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Neoclassical current drive by waves with a symmetric spectrum*

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It is shown that plasma waves need not have an asymmetric spectrum in order to produce an electric current in a plasma embedded in a curved magnetic field. For instance, in a toroidal plasma, up–down asymmetric electron–cyclotron heating drives a toroidal current even if there is no net wave–particle momentum transfer and the wave field does not interact preferentially with electrons travelling in any one direction. The resulting current drive efficiency is calculated and is found to be smaller than that of the conventional current drive mechanism in the banana regime, but not insignificant in the plateau regime.
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I. INTRODUCTION

It is well known that plasma waves can produce electric currents if the waves have an asymmetric spectrum, so that they interact preferentially with electrons traveling in one direction along the magnetic field.[1] This creates an asymmetry in the electron distribution function and thus produces a current parallel to the field. In this paper we demonstrate, somewhat surprisingly, that in a plasma confined by a *curved* magnetic field no such spectral asymmetry is necessary for current drive if the effect of collisions is properly taken into account. For instance, in a toroidal plasma an electric current can be produced by a spectrally symmetric wave field if this field is instead up–down asymmetric. More specifically, we find that pure electron cyclotron resonance heating (ECRH) without any directionality in the wave spectrum drives a toroidal current if the heating power deposition is localized above or below the magnetic midplane. In laboratory experiments the wave absorption tends indeed to be localized in this way, in particular for ECRH and ECCD (electron cyclotron current drive) in tokamaks. Although our calculations are focused on this case of ECRH/ECCD in tokamaks, the basic physical mechanism is much more general. It is of a universal neoclassical nature and applies to all wave–particle interaction in curved magnetic fields.

In order to understand how the electric current is produced, consider a toroidal plasma subject to poloidally localized ECRH somewhere above the magnetic midplane. As an electron is heated when passing through the resonance, its magnetic moment $\mu = mv_\perp^2/2B$ increases. This increases the mirror force $F_\| = -\mu \nabla_\| B$ acting on the particle, and if, say, $\nabla_\| B$ is positive at the resonance this reduces the parallel velocity of the electron and thus produces a positive electric current. Of course, once the electron has traveled half a poloidal turn around the flux surface, the mirror force has changed sign and the effect of the increased magnetic moment on the current is reversed. However, by this time collisions will partially have restored μ to its former value, and a net effect thus persists.

It follows from this physical picture that the present current drive mechanism requires both toroidicity and collisions. The current drive efficiency will therefore be poor when either the collisionality or the inverse aspect ratio $\epsilon = r/R$ are very small. Indeed, we find that in the banana regime, which is treated in Sec. II below, the current drive efficiency of the present mechanism is about a factor of $\epsilon \nu_*$ lower than the usual one for ECCD, where $\nu_* = \nu_e qR/v_{Te}\epsilon^{3/2} < 1$ is the collisionality, with q the tokamak safety factor, ν_e the electron collision frequency, and v_{Te} the electron thermal speed. On the other hand, in the plateau regime, the efficiency is found to be about 11ϵ times the conventional one; see Sec. III. Thus, the efficiency is low in the center of a typical hot tokamak plasma, but can be significant farther away from the magnetic axis where both the collision frequency and ϵ are relatively large. The present mechanism might therefore be helpful for current-profile control in advanced tokamaks, which is frequently required some distance away from the plasma center. In Sec. IV we briefly discuss the matching between the banana and plateau regimes, and determine the direction of the wave-driven current. In the following section, we present a more detailed physical picture of the current drive mechanism, and in the final section our conclusions are presented.

II. BANANA REGIME

We begin our analysis by considering the banana regime, $\nu_* \ll 1$. For simplicity, we assume that the wave field is sufficiently weak that the electron population remains close to local thermodynamic equilibrium. The electrons are then described by the linearized drift kinetic equation,

$$v_\| \nabla_\| f_1 - C(f_1) = -\mathbf{v}_d \cdot \nabla f_0 + Q(f_0) + S, \tag{1}$$

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where $\mathbf{v}_d$ is the drift velocity, C is the Coulomb collision operator linearized around a Maxwellian, f_0, and the gradients are taken at constant magnetic moment $\mu = m_e v_\perp^2/2B$. The wave–particle interaction is described by the quasilinear operator

$$Q(f_0) = \frac{1}{v_\perp}\frac{\partial}{\partial v_\perp} v_\perp \hat{D}(\theta,\mathbf{v})\frac{\partial f_0}{\partial v_\perp}, \tag{2}$$

