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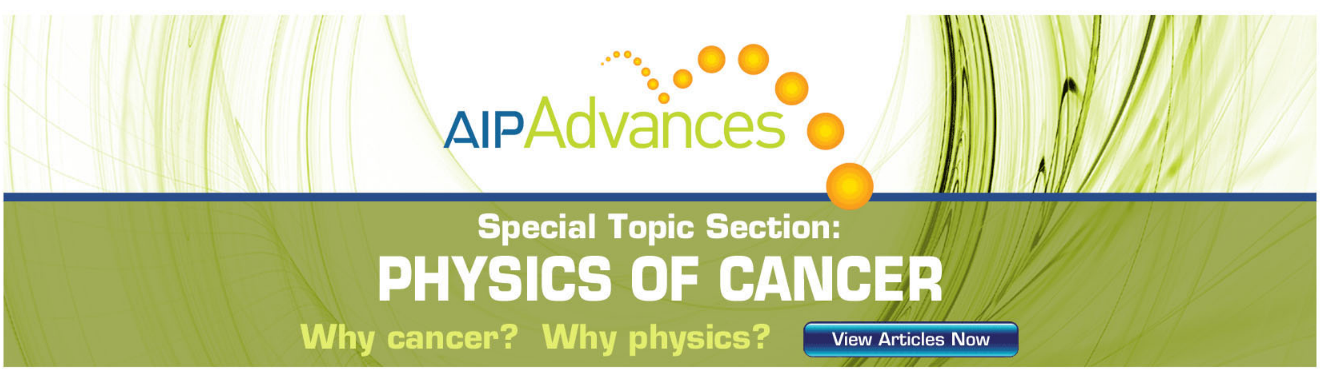

Interpretation of ion cyclotron emission from sub-Alfvénic fusion products in the Tokamak Fusion Test Reactor

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Ion cyclotron emission (ICE) has been observed during neutral beam-heated supershots in the Tokamak Fusion Test Reactor (TFTR) [Phys. Rev. Lett. **72**, 3526 (1994)] deuterium–tritium campaign at fusion product cyclotron harmonics. The emission originates from the outer midplane edge plasma, where fusion products initially have an anisotropic velocity distribution, sharply peaked at a sub-Alfvénic speed. It is shown that the magnetoacoustic cyclotron instability, resulting in the generation of obliquely propagating fast Alfvén waves at fusion product cyclotron harmonics, can occur under such conditions. The time evolution of the growth rate closely follows that of the observed ICE amplitude. Instability is suppressed if the fusion products undergo a moderate degree of thermalization, or are isotropic. In contrast, the super-Alfvénic fusion products present in the outer midplane of the Joint European Torus (JET) [Nucl. Fusion **33**, 1365 (1993)] can drive the instability if they are isotropic or have a broad speed distribution. This may help to account for the observation that fusion product-driven ICE in JET persists for longer than fusion product-driven ICE in TFTR supershots. [S1070-664X(96)01002-1]

I. INTRODUCTION

Wave activity in the tens of MHz range has been observed during recent pure deuterium and deuterium–tritium (D–T) experiments on the Tokamak Fusion Test Reactor (TFTR).[1–4] The observed spectra have a clearly defined harmonic structure, the harmonic spacings corresponding to the cyclotron frequencies of either fusion ions or injected beam ions near the outer midplane plasma edge. In high current ($\gtrsim$1 MA) discharges, for typically 100 ms after the onset of neutral beam injection (NBI), the spectral peak frequencies correspond to fusion ion cyclotron frequencies and their sequential low harmonics: at later times, the spectra become increasingly dominated by emission at beam ion harmonics. It appears reasonable to infer that the temporal evolution of these wave spectra reflects the temporal evolution of the various energetic ion populations responsible for the emission. Similar wave activity, commonly referred to as ion cyclotron emission (ICE), has been observed in previous TFTR experiments,[5] and in many other laboratory[6–8] and space[9–11] plasmas. In all cases, observations of ICE are associated with the presence of a minority population of energetic ions, arising from fusion reactions or NBI in the case of tokamaks, and from natural acceleration processes in the case of space plasmas. Observations of ICE in tokamaks have significant diagnostic consequences for alpha particle physics. In particular, models for the structure and evolution of the alpha particle population in velocity space and real space must be consistent with the observation that fusion product-driven ICE originates from the outer plasma edge, rather than the plasma center. The apparent absence of beam ion-driven ICE in the plasma center also conveys information about the velocity space structure of the beam ion population in that region.

In contrast to TFTR, ICE observed in the Joint European Torus (JET)[8] appears to be driven predominantly by fusion products, and its spectral character does not change significantly in time during the course of a single discharge. The magnetoacoustic cyclotron instability,[12–15] which involves the excitation of electromagnetic waves on the fast Alfvén and ion Bernstein branches at cyclotron harmonics of an energetic ion species, has been proposed as a mechanism for the generation of fusion product-driven ICE observed in JET. Other mechanisms have been proposed to account for the JET observations, based on relativistic mass effects,[16] and the spatial localization of Alfvénic eigenmodes.[17] The latter concept has also been invoked to account for the observed features of fusion product-driven ICE in TFTR.[18] In this paper we concentrate on the magnetoacoustic cyclotron instability. The theory of this instability, originally restricted to the case of strictly perpendicular propagation with respect to the magnetic field,[12–14] was generalized in Ref. 15 to the case of large but otherwise arbitrary propagation angles. In Refs. 8 and 15, it was proposed that ICE in JET is driven by centrally born fusion alpha particles that have undergone large drift orbit excursions to the outer midplane edge. The edge plasma is only accessible to alpha particles with a narrow range of energies and pitch angles, and so the alpha particle velocity distribution is particularly unstable in that region. This picture is consistent with early general predictions of alpha particle behavior in tokamak plasmas,[19,20] and with specific alpha particle orbit calculations.[8] The generalized theory of the magnetoacoustic cyclotron instability can account for several hitherto unexplained features of the JET data, notably the excitation of waves at the alpha particle cyclotron frequency, and the splitting of harmonic emission lines. These successes suggest strongly that a similar excitation mechanism is responsible for fusion product-driven ICE in TFTR. In the case of perpendicular propagation, the magnetoacoustic cyclotron instability can only occur if the energetic ions have a nonmonotonic distribution of speeds perpendicular to the magnetic field, strongly peaked at a super-Alfvénic value.[12–14] Simulations of deuterium–tritium experiments in TFTR indicate that the birth speed of fusion alpha particles is about one-half of the local Alfvén speed at the outer plasma edge [see Fig. 4(j) of Ref. 21]: the birth speeds of tritons (T) and helium-3 nuclei (^{3}He) are lower still. During the first 100–200 msec of high-power deuterium neutral beam experiments on TFTR, wave emission was observed at low cyclotron harmonics of ^{3}He.[2,3] These observations clearly indicate that ICE can be driven by sub-Alfvénic fusion products. In the present paper we demonstrate that the generalized theory of the magnetoacoustic cyclotron instability developed in Ref. 15 includes a viable mechanism for sub-Alfvénic fusion product-driven ICE in TFTR.

