

If the RWM is to be stabilized by some combination of Ω_{pl} and Ω_2 then its growth rate must at some stage achieve marginality, i.e., $p = i\omega$ for real ω . Inserting this into (27) and taking real and imaginary parts we can take the limit $\Omega_2 \rightarrow \infty$ to find that at marginality we must have

$$\Omega_{pl}^2 = \left(\frac{\Delta'_2}{\delta} \right) \left[\frac{(\epsilon \delta - 1) \tau_{VR} X Y - (\epsilon \Delta'_2 + 1) \delta \tau_1}{(\epsilon \Delta'_2 + 1) \tau_1 \tau_{VR}} \right]^2 \quad (29)$$

Equation (29) indicates immediately that the ‘‘topology’’ of the marginal curve in Ω_2, Ω_{pl} space will be strongly influenced by the relative signs of Δ'_2 and δ : if they are the same sign we have real solutions for Ω_{pl} and if they are not the same sign there are none. To investigate this, the cubic (27) was solved, and contours of equal growth rate plotted in Ω_2, Ω_{pl} space.

Now, Δ'_2 and δ are important parameters because, between them, they implicitly determine where the two walls are positioned radially with respect to two naturally occurring radii, r_I and r_R . These are the radii that a perfect wall has to be placed to make the ideal and resistive modes marginally stable, respectively. We now construct five illustrative examples that enumerate the various possibilities. In cases (1)–(4) we assume that the resistive mode with a perfect first wall is stable, i.e., $r_1 < r_R$ and hence $\delta > 0$.

- (1) We start by considering the case where the second wall is *outside* r_I . This means that the ideal mode is unstable even if the second wall is perfect. This in turn means that Δ'_2 will be large and negative (as r_2 goes from just inside r_I to just outside then Δ'_2 goes from large positive to large negative). In Fig. 2 we plot contours of equal growth rate for such a case ($\Delta'_2 = -100$, and ‘‘typical’’ values for the rest of the parameters $\epsilon = 0.1, \delta = 1, \tau_1 = \tau_2 = \tau_{VR} = 1, r_1 = 1.2, r_2 = 1.4, m = 2$). In this and the following figures the dashed contours represent positive (unstable) growth rates and the solid contours negative

(stable) growth rates. We see immediately that all growth rates are positive, and that stabilization of the RWM is impossible in this case for any combination of Ω_{pl} and Ω_2 (notated OMEGA_PL and OMEGA_2 in the figures). This is not surprising as the ideal mode is not really a RWM, but an ideal mode ‘‘in its own right’’ as $r_2 > r_I$. The symmetry evident in the figure is a straightforward consequence of the model geometry.

- (2) Now let us reverse this condition and move r_2 inside r_I . Δ'_2 will now be generically large and positive. Figure 3 shows the results for this case ($\Delta'_2 = 5$; all other parameters are the same as Fig. 2). Note the appearance of stable regions. However, access to each of the stable regions requires a sufficient amount of plasma rotation. As the second wall is moved farther toward the plasma then Δ'_2 drops and so do the amounts of Ω_2, Ω_{pl} required for stabilization.
- (3) As the wall is moved farther in then the next radius of importance it encounters is r_R , the marginal radius of the resistive mode. At this point, of course, $\Delta'_2 = 0$. Figure 4 shows the case where r_2 has just moved inside r_R and $\Delta'_2 = -0.3$ (all other parameters are the same as in Figs. 2 and 3). We see that there has been a topology change due to the change of sign of Δ'_2 , and now it is possible to access a stable region with *no* plasma rotation present.
- (4) As the second wall is now moved toward r_1 , although RWM stabilization is still possible with $\Omega_{pl} = 0$ it becomes increasingly more difficult in terms of the ampli-

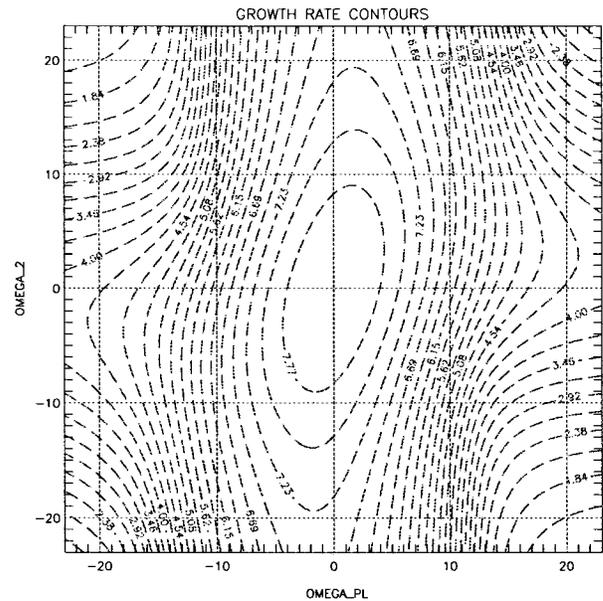


FIG. 2. The second rotating wall is outside the ideal marginal radius and stabilization of the ‘‘RWM’’ is impossible.

and r_R (these are, respectively, the radii at which a perfect wall must be placed to make the ideal and resistive modes marginally stable). RWM stabilization is impossible if $r_2 > r_I$, but possible with finite plasma rotation if $r_R < r_2 < r_I$. Further, the rotation rates required are “slow” in the sense that they are of the order of the inverse wall time of the least conducting wall. If $r_2 < r_R$ then stabilization is possible even in the absence of plasma rotation. (However, as r_2 approaches r_1 stabilization becomes increasingly more difficult, and there is an optimization problem.)

This scheme was first considered for the RFP, where the TITAN power plant design used a flowing lithium blanket.^{8,9} However, it was later realized that a secondary rotating wall could be “faked” by a suitable array of external sensors and active coils.¹⁰ What is more, such a wall is “projectable” and need not reside at the actual location of the coils, a property that may be required in power plant designs.¹⁷ Reference 17 also stated that the gain, bandwidth, current, and total power requirements of the feedback system could be estimated as less than a hundred, a few Hz, a few tens of kA, and a few MW, respectively. These requirements are within the scope of present technology. This scheme, together with that of the “intelligent” shell¹⁸ (which seeks to directly simulate an ideal wall with external sensors and coils), appears to form a useful basis for stabilizing the RWM in fusion tokamak power plants.

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