where $\hat{D} = D\,\delta(\theta - \pi/2)x_\perp^{2(l-1)}$ for ECRH at the lth harmonic right above the midplane, D is a constant, θ the poloidal angle, and we have introduced the dimensionless velocity vector $\mathbf{x} = \mathbf{v}/v_{Te}$, with $v_{Te} = (2T_e/m_e)^{1/2}$ the thermal speed. Note that this operator describes heating in the perpendicular direction, but operates symmetrically with respect to the parallel direction. Thus, the wave field does not impart any net momentum to the plasma and does not interact preferentially with electrons traveling in any particular parallel direction. On the right-hand side of Eq. (1) there also appears a term S, which accounts for any additional sources and losses, e.g., caused by anomalous transport across the confining magnetic field. In a steady state, these losses must balance the heat input from $Q(f_0)$, as follows from taking the energy moment and flux-surface average of both sides of Eq. (1),

$$\left\langle \int \frac{m_e v^2}{2}(Q(f_0)+S)d^3v \right\rangle = 0,$$

where we use the notation

$$\langle \cdots \rangle = \oint \frac{(\cdots)d\theta}{\mathbf{B}\cdot\boldsymbol{\nabla}\theta} \bigg/ \oint \frac{d\theta}{\mathbf{B}\cdot\boldsymbol{\nabla}\theta}.$$

In order to keep the analysis as simple as possible while still retaining the essential physics of the problem, we assume that the effective ion charge is high, $Z_{\text{eff}} \gg 1$. Electron–electron collisions can then be ignored, and the collision operator becomes

$$C(f_1) = \frac{\nu_{ei}(v)}{2}\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial f_1}{\partial\xi}, \tag{3}$$

where $\nu_{ei} = 3\pi^{1/2}/4\tau_{ei}x^3$ is the electron–ion collision frequency and $\tau_{ei} = 3(2\pi)^{3/2}\epsilon_0^2 m_e^{1/2} T_e^{3/2}/n_e Z_{\text{eff}} e^4 \ln\Lambda$ the collision time.

It is useful to split the heating operator into terms that are even and odd in the poloidal angle, $-\pi < \theta \leq \pi$,

$$Q(f_0) = Q_+(f_0) + Q_-(f_0),$$

where $Q_\pm(\theta) = [Q(\theta) \pm Q(-\theta)]/2$. Since Eq. (1) is a linear equation for f_1 and the driving terms on the right appear additively, the solution f_1 consists of a sum of the separate contributions from these terms. Our primary interest in this paper is to find the current driven by the up–down asymmetric drive $Q_-(f_0)$. This current is normally neglected in theories of ECCD proceeding from the orbit average of the kinetic equation (1) since this average of $Q_-(f_0)$ vanishes. We shall not be concerned with the current produced by the symmetric term $Q_+(f_0)$, since this has already been calculated in numerous papers (see, e.g., Refs. 1–4 and references therein), or the loss term S, which we also assume to be up–down symmetric. Nor do we need to consider the drift term $-\mathbf{v}_d \cdot \boldsymbol{\nabla} f_0$, which produces the usual bootstrap current.[5–8]

Thus, we only need to include the up–down asymmetric piece,

$$Q_-(f_0) = \frac{m_e D}{T_e}\left[\delta\left(\theta - \frac{\pi}{2}\right) - \delta\left(\theta + \frac{\pi}{2}\right)\right] x_\perp^{2(l-1)}(x_\perp^2 - l)f_0,$$

of the heating operator (2) on the right-hand side of Eq. (1). Since the orbit average of this term vanishes it can be written as a derivative taken along a particle orbit,

$$Q_-(f_0) = v_\parallel \boldsymbol{\nabla}_\parallel h,$$

where the gradient is taken at fixed x and Λ, and the function h is defined by

$$h = -\frac{2qR_0 D f_0}{v_{Te}^3}(x^2\Lambda - l)\frac{x^{2(l-1)}\Lambda^{l-1}}{\sigma\sqrt{1-\Lambda}}H_{\text{out}}(\theta), \tag{4}$$

for $\Lambda < 1$ and $h = 0$ for $\Lambda > 1$. Here $\Lambda = v_\perp^2 B_0/v^2 B$, $\sigma = v_\parallel/|v_\parallel|$, and B_0 is a reference magnetic field, which we choose to be the field at the poloidal location of the heating, $\theta = 90°$. Thus, particles with $\Lambda = 1$ are trapped on the outboard side of the torus and are reflected right at the heating resonance. The trapped–passing boundary is located at $\Lambda = B_0/B_{\max}$, where $B_{\max}$ is the maximum value of B on the flux surface. R_0 is defined by $qR_0 = 1/\boldsymbol{\nabla}_\parallel\theta$ at $\theta = \pi/2$ and becomes equal to the major radius of the magnetic axis in a standard, large-aspect-ratio, circular equilibrium. The even function $H_{\text{out}}(\theta)$ is defined to be equal to unity on the outboard side of the flux surface and to vanish on the inboard side,

$$H_{\text{out}}(\theta) = H\left(\theta + \frac{\pi}{2}\right) - H\left(\theta - \frac{\pi}{2}\right), \quad -\pi < \theta \leq \pi,$$

where H is the Heaviside function. Note that we have defined h so that it is continuous at the bounce points for trapped particles. In fact, h vanishes at these points: if the bounce points are on the outboard side then $\Lambda > 1$ so that $h = 0$, and if they are on the inboard side then $H_{\text{out}}(\theta) = 0$.