In Ref. 4 we proposed a predominantly electrostatic mechanism for the observed beam-driven component of ICE in TFTR, which requires the (greatly sub-Alfvénic) beam ions to have an extremely narrow distribution of speeds parallel to the magnetic field. It appears reasonable to invoke such distributions for the beam ions in the edge plasma, close to the point of beam injection: this provides a further example of the diagnostic application of ICE. We point out in Sec. II that centrally born fusion ions that undergo large excursion orbits to the edge plasma will inevitably have a nonnegligible spread of parallel speeds, even before they have undergone significant collisional relaxation. The instability identified in Ref. 4 cannot, therefore, account for the observed generation of ICE by sub-Alfvénic fusion products in TFTR, and we are thus led to a reexamination of the magnetoacoustic cyclotron instability. Following a summary of the theory of this instability in Sec. III, we present in Sec. IV numerical and analytical solutions of the appropriate dispersion relation, which demonstrate that the magnetoacoustic cyclotron instability can be driven by an anisotropic distribution of sub-Alfvénic fusion ions. In Sec. IV we also examine the dependence of the instability growth rate on the width in velocity space of the fusion ion distribution, and demonstrate that the time evolution of the maximum growth rate closely follows that of the ICE amplitude observed in several TFTR supershot discharges. By proving in Sec. V that the instability cannot occur if the fusion ions are both sub-Alfvénic and isotropic, we obtain a further constraint on the distribution function of those ions in the edge plasma. In Sec. VI we conclude that our analysis provides an explanation of the fact that fusion product-driven ICE in JET persists throughout Ohmic and NBI-heated discharges,[8] whereas fusion product-driven ICE in TFTR supershot discharges is essentially transient.

II. FUSION ION DISTRIBUTIONS IN THE EDGE PLASMA

The maximum radial excursion $r_{\max}$ of a centrally born fusion product is given approximately by[19,22]

$$r_{\max} \simeq R_0 (2.9 \rho q / R_0)^{2/3}, \tag{1}$$

where R_0 is the plasma major radius, ρ is the fusion product Larmor radius, and q is the tokamak safety factor, assumed to be independent of minor radial distance r.[19] Thus, we may take

$$q \simeq \frac{a B_\phi}{R_0 B_\theta} = \frac{2\pi a^2}{R_0} \frac{B_\phi}{\mu_0 I_p}, \tag{2}$$

where a is the plasma minor radius, B_ϕ, B_θ are the toroidal and poloidal components of the magnetic field, I_p is the plasma current, and μ_0 is the vacuum permeability. Equation (2) provides a somewhat crude average value for q, since in reality it is strongly r dependent, ranging from approximately unity on the magnetic axis to about 5–6 at the plasma edge [see Fig. 1(k) of Ref. 21]. For the deuterium–tritium experiments in TFTR the relevant parameters are $a = 0.87$ m, $R_0 = 2.52$ m, $B_\phi = 5.0$ T, and $I_p = 2.0$ MA,[1] and so, according to Eq. (2), we have $q \simeq 3.75$. Fusion products that undergo large radial excursions have pitch angles ψ lying close to the trapped–passing boundary, so that $\sin \psi \simeq 1$,[8,19,20] and therefore ρ is evaluated in Eq. (2) for $v_\perp \simeq v_\alpha$, where $v_\perp$ is the velocity component perpendicular to the magnetic field and v_α is the birth speed. Given that tritons, helium-3 nuclei, and alpha particles are born with energies of 1.0, 0.82, and 3.6 MeV, respectively,[10] and using the values of B_ϕ, R_0, a, and q given above, we can infer from Eq. (1) that $r_{\max} \simeq 1.05a$ (T), $0.62a$ (^{3}He), and $1.11a$ (^{4}He). This implies that newly born tritons and alpha particles can undergo large excursion orbits from the plasma center to the plasma edge, while helium-3 nuclei cannot: a similar conclusion was reached in the case of D–T experiments in JET.[8] We note, however, that the figures quoted above are based on the model approximation of constant q, and we cannot rule out the possibility of centrally born helium-3 nuclei reaching the edge plasma. In any case, the fusion ion production profile, although peaked on the magnetic axis, has a finite radial width, and, in view of the monotonic increase of q with r, it is reasonable to assume that the radial excursions of helium-3 nuclei created off axis will be larger than those created on axis.

The values obtained above for $r_{\max}$, and the finite width of the fusion ion production profile, suggest that fusion tritons and alpha particles with a broad range of energies below their respective birth energies can reach the edge plasma of TFTR. The growth rate of the magnetoacoustic cyclotron instability depends critically on the narrowness of the fusion ion velocity distribution f_α.[15] Immediately after the onset of neutral beam injection, fusion ions will be created in the plasma with a very narrow range of energies, centered on the birth energies quoted above. As the discharge progresses, the fusion reactivity continues to rise steadily for about 400 ms [see Fig. 2(a) of Ref. 1], providing a continuous supply of essentially monoenergetic fusion ions that can penetrate the edge plasma. At the same time, fusion ions created earlier in the discharge will have undergone collisional degradation, and the resulting spread in f_α will tend to suppress the magnetoacoustic cyclotron instability. There are two critical time scales here: τ_N, the rise time of the neutron emission; and $\tau_{\rm slow}$, the fusion ion slowing-down time. In early generic discussions of alpha particle-driven instabilities, Sigmar,[23] Kolesnichenko,[24] and Cordey *et al.*[25] pointed out that if $\tau_{\rm slow}<\tau_N$, the energy distribution of fusion products will be broadened by collisions before it can be replenished by newly created monoenergetic ions, thus switching off any velocity–space instability. In the course of a single large excursion orbit, a fusion ion spends most of its time close to the banana tips, far from the plasma center (see, e.g., Fig. 14 of Ref. 8). The collisional degradation of such ions is thus determined by the value of $\tau_{\rm slow}$ close to the plasma edge: from Fig. 4(i) of Ref. 21, we find that $\tau_{\rm slow}\simeq 150$ ms for alpha particles in the edge plasma of TFTR. The neutron emission rise time can be estimated from the slope of Fig. 2(a) in Ref. 1. Defining τ_N to be $F_N(t)/\dot{F}_N(t)$, where F_N is the neutron flux, we find that τ_N is initially about 50 ms, and increases to about 500 ms in the phase where F_N rises toward its peak value, before tending to infinity as F_N approaches quasi-steady state. The neutron emission rise time is less than the alpha particle slowing-down time for about the first 100 ms of the discharge, during which fusion ion-driven ICE is observed. This suggests that the temporal evolution of the ICE is indeed determined by the competing effects of fusion reactivity and collisional degradation.

We can place the above argument on a more quantitative footing as follows by considering a specific, simple model for the temporal evolution of f_α in the region of fusion ion production. Neglecting temporal variations in the bulk plasma parameters, particle losses, departures from isotropy, and diffusion in v space, and assuming that the fusion ions are created with a unique speed v_α, we can follow Sigmar[23] in writing a model distribution,

$$f_\alpha(v,t)=\frac{\tau_s}{4\pi}\,\frac{S_0[t-T(v)]}{v^3+v_c^3}\,H[v-V(t)]H(v_\alpha-v), \quad (3)$$

where S_0 is the production rate of fusion ions per unit volume; H is the Heaviside function; $T(v)$ and $V(t)$ are, respectively, given by

$$T(v)=\frac{\tau_s}{3}\log\left(\frac{v_\alpha^3+v_c^3}{v^3+v_c^3}\right), \quad (4)$$

$$V(t)=[(v_\alpha^3+v_c^3)\exp(-3t/\tau_s)-v_c^3]^{1/3}; \quad (5)$$

and expressions for the collision time τ_s and critical velocity v_c can be found in Ref. 26. The speed distribution at a given t depends on the fusion ion production rate at the retarded time $t-T(v)$. An appropriate analytical representation of the function $S_0(t)$ is one that is broadly consistent with the observed neutron emission. Figure 1 shows the time evolution of the 14 MeV neutron flux F_N measured during a D–T neutral beam-heated discharge in TFTR.[1] The dashed line in this figure is the function $R_0\sin^2(\pi t/2\tau_0)$, with R_0 equal to the maximum observed neutron flux and $\tau_0=500$ ms. Given that $F_N(t)\propto S_0(t)$, we infer from Fig. 1 that $S_0(t\geqslant 0)\propto\sin^2(\pi t/2\tau_0)$ is a fairly accurate representation of the alpha particle production rate in TFTR. We thus have, according to Eq. (3), a model fusion product distribution of the form

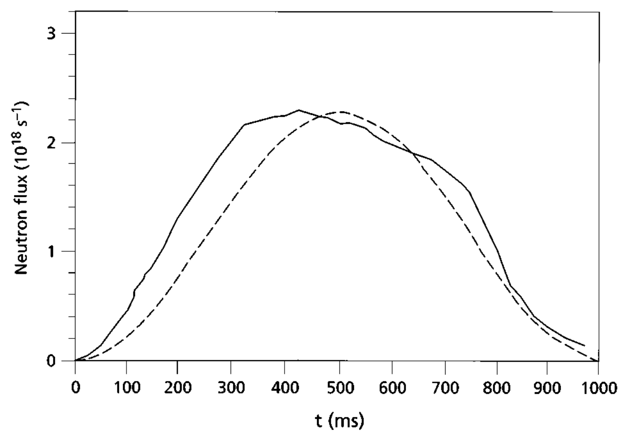

FIG. 1. Solid line: 14 MeV neutron flux measured during a D–T neutral beam-heated discharge in TFTR (Ref. 1). Dashed line: the function $R_0\sin^2(\pi t/2\tau_0)$, with R_0 equal to the maximum observed neutron flux and $\tau_0=500$ ms. The neutral beams were switched on at $t=0$.

$$f_\alpha(v,t)\propto\frac{\sin^2[\pi[t-T(v)]/2\tau_0]}{v^3+v_c^3}\,H[v-V(t)]H(v_\alpha-v). \quad (6)$$

We have evaluated this expression for the case of alpha particles in TFTR, with $v_\alpha\simeq 1.3\times 10^7$ ms^{-1}. The parameter v_c depends on the electron temperature T_e, while τ_s depends on both T_e and the electron density n_e. As we discussed earlier in this section, the collisional evolution of those fusion ions that can reach the edge plasma is determined predominantly by the values of plasma parameters close to $r=a$. Thus, following Figs. 1(c) and 1(d) in Ref. 21, we set $T_e=3$ keV and $n_e=2\times 10^{19}$ m^{-3}. With τ_0 set equal to 500 ms, we then have all the parameters we require in order to evaluate f_α in Eq. (6). Figure 2 shows the alpha particle speed distribution at various times after $t=0$. As expected, f_α at $v=v_\alpha$ rises rapidly, but the width of the distribution also increases. Defining Δv to be the full width of $f_\alpha(v)$ at half-maximum, we find in the case of Fig. 2 that Δv rises from $0.03v_\alpha$ at $t=50$ ms to $0.26v_\alpha$ at $t=300$ ms. In general, Δv depends on both collisional effects (τ_s and v_c) and the fusion reactivity rise time (τ_0 in our model).

The argument outlined above is somewhat crude, neglecting as it does many important effects. For example, we have neglected the velocity spread in the fusion product population arising from the finite temperature of the reacting nuclei. Also, we would expect the electron temperature to rise rapidly during the first few hundred milliseconds of the discharge, and so the values adopted above for τ_s and v_c should be regarded as temporal and spatial averages. We could, in principle, take into account other effects, such as particle losses and velocity–space diffusion. However, our aim here is to establish whether or not the essential features of fusion product-driven ICE in TFTR can be understood in terms of fusion product populations which are described in terms of a very small number of free parameters (T_e, n_e and

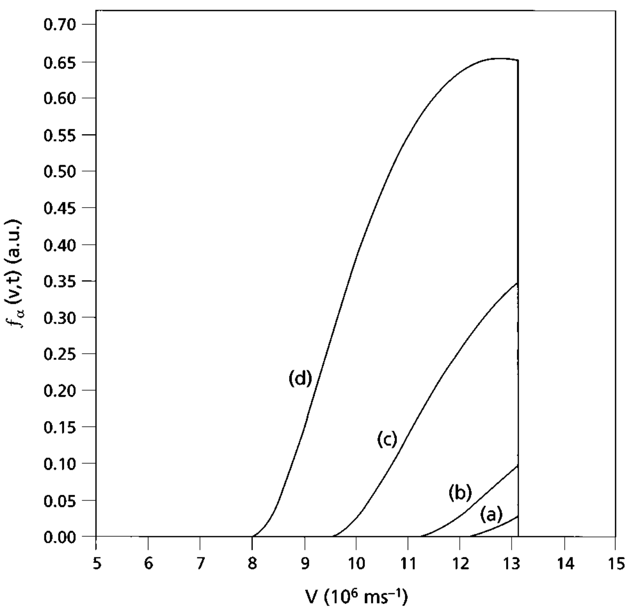

FIG. 2. Alpha particle speed distribution at various times, according to Eq. (6): (a) $t=50$ ms; (b) $t=100$ ms; (c) $t=200$ ms; and (d) $t=300$ ms. The relevant parameters are $T_e=3$ keV, $n_e=2\times10^{19}$ m^{-3}, and $\tau_0=500$ ms.