The kinetic equation now assumes the form

$$v_\parallel \boldsymbol{\nabla}_\parallel (f_1 - h) = C(f_1),$$

which is familiar from neoclassical transport theory.[9] Expanding in the smallness of the collision frequency in the usual way, $f_1 = f_1^0 + f_1^1 + \ldots$, gives

$$v_\parallel \boldsymbol{\nabla}_\parallel (f_1^0 - h) = 0,$$

in lowest order, which implies $f_1^0 = g + h$, where g only depends on constants of motion, $g = g(v, \Lambda, \psi, \sigma)$, and is determined by the constraint

$$\oint C(g+h)dt = 0, \tag{5}$$

where the integral is taken over one poloidal turn around the orbit. It follows from a conventional argument that g vanishes in the trapped domain: on the one hand g must be an even function of σ in the trapped region to make f_1^0 continuous at the bounce points; on the other hand g must be odd in order to satisfy the constraint (5). Thus, we only need to solve this equation in the passing domain, where it can be written in terms of the flux-surface average as

$$\left\langle \frac{B}{v_\parallel} C(g+h) \right\rangle = 0.$$

If the Lorentz approximation (3) is used for the collision operator, this equation is easily integrated once to give

$$\begin{aligned} \frac{\partial g}{\partial \Lambda} &= -\frac{\langle v_\parallel \partial h/\partial \Lambda \rangle}{\langle v_\parallel \rangle} \\ &= -\frac{2qR_0 D f_0}{v_{Te}^3} \frac{\langle v_\parallel H_{\rm out}(\theta)\rangle}{\langle v_\parallel \rangle} \frac{x^{2l-3}\Lambda^{l-1}}{\sigma\sqrt{1-\Lambda}} \\ &\quad \times \left[x^2 + (x^2\Lambda - l)\left(\frac{l-1}{\Lambda} + \frac{1}{2(1-\Lambda)} \right) \right], \end{aligned} \tag{6}$$

in the passing domain.

We are now in a position to calculate the current. A straightforward calculation shows that the parallel velocity moment of h vanishes everywhere since

$$\int v_\parallel h d^3 v \propto \int_0^1 \frac{\Lambda^{l-1} d\Lambda}{\sqrt{1-\Lambda}} \int_0^\infty (x^2\Lambda - l) x^{2l} e^{-x^2} dx = 0,$$

for any l, because of the separable, number-conserving form of our quasilinear operator Q. Notice, however, that even though h gives no net contribution to the current density, the individual contributions of the trapped and passing do not vanish separately. Therefore, the passing electron current contribution in h is exactly cancelled by the trapped response. However, the passing electrons make an additional contribution to the current density through g that is given by

$$j_\parallel = -\int e v_\parallel g \, d^3 v = 2\pi e B \int_0^\infty v^3 dv \int_0^{B/B_{\rm max}} \Lambda \frac{\partial g}{\partial \Lambda} d\Lambda. \tag{7}$$

We have not yet made any assumptions about the geometry of the magnetic equilibrium, but it is now convenient to simplify the algebra by assuming that the aspect ratio is large and the flux surfaces are circular, $\epsilon = r/R_0 \ll 1$. Substituting the solution (6) in the expression (7) for the current then gives

$$\begin{aligned} j_\parallel &= \frac{qen_e m_e R_0 D}{4\pi^{1/2} T_e} \Gamma\left(l + \frac{1}{2} \right) \int_0^{1-\epsilon} \frac{\langle v_\parallel H_{\rm out}(\theta)\rangle}{\langle v_\parallel \rangle} \frac{d\Lambda}{(1-\Lambda)^{3/2}} \\ &\quad + O(\epsilon^{1/2}), \end{aligned} \tag{8}$$

where n_e is the electron density associated with f_0. Most of the contribution to this integral comes from the region close to the trapped–passing boundary, $\Lambda \simeq 1-\epsilon$. In the standard circular equilibrium we have assumed, the flux surface averages appearing in Eq. (8) are equal to

$$\frac{\langle v_\parallel H_{\rm out}(\theta)\rangle}{\langle v_\parallel \rangle} = \frac{E(k,\pi/4)}{E(k,\pi/2)},$$

where E denotes the elliptic integral of the second kind and $k^2 = 2\epsilon\Lambda/(1-\Lambda+\epsilon\Lambda)$ is the trapping parameter. Using this result to numerically evaluate the remaining integral in Eq. (8) finally gives the current density,

$$j_\parallel = 0.99\ \Gamma\left(l + \frac{1}{2} \right) \frac{qen_e m_e R_0 D}{2\pi\epsilon^{1/2} T_e}, \tag{9}$$

where

$$0.99 \simeq 2\pi^{1/2} \int_0^1 \frac{E(k,\pi/4)}{E(k,\pi/2)} \frac{dk}{(2-k^2)^{3/2}}.$$

The flux-surface average of the absorbed ECRH power density is

$$\langle P \rangle = \left\langle \int \frac{m_e v^2}{2} Q_+(f_0) d^3 v \right\rangle = \frac{\Gamma(l+1) m_e n_e |D|}{\pi}, \tag{10}$$

so the local current drive efficiency becomes

$$\frac{|j_\parallel|}{\langle P \rangle} = 0.99 \frac{\Gamma(l+1/2)}{2\Gamma(l+1)} \frac{eqR_0}{\epsilon^{1/2} T_e}. \tag{11}$$

Note that the current drive efficiency is independent of the collision frequency in the banana regime although collisions play in instrumental role in producing the current. The present current drive mechanism is similar to the bootstrap current in this respect. A widely used figure of merit for current drive efficiency is[10]

$$\eta = \frac{n_e I R_0}{P_{\rm ECRH}}, \tag{12}$$

where I is the total current driven and $P_{\rm ECRH}$ the total power deposited by the waves, so that $I/P_{\rm ECRH} = \langle j_\parallel \rangle / 2\pi R_0 \langle P \rangle$ if the current drive occurs only in the vicinity of one particular flux surface. For conventional current drive, η can be estimated by assuming that the momentum $m_e \Delta v_\parallel$ imparted to an electron by the wave field typically lasts for about a collision time τ_{ei}. Since the expended energy is $m_e v_\parallel \Delta v_\parallel$, intuitively the current drive efficiency becomes $j_\parallel / P \sim e\tau_{ei}/m_e v_{Te}$, so the figure of merit for ordinary current drive is

$$\eta_0 = \frac{n_e e \tau_{ei}}{2\pi\sqrt{2m_e T_e}} = \frac{18}{\ln\Lambda} \frac{T_{\rm keV}}{Z_{\rm eff}}\, 9\cdot 10^{17} \text{A/W m}^2,$$

where $T_{\rm keV}$ is the electron temperature in keV. In many experiments (e.g., Refs. 11 and 12) the efficiency approaches η_0. In contrast, the result (11) can be expressed as

$$\eta_{\rm ban} = 0.99 \frac{\Gamma(l+1/2)}{\Gamma(l+1)} \epsilon \nu_* \eta_0, \tag{13}$$

where both the collisionality $\nu_* = qR_0/(v_{Te}\tau_{ei}\epsilon^{3/2})$ and ϵ have been assumed to be small. The current drive efficiency of the present mechanism is therefore modest in the banana regime.

III. PLATEAU REGIME

We now turn our attention to the plateau regime, which is defined by $1 \ll \nu_* \ll \epsilon^{-3/2}$. In experiments, this regime is frequently realized very close to the magnetic axis, where ϵ is small, and far from the axis, where the temperature is lower than in the center of the discharge. In order to calculate the current drive efficiency in this regime, we find it more convenient to use the adjoint method[13] than to solve Eq. (1) directly. (Of course, the adjoint method can also be used in the banana regime and then produces exactly the same result as we found in the previous section.) In this method, the adjoint equation,

$$v_\parallel \nabla_\parallel G + C(G) = v_\parallel f_0, \tag{14}$$

is first solved, and the current is then calculated from

$$\langle j_\parallel \rangle = e \left\langle \int \frac{G}{f_0} Q_-(f_0) d^3 v \right\rangle. \tag{15}$$

To derive this relation, one multiplies the original kinetic equation (1) by G/f_0, integrates over velocity space, and takes the flux surface average, to obtain

$$\left\langle \int \frac{G}{f_0} (v_\parallel \nabla_\parallel f_1 - C(f_1) - Q_-(f_0)) d^3 v \right\rangle = 0,$$

where, again, we have only kept the asymmetric term on the right-hand side of Eq. (1). When this equation is added to the analogous one that is obtained by multiplying Eq. (14) by f_1/f_0, one obtains the expression (15) for the current.