τ_0). In the next three sections we examine quantitatively the possibility that sub-Alfvénic fusion products with a sharply peaked velocity distribution can drive the magnetoacoustic cyclotron instability, and thus be responsible for ICE.

III. THEORY OF INSTABILITY FOR OBLIQUE PROPAGATION

Fast Alfvén (magnetoacoustic) waves propagating at large angles with respect to the magnetic field can be excited at frequencies lying close to harmonics of the cyclotron frequency of an energetic ion population whose velocity-space distribution is non-Maxwellian: hence the term "magnetoacoustic cyclotron instability." In Ref. 15, numerical and analytical solutions of the appropriate dispersion relation were obtained for the case of a normalized energetic ion distribution of the form

$$f_\alpha=\frac{1}{2\pi^{3/2}uv_r}\exp\left(-\frac{(v_\parallel-v_d)^2}{v_r^2}\right)\delta(v_\perp-u), \tag{7}$$

where $v_\parallel$ is the parallel velocity component and u, v_d, v_r are constants that, respectively, define the perpendicular speed, average parallel drift speed, and parallel velocity spread of the energetic ions. Equation (7) appears to contain a sufficiently large number of free parameters to provide a reasonable first approximation to the fusion ion velocity distribution in the edge plasma of TFTR. Calculations of large excursion orbits in JET[8] indicate that f_α in the edge plasma is nonzero in a narrow, wedge-shaped region of $(v_\parallel,v_\perp)$ space, centered on a finite negative value of $v_\parallel$ (also see Ref. 20). We thus have $v_d\neq0$, and v_r gives a measure of the spread of parallel velocities within the large excursion wedge. If $r_{\max}\lesssim a$, as is the case for fusion products in JET, the wedge is always small, since the edge plasma is only accessible to ions with pitch angles lying close to the trapped–passing boundary and energies lying close to the birth energy (see Fig. 15 in Ref. 8). If, on the other hand, $r_{\max}>a$, which we have shown to be the case for alpha particles and tritons in TFTR, the large excursion wedge occupies a larger area of $(v_\parallel,v_\perp)$ space. If the energy distribution of fusion ions in the center of the plasma is sufficiently broad, the large excursion wedge will be evenly populated. These considerations, coupled with the arguments presented in Sec. II, suggest that the effective value of v_r increases with time.

The results presented in Ref. 15 are based on the approximation that the wave electric field is polarized in the plane perpendicular to the ambient magnetic field. This approximation excludes the possibility of Landau damping, and means that the results are only strictly self-consistent if the propagation angle of the wave relative to the magnetic field direction is large. However, bulk ion cyclotron damping and electron transit time damping are included in the analysis. It is assumed that the bulk plasma consists of electrons and a single ion species, both of which have Maxwellian distributions. Strictly speaking, the theory must be modified if non-Maxwellian beam ions (tritons or deuterons) are present in the plasma. Since, however, beam ions constitute only a small fraction of the ion density in the edge plasma of TFTR supershots [see, e.g., Fig. 1(d) of Ref. 21], their contribution to bulk plasma damping can be safely neglected. In the limit of small $\xi\equiv n_\alpha/n_i$, where n_α,n_i are the energetic and bulk ion number densities, one can obtain the following expression for the growth or damping rate γ corresponding to Eq. (7):

$$\gamma=\frac{\omega_{p\alpha}^2}{\omega_{pi}^2}\frac{\Omega_i^4}{[\Omega_i+(\omega-\Omega_i)N_\parallel^2][\Omega_i-(\omega+\Omega_i)N_\parallel^2]}\times\left(\frac{l\Omega_\alpha}{k_\parallel v_r}M_l-\frac{2u^2}{v_r^2}\eta_lN_l\right)\frac{\sqrt{\pi}}{2\omega}e^{-\eta_l^2}. \tag{8}$$

In this expression, cyclotron and plasma frequencies are denoted by Ω and ω_p, the subscripts i and α referring as before to bulk and energetic ions; $N_\parallel=k_\parallel c_A/\omega$, where $k_\parallel$ is the parallel component of the wave vector, c_A is the Alfvén speed, and ω is the wave frequency; $\eta_l=(\omega-k_\parallel v_d-l\Omega_\alpha)/k_\parallel v_r$; l is a positive integer such that $l\Omega_\alpha$ is the energetic ion cyclotron harmonic lying closest to ω; and M_l,N_l are given by

$$M_l=2l\frac{\omega}{\Omega_i}\left(J_l'^2+\frac{1}{z_\alpha^2}(l^2-z_\alpha^2)J_l^2\right)-2\frac{\omega^2-\Omega_i^2}{\Omega_i^2}\frac{J_lJ_l'}{z_\alpha}[l^2N_\perp^2-(z_\alpha^2-2l^2)N_\parallel^2]+\frac{2J_lJ_l'}{z_\alpha}(z_\alpha^2-2l^2), \tag{9}$$

$$N_l=-2l\frac{\omega}{\Omega_i}\frac{J_lJ_l'}{z_\alpha}+\frac{\omega^2-\Omega_i^2}{\Omega_i^2}\left[N_\parallel^2\left(\frac{l^2J_l^2}{z_\alpha^2}+J_l'^2\right)+N_\perp^2\frac{l^2J_l^2}{z_\alpha^2}\right]+\frac{l^2J_l^2}{z_\alpha^2}+J_l'^2, \tag{10}$$

where $N_\perp = k_\perp c_A/\omega$ and J_l is the Bessel function of order l, with argument $z_\alpha = k_\perp u/\Omega_\alpha$, $k_\perp$ being the perpendicular component of the wave vector. The wave frequency is given by

$$\omega^2 = \frac{1}{2} c_A^2 \left[k^2 + k_\parallel^2 + k^2 k_\parallel^2 \frac{c_A^2}{\Omega_i^2} + \sqrt{\left(k^2 + k_\parallel^2 + k^2 k_\parallel^2 \frac{c_A^2}{\Omega_i^2} \right)^2 - 4k^2 k_\parallel^2} \right], \tag{11}$$

where $k^2 = k_\parallel^2 + k_\perp^2$. Positive values of γ imply instability. The quantity N_l is usually positive, and therefore destabilizing, only if $\eta_l < 0$, i.e. $\omega - k_\parallel v_d < l\Omega_\alpha$. The sign of the M_l term, on the other hand, depends on the values of l and z_α, but for any particular harmonic is independent of the sign of η_l.[15] Let us now examine whether or not linear instability can occur in the régime of interest to TFTR, using both numerical solutions of the full dispersion relation given in Ref. 15 and the approximate expressions given above.