To solve the adjoint equation (14), it is convenient to write $G = -v_\parallel f_s(v) + k$, where f_S is the Spitzer function defined by $C(v_\parallel f_s) = -v_\parallel f_0$. When substituted in Eq. (15), the first term in G then gives the current which arises in a straight magnetic field, while k gives the correction due to toroidicity. The equation for k becomes

$$v_\parallel \nabla_\parallel k + C(k) = v_\parallel \nabla_\parallel (v_\parallel f_s) = -\frac{\epsilon v_\perp^2}{2qR_0} f_s(v) \sin\theta,$$

which has a well-known solution in the plateau regime,[9]

$$k = y \epsilon v \frac{1-\xi^2}{2} f_s(v) \int_0^\infty \exp(-x^3/3) \sin(\theta + y\xi x) dx,$$

where $\xi = v_\parallel / v$ and $y = (2v/\nu_{ei} q R_0)^{1/3} \gg 1$. Only the even (in ξ) part of k contributes to the current (15) and is given by

$$k_{\mathrm{even}} = (\pi/2) \epsilon v f_s(v) \sin\theta\, \delta(\xi),$$

in the plateau limit $y \to \infty$. Inserting this result in Eq. (15) shows that the current associated with k arises from the up–down asymmetry of the heating and becomes

$$j_\parallel = \epsilon n_e e D \sqrt{\frac{2m_e}{\pi T_e}} \int_0^\infty \frac{f_s}{f_0} x^{2l+1} (x^2 - l) e^{-x^2} dx. \tag{16}$$

In the Lorentz limit $Z_{\mathrm{eff}} \gg 1$, where the collision operator is given by Eq. (3), the Spitzer function is equal to $f_s(v) = f_0(v)/\nu_{ei}(v)$ and the remaining integral in Eq. (16) can easily be calculated analytically to give

$$j_\parallel = \frac{5}{3\pi} \Gamma\left(l + \frac{5}{2}\right) \sqrt{\frac{2m_e}{T_e}}\, \epsilon n_e e \tau_{ei} D.$$

The current drive efficiency is obtained by dividing this result by the heating power (10),

$$\frac{|j_\parallel|}{\langle P \rangle} = \frac{5\sqrt{2}}{3} \frac{\Gamma(l+5/2)}{\Gamma(l+1)} \frac{\epsilon e \tau_{ei}}{\sqrt{m_e T_e}}. \tag{17}$$

In terms of the parameter (12) the efficiency is

$$\eta_{\mathrm{plat}} = \frac{5}{3\pi} \frac{\Gamma(l+5/2)}{\Gamma(l+1)} \frac{\epsilon e n_e \tau_{ei}}{\sqrt{2m_e T_e}} = \frac{10}{3} \frac{\Gamma(l+5/2)}{\Gamma(l+1)} \epsilon \eta_0. \tag{18}$$

For instance, $\eta_{\mathrm{plat}} \simeq 11 \epsilon \eta_0$ for heating at the first harmonic ($l=1$), and $\eta_{\mathrm{plat}} \simeq 19 \epsilon \eta_0$ at the second harmonic ($l=2$). This suggests that the current drive efficiency can be substantial in the plateau regime although ϵ is formally a small parameter. (Of course, very close to the magnetic axis the efficiency will be quite small.)

If the effective ion charge is not very high, $Z_{\mathrm{eff}} = O(1)$, it necessary to account for electron–electron collisions by evaluating the integral in Eq. (16) using the proper Spitzer function. Accurate approximations to this function have been published, e.g., by Hirshman,[14] which enables the integral to be calculated numerically. In a pure plasma, $Z_{\mathrm{eff}} = 1$, the resulting current drive efficiency becomes $\eta_{\mathrm{plat}} \simeq 5 \epsilon \eta_0$ for heating at the first harmonic, and $\eta_{\mathrm{plat}} \simeq 7 \epsilon \eta_0$ at the second harmonic.

IV. DISCUSSION

The presence of the large factor $10\Gamma(l+5/2)/3\Gamma(l+1) \gg 1$ in Eq. (18) makes it difficult to match the banana and plateau regimes, and may indicate that the banana–plateau transition only occurs at rather high collisionality, so that the plateau regime is narrow for realistic values of ϵ. On the other hand, the banana–plateau transition is also sensitive to the location of the heating. As shown in the Appendix, the current drive efficiency increases in the banana regime if the heating resonance is moved toward the inboard side of the torus, while this decreases the efficiency in the plateau regime. Thus the banana and plateau regimes connect more smoothly if the heating occurs at poloidal locations $90° < \theta < 180°$, and the current drive efficiency can then considerably exceed the somewhat pessimistic result (13).

In order to determine the direction of the wave-driven current, it is convenient to assume that the total plasma current is in the clockwise direction when viewed from above. (If in an actual experiment the current is in the opposite direction then the vertical direction may simply be reversed.) In the analysis above, we have implicitly assumed that $\nabla_\parallel \theta = 1/qR_0$ is positive since $q > 0$. Thus, $B_\theta > 0$ and the heating occurs above (below) the midplane if D is positive (negative). According to our results (9) and (16) the sign of the parallel wave-driven current $j_\parallel$ coincides with the sign of D. Thus, if the heating occurs above the midplane ($D > 0$) the driven current is in the same direction as the toroidal magnetic field, and in the opposite direction if the heating is below the midplane ($D < 0$). In order to relate the direction of $j_\parallel$ to that of the total plasma current, we note that the electron drift is in the direction of $-\mathbf{B} \times \nabla B$, which is upward if the magnetic field is in the same direction as the plasma current and downward if the magnetic field and the current are in opposite directions. The rf-driven current $j_\parallel$ is therefore in the same direction as the total plasma current if the electron drift is directed toward the heated side of the flux surface, and $j_\parallel$ opposes the total current if the drift is the opposite direction.