IV. STABILITY OF ANISOTROPIC FUSION IONS IN TFTR

We noted in the Introduction that the birth speed v_α of fusion alpha particles is smaller than c_A at the outer edge of the TFTR plasma. Since the parameter u defined in Eq. (7) cannot be greater than v_α, it follows that $u < c_A$. More precisely, from Fig. 4(j) of Ref. 21, we find that $u \lesssim 0.55 c_A$. When the magnetoacoustic cyclotron instability occurs, it does so at wave numbers such that the lth cyclotron harmonic of the fusion ions lies close to the fast magnetoacoustic wave frequency, which, according to Eq. (11), is approximately equal to $k_\perp c_A$. The corresponding value of the parameter z_α is lu/c_A. Equation (8) indicates that the magnitude of any growth or damping is determined essentially by the values of M_l and N_l at $z_\alpha \simeq lu/c_A$: for $u/c_A \lesssim 0.55$ and $l=4$, say, we have $lu/c_A \lesssim 2.2$, in which case both M_4 and N_4 are very small (see Fig. 5 of Ref. 15). In general, although both growth and damping can occur at any cyclotron harmonic, the peak magnitude of γ falls off extremely rapidly with l when u is significantly smaller than c_A.

We now proceed to compute the growth rates of obliquely propagating fast Alfvén waves for the case of sub-Alfvénic fusion ions in TFTR. For any given harmonic, γ/Ω_α depends on the following parameters:[15] the wave propagation angle θ; the dimensionless perpendicular wave number z_α; the bulk ion plasma beta β_i; the concentration of fusion ions $\xi \equiv n_\alpha/n_i$; and the velocity ratios u/c_A, v_d/u, and v_r/u. We set $\xi = 10^{-4}$, $v_d/u = -0.5$, and $v_r/u = 0.05$ for all three fusion ion species. Given that $c_A \simeq 2.2 \times 10^7$ ms^{-1} in the TFTR edge plasma [see Fig. 4(j) of Ref. 21], and using the fusion ion birth energies quoted in Sec. II, we infer that $u/c_A \simeq 0.3$ (T), 0.27 (^{3}He), and 0.5 (^{4}He). If the Doppler shift $k_\parallel v_d$ is sufficiently large, the growth rates are insensitive to β_i, which we set equal to 10^{-4} [see Fig. 4(g) of Ref. 21]. The bulk plasma is assumed to be pure deuterium.

Figure 3 shows γ as a function of $\omega' \equiv \omega - k_\parallel v_d$ in the vicinity of $\omega' \simeq 2\Omega_\alpha$ for the case of sub-Alfvénic alpha particles. The propagation angle is 80°. The dotted line was

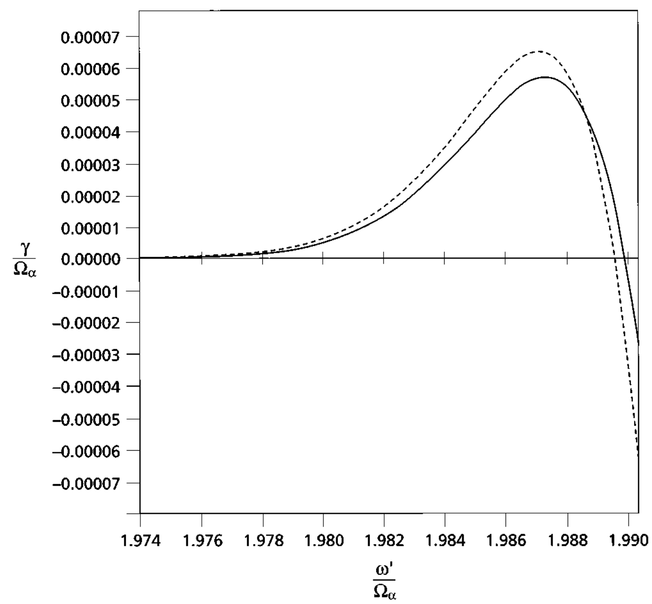

FIG. 3. Growth rate of obliquely propagating waves as a function of $\omega' \equiv \omega - k_\parallel v_d$ for the case of sub-Alfvénic alpha particles in the edge plasma of TFTR. Numerical results are indicated by a solid line, and analytical results by a dashed line. The wave propagation angle is 80° and the distribution function parameters are the following: $\beta_i = 10^{-4}$, $\xi = 10^{-4}$, $u/c_A = 0.5$, $v_d/u = -0.5$, and $v_r/u = 0.05$.

obtained from Eq. (8), and the solid line is an exact numerical solution of the full dispersion relation. Equation (8) clearly provides a good approximation to the growth rate, which, in the rest frame of the fusion ion guiding centers, peaks at a frequency lying slightly below the cyclotron harmonic. Heavy damping generally occurs on the high-frequency side of the Doppler-shifted harmonic, which can be understood in terms of Eq. (8) by recognizing that M_l is generally negative and N_l is generally positive (at least for $l > 1$) when $z_\alpha \simeq lu/c_A$ and $u < c_A$. The peak value of γ in Fig. 3 corresponds to a growth time of about 0.1 ms, which is much smaller than the observed neutron emission rise time (see Fig. 1).

In Table I we list the numerically determined maximum growth rates in $|\mathbf{k}|$ space of the first three cyclotron harmonics of T, ^{3}He, and ^{4}He for a propagation angle of 80°. The plasma parameters are identical to those used in Fig. 2. Table I illustrates the point made above, that γ falls off extremely rapidly with l whenever u is significantly less than the Alfvén speed. This is particularly true of T and ^{3}He. When

TABLE I. Maximum growth rates of the first three cyclotron harmonics of T, ^{3}He, and ^{4}He. The wave propagation angle θ is 80° and the distribution function parameters are the following: $\beta_i = 10^{-4}$, $\xi = 10^{-4}$, $u/c_A = 0.3$ (T), 0.27 (^{3}He), and 0.5 (^{4}He), $v_d/u = -0.5$, and $v_r/u = 0.05$. For $\theta = 100°$, instability occurs close to the cyclotron frequency ($l=1$) of ^{4}He.

Species	$\gamma_{max}/\Omega_\alpha$ $l=1$	$l=2$	$l=3$
T	1.6×10^{-4}	7.1×10^{-8}	0
^{3}He	5.1×10^{-5}	3.8×10^{-7}	0
^{4}He	0	5.7×10^{-5}	2.6×10^{-6}

$\theta=80°$ there is no instability at frequencies lying close to the alpha particle cyclotron frequency because $N_l<0$ at $z_\alpha \simeq u/c_A$. This inequality is reversed, however, when $\theta>90°$, in which case instability is possible: setting $\theta=100°$, for example, we find a maximum growth rate at $\omega \simeq \Omega_\alpha$ of the order of $10^{-5}\Omega_\alpha$.

For the parameter sets used to obtain Table I, one can easily verify that the transit time damping rate is completely negligible [see Eq. (31) in Ref. 15]. We infer from this that our neglect of electron Landau damping is justified.[15] The threshold for instability might be determined instead by collisional damping, which can be incorporated into a linear stability analysis by adding terms of the form $i\nu$ to the resonant denominators appearing in the dielectric tensor integrands.[27] The collision frequency ν is a complicated function of velocity, and so the bulk plasma contributions to the dielectric tensor can no longer be written in terms of the plasma dispersion function, as in Ref. 15. For this reason, a completely rigorous treatment of collisional damping would require a rather detailed analysis, which lies well beyond the scope of the present paper. However, it has been shown in Ref. 27 that the effect of particle collisions on various wave modes is reduced by a near-cancellation of collision terms in the cold plasma susceptibility. A straightforward calculation shows that this result holds in the case of fast Alfvén waves. Collisional damping is thus expected to be a second-order effect, smaller than the characteristic particle collision frequency. Evaluating the electron–ion collision frequency given in Ref. 28, for example, using appropriate values for n_e and the bulk ion temperature T_i in the edge plasma of TFTR during D–T experiments (10^{19} m^{-3} and 2 keV), we obtain $\sim 10^{-6}\Omega_\alpha$. Since the collisional damping rate is expected to be less than this, we conclude that the growth rates listed in Table I are unlikely to be significantly affected by collisions. It should be stressed, moreover, that the growth rate depends linearly on ξ, which may increase rapidly toward the plasma edge, because of the steep gradient in n_i: the value adopted for ξ in Fig. 3 and Table I may thus be an underestimate. Finally, it is worth pointing out that the growth rate is a sensitive function of θ, and that there is nothing significant about the value of θ used to obtain Fig. 3 and Table I: 80° is not the maximally unstable propagation angle.

We have also computed linear growth rates corresponding to fusion ion distributions of the form

$$f_\alpha = \frac{1}{2\pi^2 u v_{r\parallel} v_{r\perp}} \exp\left(-\frac{(v_\parallel - v_d)^2}{v_{r\parallel}^2} - \frac{(v_\perp - u)^2}{v_{r\perp}^2} \right). \quad (12)$$

This is similar to Eq. (7), except that there is now a finite spread of velocities in the perpendicular direction. Figure 4 shows γ as a function of ω' in the vicinity of $\omega' \simeq 2\Omega_\alpha$ for the case of an alpha particle velocity distribution given by Eq. (12). The wave propagation angle is again 80°, and the distribution function parameters other than $v_{r\parallel}/u$ and $v_{r\perp}/u$ are identical to those assumed in Fig. 3. In each case $v_{r\parallel}=v_{r\perp}$, the solid, dashed, and dotted lines corresponding, respectively, to $v_{r\parallel}/u=0.04$, 0.05, and 0.06. We note that a small increase in the alpha particle velocity spread causes a massive reduction in the maximum growth rate. As $v_{r\parallel}$ and $v_{r\perp}$ are increased further, the maximum value of γ rapidly tends to zero. The instability is much more robust, however, if the fusion ions are super-Alfvénic. Setting u/c_A equal to 1.5, which is representative of alpha particles in the edge plasma of JET,[8] we find linear growth rates of the order of $10^{-5}\Omega_\alpha$, even when $v_{r\parallel}/u=v_{r\perp}/u=0.2$. Figure 5 shows γ as a function of ω' for the case considered in Figs. 3 and 4, with $u/c_A=1.5$ and $v_{r\parallel}/u=v_{r\perp}/u=0.1$ (solid line), 0.125 (dashed line), and 0.15 (dotted line). As we discussed earlier, the

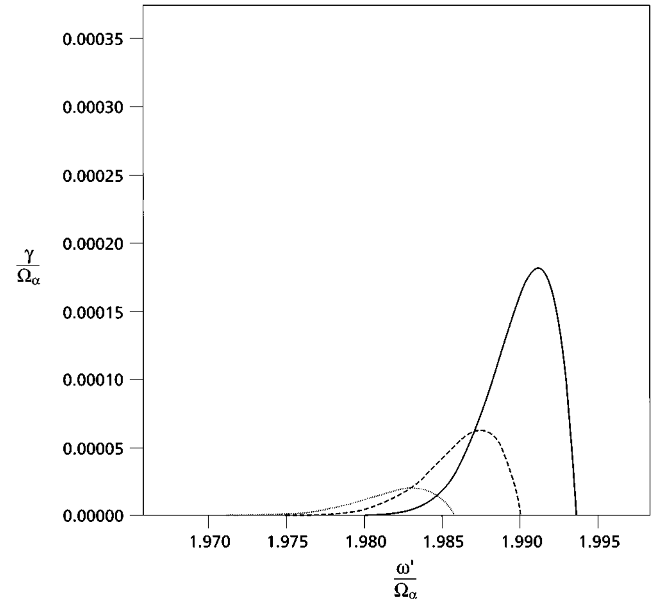

FIG. 4. Numerically computed growth rate as a function of $\omega' \equiv \omega - k_\parallel v_d$ for the case of an alpha particle velocity distribution given by Eq. (12). Only positive values of γ are shown. The wave propagation angle is again 80° and the distribution function parameters other than $v_{r\parallel}/u$ and $v_{r\perp}/u$ are identical to those assumed in Fig. 3. In each case $v_{r\parallel}=v_{r\perp}$, with $v_{r\parallel}/u=0.04$ (solid line), 0.05 (dashed line), and 0.06 (dotted line).

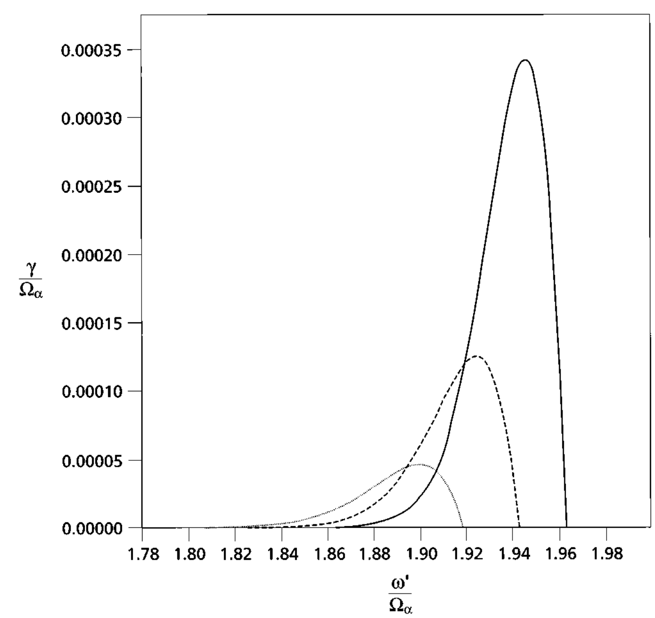

FIG. 5. The same as Fig. 4, except that $u/c_A=1.5$ and $v_{r\parallel}/u=v_{r\perp}/u=0.1$ (solid line), 0.125 (dashed line), and 0.15 (dotted line).