Throughout our analysis, we have neglected any poloidal electric field that might arise as a consequence of the heating. Hsu *et al.*[15] have demonstrated that sufficiently strong ECRH leads to an accumulation of electrons on the low-field side of the torus and a poloidal variation in the electrostatic potential of the order

$$\frac{e\tilde{\Phi}}{T_e} \sim \frac{m_e D \epsilon}{\nu_e T_e}.$$

If this quantity becomes comparable to ϵ then the poloidal electric field affects the particle trapping and should be taken into account in the banana regime. Relating D to the current (9), we conclude that our neglect of the poloidal electric field is justified as long as the driven current is sufficiently small,

$$j_\parallel \ll \epsilon^{-1/2} n_e e \nu_e q R.$$

If the entire plasma current were driven by the ECRH, so that $j_\parallel \sim B_\theta / \mu_0 r$, this means that the inequality

$$\frac{\rho_{e\theta}/\epsilon r}{\beta_{e\theta}\nu_*} \ll 1$$

should be satisfied to justify the neglect of the poloidal electric field. Here $\rho_{e\theta} = m_e v_{Te}/eB_\theta$ is the poloidal electron gyroradius and $\beta_{e\theta} = 2\mu_0 n_e T_e / B_\theta^2$ is a measure of the poloidal beta for electrons.

V. PHYSICAL PICTURE

It is possible to understand the physics behind the results obtained in the two previous sections by elaborating on the physical picture given in the Introduction, following an argument related to that by Fisch and Boozer in Ref. 16. To this end, we consider the motion of a single electron as it interacts with the wave field and collides with other electrons and ions in the plasma. When the electron passes through the cyclotron resonance, its velocity vector receives a random change, $v \rightarrow v + \Delta v$, $\Lambda \rightarrow \Lambda + \Delta\Lambda$. Since the parallel velocity at the resonance, $v_{\parallel 0} = \pm v\sqrt{1-\Lambda}$, is not affected by the heating, the changes in the variables v and Λ are related by

$$\frac{\Delta\Lambda}{1-\Lambda} = \frac{2\Delta v}{v}.$$

The parallel velocity elsewhere along the orbit is given by

$$v_\parallel^2 = v_{\parallel 0}^2 \left(1 + \frac{\epsilon\Lambda\cos\theta}{1-\Lambda}\right),$$

and thus changes by the amount

$$\Delta v_\parallel = \frac{\epsilon v \cos\theta}{v_\parallel} \Delta v,$$

when the electron has interacted with the wave field. In addition, $v_\parallel$ continuously changes because of collisions so that, following the wave–particle interaction, the perturbation $\Delta v_\parallel$ caused by the wave gradually fades away on the effective collision time scale $\tau_{\mathrm{eff}} = (v_\parallel / v)^2 \tau_{ei}$. (Of course, because of the random nature of collisions this decay only occurs in an average sense: if many electrons are given the same $\Delta v_\parallel$ by the wave, the average of these perturbations decays on the collision time scale.) If we model this decay by an exponential, we obtain the average current associated with the electron during some time interval $\Delta t \gg \tau_{\mathrm{eff}}$,

$$J \sim -\frac{e}{\Delta t}\int_0^{\Delta t} \Delta v_\parallel(\theta) e^{-t/\tau_{\mathrm{eff}}} dt.$$

Here $t=0$ corresponds to the time of wave–particle interaction. Since $\Delta v_\parallel$ is proportional to $\cos\theta$, the integrand oscillates on the transit time scale, τ_θ, required to complete a poloidal orbit. To evaluate the integral when the collisionality is low, it is thus appropriate to use the following relation, which holds for any τ_θ-periodic function $F(t)$:

$$\int_0^\infty F(t) e^{-t/\tau_{\mathrm{eff}}} dt = \frac{1}{1-e^{-\tau_\theta/\tau_{\mathrm{eff}}}} \int_0^{\tau_\theta} F(t) e^{-t/\tau_{\mathrm{eff}}} dt$$

$$\simeq \tau_{\mathrm{eff}} \langle F(t) \rangle_t - \langle F(t) t \rangle_t,$$

in the low-collisionality limit $\tau_\theta \ll \tau_{\mathrm{eff}}$. Here

$$\langle F(t) \rangle_t = \tau_\theta^{-1} \int_0^{\tau_\theta} F(t) dt$$

denotes the time average over an orbit. Hence we conclude that the average current carried by an electron following an interaction with the wave is of the order

$$J \sim -\epsilon e v \frac{\Delta v}{\Delta t} \left(-\tau_{\mathrm{eff}} \left\langle \frac{\cos\theta}{v_\parallel} \right\rangle_t + \left\langle \frac{t\cos\theta}{v_\parallel} \right\rangle_t \right).$$