values of $v_{r\parallel}$ and $v_{r\perp}$ depend on the size of the large excursion wedge, and also on the degree to which fusion ions in the center of the plasma have become thermalized. In the case of JET, the large excursion wedge is relatively small.[8] The fusion ion distribution in the outer plasma edge can thus remain sufficiently narrow in velocity space for instability to persist, despite collisional broadening of f_α in the plasma center. In TFTR the large excursion wedge is larger, and in the course of time becomes more evenly populated as f_α in the plasma center is broadened by collisions. The effective values of $v_{r\parallel}$ and $v_{r\perp}$ thus rise appreciably, and, according to Fig. 4, the instability is rapidly switched off.

To demonstrate this effect, we combine our linear stability analysis with Eq. (6), assuming for simplicity that the latter gives the width in velocity space of f_α in the plasma edge. In that case, equating the full width at half-maximum of the distribution given by Eq. (6) with that of the distribution given by Eq. (12), and assuming $v_{r\parallel}=v_{r\perp}$, we infer that $v_{r\parallel}=\Delta v/2(\log 2)^{1/2}\simeq 0.6\,\Delta v$. If particle losses are negligible, n_α is equal to the time-integrated fusion ion production rate, and therefore

$$\xi \equiv \frac{n_\alpha}{n_i} = \frac{R_0}{n_i}\int_0^t \sin^2\left(\frac{\pi t'}{2\tau_0}\right) dt' = \xi_0\left(\frac{t}{\tau_0} - \frac{1}{\pi}\sin\frac{\pi t}{\tau_0}\right), \tag{13}$$

where ξ_0 is a constant. We consider the specific case of wave emission observed[2] at the ^{3}He cyclotron frequency in six deuterium beam-heated TFTR discharges (shot numbers 73451, 73455, 73456, 73458, 73459, and 73460), in which the neutral beam power was 20–22 MW, the plasma current was 1.8 MA, and the toroidal magnetic field was 4.8 T. We choose the case of emission at the ^{3}He cyclotron frequency because it can be attributed unambiguously to the presence of fusion products, rather than beam ions (proton cyclotron harmonics, for example, are degenerate with even deuteron harmonics, and therefore ICE driven by fusion protons could be confused with ICE driven by beam deuterons). In Fig. 6 the data points joined by dashed lines represent the average wave amplitude observed in the six discharges during the 150 ms following the start of beam injection. The amplitude scale is linear, and the units are arbitrary. The noise level in the original ICE intensity data is about ± 5 dB. The solid curve in Fig. 6 shows $\gamma_{\max}$, the maximum growth rate in $|\mathbf{k}|$ space of waves with $\omega' \simeq \Omega_\alpha$ and propagation angle $\theta=80°$. The parameter Δv, from which we obtain $v_{r\parallel}$ and $v_{r\perp}$, was computed as a function of time from Eq. (6), using the same values of n_e, T_e, and τ_0 as those used to obtain Fig. 2, namely 2×10^{19} m^{-3}, 3 keV, and 500 ms. The ^{3}He concentration was obtained from Eq. (13), with $\xi_0=10^{-2}$. The model parameters other than ξ, $v_{r\parallel}$, and $v_{r\perp}$ are identical to those assumed in Table I. There is a striking correlation between the time evolution of our computed $\gamma_{\max}$ and the average observed ICE amplitude: the growth rate peaks at the same time as the average ICE amplitude, and the two quantities evolve on very similar time scales. Several assumptions and approximations have been made in order to compute $\gamma_{\max}$ as a function of time, and we have so far omitted from our analysis any consideration of wave propagation and nonlinear effects, upon which the time evolution of ICE amplitude must partly depend. The growth rates shown in Fig. 6 are sufficiently high that wave amplification to 30 dB above the continuum noise level, as observed in D–T experiments on TFTR,[2,3] can occur in a fraction of a millisecond: $\gamma_{\max}$ in Fig. 6 rises and decays on a much longer time scale. At any instant, it appears that the observed wave amplitude is closely correlated with the linear growth rate corresponding to the plasma and fusion product parameters at that instant. It is possible that a nonlinear damping process (involving, for example, wave–wave coupling) rapidly dissipates the wave energy in the system at any particular time, so that a continuous drive is required in order to maintain the ICE amplitude over time scales of the order of 100 ms. One would then expect a temporal correlation between linear growth rate and wave amplitude, of the type that is apparent in Fig. 6.

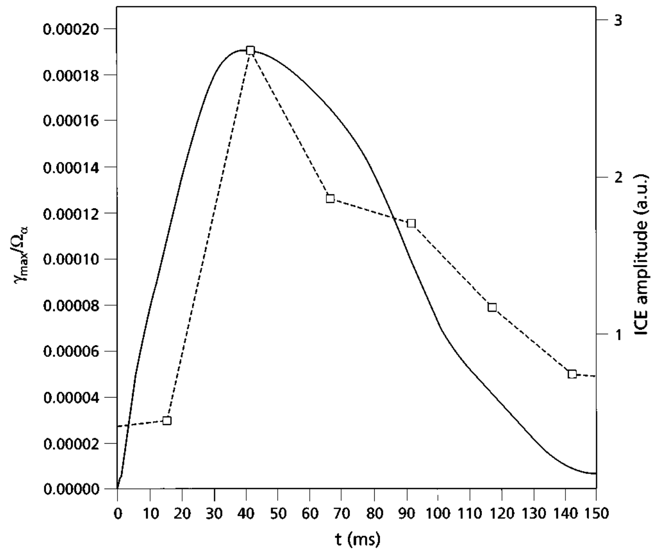

FIG. 6. Data points: time evolution of ICE amplitude at the ^{3}He cyclotron frequency, averaged over several deuterium beam-heated TFTR discharges (shot numbers 73451, 73455, 73456, 73458, 73459, and 73460: see Ref. 2). Solid curve: maximum growth rate of obliquely propagating waves at the same frequency. The wave propagation angle is 80°, and the distribution function parameters ξ and $v_{r\parallel}/u$ (the latter assumed equal to $v_{r\perp}/u$) are computed as a function of time, in accordance with the model described in Ref. 21, using $n_e=2\times 10^{19}$ m^{-3}, $T_e=3$ keV, and $\tau_0=500$ ms. The ^{3}He concentration was evaluated using Eq. (13), with $\xi_0=10^{-2}$. The parameters other than ξ, $v_{r\parallel}$, and $v_{r\perp}$ are identical to those assumed in Table I. Note that the ICE amplitude is plotted on a linear scale.