Here the first term is larger than the second one, but since it is odd in $v_\parallel$ its contribution to the total current is cancelled by the corresponding term from an electron traveling in the opposite direction. However, the second term is not odd if the heating is up–down asymmetric, and thus gives a net contribution to the current. (The first term is odd since two particles at the same θ but with opposite $v_\parallel$ cancel. In the second term, the time t elapsed since the interaction with the wave will be different for these particles.) The net current is thus of the order

$$J \sim \frac{\epsilon e q R P}{m_e n_e v_\parallel^2}, \tag{19}$$

where we have approximated $t \sim \tau_\theta$ by $qR_0/v_\parallel$ and noted that the heating power per particle is $P/n_e = m_e v \Delta v / \Delta t$.

It might perhaps be thought that the total current density could now be obtained by integrating the estimate (19) over velocity space. However, there is an additional subtlety which has to do with the collisional equilibrium between trapped and untrapped particles, which amplifies the current carried by the latter. It follows from the expression (19) that the largest current is carried by trapped (and barely passing) particles. Their typical contribution per particle is of the order

$$J_{\rm tr}\sim\frac{eqRP}{n_eT_e}, \qquad (20)$$

since for these particles $v_\parallel\sim\epsilon^{1/2}v_{Te}$. Lest it be thought that trapped particles cannot carry any current at all, we remark that the wave field and collisions play a key role in the present current drive mechanism. While it is certainly true that no current is associated with unperturbed trapped orbits, in the present circumstances these orbits are continually disturbed by the wave field and by Coulomb collisions. Looking back at our kinetic calculation, it can be verified that the perturbation (4) of the distribution function is largest in the trapped region and that the trapped particles do carry a current (although the total current associated with h happens to vanish). The point is that, according to Eqs. (4) and (20), there is a surplus of counter-moving trapped electrons and a deficit of co-moving ones, so that their corresponding current contributions are perturbed by the fraction

$$\frac{\Delta n_e}{n_e}\sim\frac{J_{\rm tr}}{ev_{Te}\epsilon^{1/2}},$$

and the trapped population thus experiences a net average drift given by Eq. (20). The passing particles can only be in collisional equilibrium with the trapped ones if they develop a similar asymmetry. The average current per passing particle then becomes $J_{\rm pass}\sim ev_{Te}\Delta n_e/n_e$, and it follows that the total current density is of the order

$$j_\parallel\sim n_eJ_{\rm pass}\sim\frac{eqRP}{\epsilon^{1/2}T_e},$$

in agreement with the result (11). Note that the contribution from the circulating particles is larger than Eq. (19) suggests since the single-particle picture does not properly account for the collisional equilibrium between the trapped and passing populations.

In the plateau regime, the key role is played not by trapped particles (whose orbits are interrupted by collisions), but by ''resonant'' ones,[9] for which the effective collision time, $\tau_{\rm eff}=(v_\parallel/v)^2\tau_{ei}$, is comparable to the period of the orbit, $\tau_\theta=qR/v_\parallel$, so that

$$v_\parallel\sim v_{\parallel\rm res}=\left(\frac{v^2qR}{\tau_{ei}}\right)^{1/3}.$$

These particles are passing; they are situated well away from the trapped passing boundary, but have relatively small parallel velocity, $\epsilon^{1/2}\ll v_{\parallel\rm res}/v\ll 1$. Again, the average current (19) carried by these particles corresponds to an asymmetry $\Delta n_e/n_e$ between the co- and counter-moving populations, which can be found from the requirement

$$J_{\rm res}=\frac{\epsilon eqRP}{m_en_ev_{\parallel\rm res}^2}=\frac{\Delta n_e}{n_e}ev_{\parallel\rm res}.$$

Hence $\Delta n_e/n_e\sim\epsilon\tau_{ei}P/m_en_ev_{Te}^2$, and the current density becomes

$$j_\parallel\sim\Delta n_eev_{Te}\sim\frac{\epsilon e\tau_{ei}P}{\sqrt{m_eT_e}},$$

in agreement with our earlier result (17).

VI. SUMMARY

It is well known that in a *straight* magnetic field, plasma waves can only produce an electric current if they have a spectral asymmetry, so that they either impart net parallel momentum to the electrons (as in lower-hybrid current drive) or preferentially heat the electrons traveling in one particular direction (as in ECCD). Here, we have demonstrated that when the plasma is confined by a *curved* magnetic field with $\nabla_\parallel B\neq 0$, neoclassical effects enable an electric current to be driven purely by a spatial asymmetry of the wave field. For instance, pure electron cyclotron heating (operating symmetrically in velocity space) in a tokamak gives rise to a toroidal current if the wave field is up–down asymmetric, which is normally the case in experiments.

The efficiency of this ''neoclassical'' current drive mechanism is given by Eq. (13) in the banana regime and by Eq. (18) in the plateau regime. Since in these regimes $\nu_*\ll 1$ and $\epsilon\ll 1$, respectively, the efficiency is always lower than the typical efficiency η_0 associated with ''classical'' current drive. Nevertheless, since the additional neoclassical current drive comes at no extra expense, it could perhaps be usefully exploited in cases where the classical efficiency is poor (e.g., when it is difficult to achieve the necessary spectral asymmetry). The theoretical prediction is that the neoclassically driven current is in the same direction as the plasma current if the heating occurs on that side of the flux surface towards which the electron magnetic drift points, and opposes the plasma current if the opposite side of the flux surface is heated.

The possibility of neoclassical currents arising in response to localized heating is not limited to tokamaks. For instance, undesirable currents could arise in this way in a stellarator with ECRH. More generally, the phenomenon should occur whenever there is a spatially varying electron heat source (or sink) in a magnetized plasma with $\nabla_\parallel B\neq 0$.

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APPENDIX: GENERAL HEATING LOCATION

The calculation given in the main body of this paper treats the case when the heating occurs right above the magnetic axis, at the poloidal angle $\theta_*=\pi/2$. In this appendix, we calculate the current when the heating position θ_* is general, so that the up–down asymmetric part of the quasilinear operator is

$$Q_-(f_0)=\frac{m_eD}{T_e}[\delta(\theta-\theta_*)-\delta(\theta+\theta_*)]x_\perp^{2(l-1)}(x_\perp^2-l)f_0. \tag{A1}$$

In the banana regime, the function h defined in Eq. (4) is modified by the replacement $\Lambda\to\Lambda_*$, where $\Lambda=\Lambda B_*/B_0$, $B_*=B(\theta_*)$, and a simple re-definition of $H_{\text{out}}(\theta)$,

$$H_{\text{out}}(\theta)=H(\theta+\theta_*)-H(\theta-\theta_*).$$

With these modifications, the other part of the distribution function $f_1=g+h$ is still given by the expression (6). The current carried by h still vanishes, and that carried by g becomes

$$j_\parallel=\frac{qen_em_eR_0D}{4\pi^{1/2}T_e}\Gamma\left(l+\frac{1}{2}\right)\int_0^{B_*/B_{\max}}\frac{\langle v_\parallel H_{\text{out}}(\theta)\rangle}{\langle v_\parallel\rangle}\frac{d\Lambda_*}{(1-\Lambda_*)^{3/2}}+O(\epsilon^{1/2}),$$

where $B_*/B_{\max}=1-\epsilon(1+\cos\theta_*)$. Hence

$$j_\parallel=\frac{qen_em_eR_0D}{\pi^{1/2}T_e}\Gamma\left(l+\frac{1}{2}\right)\times\int_0^1\frac{E(k,\theta_*/2)}{E(k,\pi/2)}\frac{dk}{[2-(1-\cos\theta_*)k^2]^{3/2}},$$

and it follows that the current drive becomes more efficient with increasing $\theta_*<\pi$. In the limit $\theta\to\pi$ the efficiency becomes

$$\frac{j_\parallel}{\langle P\rangle}\to\frac{\Gamma(l+1/2)}{\Gamma(l+1)}\sqrt{\frac{\pi}{2\epsilon}}\frac{eqR_0}{T_e(\pi-\theta_*)}.$$

The physical reason for the high efficiency achieved in this limit is the dominant role played by electrons close to the trapped–passing boundary since the current (19) associated with these particles becomes very large. Of course, the singularity at $\theta_*=\pi$ is not real but is resolved by a small collisionality ν_*, which restricts the validity of the banana regime ordering close to the trapped–passing boundary and thus limits the current drive efficiency.

In the plateau regime, the current is obtained by inserting Eq. (A1) in the adjoint expression (15). The result is that the driven current is simply multiplied by $\sin\theta_*$, so that the current drive efficiency is reduced by this factor. For example, instead of Eq. (17), we obtain

$$\frac{|j_\parallel|}{\langle P\rangle}=\frac{5\sqrt{2}}{3}\frac{\Gamma(l+5/2)}{\Gamma(l+1)}\frac{\epsilon e\tau_{ei}}{\sqrt{m_eT_e}}\sin\theta_*,$$

in the Lorentz limit. Thus, in the plateau regime, the current drive efficiency peaks at $\theta_*=\pi/2$.

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