Further clear evidence that the intensity and duration of fusion product-driven ICE depends critically on the value of u/c_A in the outer midplane has emerged recently from TFTR D–T experiments in which L modes were induced by He gas puffing.[29] The effect of gas puffing was to reduce c_A in the edge plasma, causing 3.6 MeV alpha particles to become super-Alfvénic in that region. At the same time, a very large ($\sim$30 dB) enhancement was observed in the intensity of alpha particle-driven ICE. High levels of ICE have also been observed from L mode discharges in which alpha particles remained super-Alfvénic throughout the duration of neutral beam injection.[29] For alpha particles with energies close to 3.6 MeV, the values of u/c_A in the edge plasmas of JET and TFTR L modes are similar, and so the magnetoacoustic cyclotron mode is strongly unstable in both machines, notwithstanding the fact that the large excursion wedge in TFTR becomes more evenly populated during the course of the discharge.

Although this paper is concerned primarily with the physics of the ICE emission mechanism in the localized source region, let us consider briefly the question of wave propagation. The ICE detectors on TFTR are mounted in the vertical plane of the magnetic axis, i.e. at poloidal angles of roughly $\pm 90°$ relative to the outer midplane. The spectral peak frequencies, however, indicate quite clearly that the emission originates from the outer midplane edge. We infer that the observed waves propagate predominantly in the poloidal direction, rather than the radial direction, and, furthermore, can propagate without being strongly damped in regions of the plasma that do not contain significant numbers of fusion products. These facts are entirely consistent with wave excitation via the magnetoacoustic cyclotron instability. As we noted above, transit time damping and electron Landau damping are negligible if $k_\parallel$ is sufficiently small. Bulk ion cyclotron damping is not negligible, and could attenuate the wave to some extent if it passes through a region in which $\omega \simeq s\Omega_i$, for some integer s. In the case of alpha particles in a deuterium plasma, $\Omega_\alpha = \Omega_i$, and, since the magnetic field has a minimum value at the outer midplane, a poloidally propagating wave that is generated in that region at $\omega < \Omega_\alpha = \Omega_i$ is unaffected by bulk cyclotron damping. Given the aspect ratio of the TFTR plasma ($R_0/a \simeq 3$), it is straightforward to show also that a wave generated in the outer midplane at $\omega \lesssim 2\Omega_\alpha$ never encounters a region in which $\omega \simeq \Omega_i$, and so the second harmonic is not affected by cyclotron damping either. A wave generated at the third alpha particle harmonic, on the other hand, can undergo second harmonic bulk cyclotron damping during the course of its poloidal circuit. At higher harmonics, bulk cyclotron damping is generally weak, and is only significant over a narrow range of frequencies in the vicinity of $s\Omega_i$ (see, for example, Fig. 2 in Ref. 15). Thus, damping will only occur in a thin layer of plasma, within which ω lies sufficiently close to $s\Omega_i$, and the aggregate bulk cyclotron damping in one circuit of the poloidal cross section will generally be small. We noted from Fig. 3 that heavy cyclotron damping, due to the fusion products themselves, frequently occurs on the high-frequency side of the Doppler-shifted cyclotron harmonic. This will have no effect on a poloidally propagating wave with $\omega - k_\parallel v_d \lesssim l\Omega_\alpha$ in the outer midplane edge, since such a wave never encounters a region containing significant numbers of fusion products in which the opposite inequality is satisfied.

On both experimental and theoretical grounds, it is thus reasonable to suppose that waves excited by the magnetoacoustic cyclotron instability in the outer midplane can propagate many times around the poloidal cross section, undergoing successive cumulative increments as they pass through the region populated by fusion products, i.e. the outer midplane. In this region, the magnetic field varies only weakly with poloidal angle, and so the cyclotron resonance condition $\omega - k_\parallel v_d = l\Omega_\alpha$ will be approximately satisfied over a sufficiently large section of the ray to give significant amplification.[4] A wave propagating radially outward, on the other hand, would traverse only once a narrow region of diminishing magnetic field, in which the density is also rapidly varying. We would not expect such a wave to undergo significant amplification in a single pass through the plasma edge. The wave could be reflected near the plasma edge, due to the fast wave cutoff, but it would then propagate to the center of the plasma, where strong cyclotron damping would occur: multiple passes of the outer midplane edge are not possible in such cases. Some radial propagation must occur in order for the wave to reach the detector outside the plasma, but this could occur when the wave has already been amplified to the observed level.

It appears likely that ICE originates from the outer midplane edge, essentially because of the existence of large excursion orbits for centrally born alpha particles (see, for example, Fig. 15 of Ref. 8), and the consequently unstable character of the local fusion product velocity distribution at the outer midplane. However, the line profile asymmetry that is apparent in Fig. 3 may also have some bearing on radial localization. Although, as we have seen, the most favorable conditions for instability occur at the outer midplane edge, it is possible that wave excitation might occur deep inside the plasma if the local fusion product velocity distribution were sufficiently narrow. This condition is unlikely to be satisfied (see, for example, the measured core alpha particle velocity distribution shown in Fig. 14 of Ref. 30). However, even if f_α deep inside the plasma were sufficiently narrow for waves to be generated, such waves would generally encounter a diminishing magnetic field as they propagated to the plasma edge, and consequently would undergo strong cyclotron damping in regions where $\omega - k_\parallel v_d \gtrsim l\Omega_\alpha$.[15] The observed radial localization of the wave source may thus reflect not only the locally unstable nature of the fusion product velocity distribution in that region, but also the fact that any waves generated deep inside the plasma would be strongly damped because of propagation effects. One could, in principle, use the observed width in frequency space of ICE harmonics to deduce information about the radial or poloidal extent of the wave source, but in the case of $k_\parallel \neq 0$ there is an intrinsic line width arising from Doppler broadening,[15] which would exist even if the plasma were homogeneous, and which may explain the observed doublet structure of the ICE spectral peaks observed in JET. The existence of Doppler broadening makes it difficult to infer information about radial propagation from the observed emission profile.

A more quantitative discussion of propagation effects would require a full wave calculation, and is beyond the scope of the present paper. Our main aim here has been to establish the extent to which fusion product-driven ICE in TFTR D–T discharges can be understood in terms of linear theory in the locally uniform approximation, without having to consider spatial variations in plasma parameters.

Wave emission has been observed at several sequential cyclotron harmonics of the fusion product ^{3}He in pure deuterium beam-heated discharges in TFTR:[2,3] there is no clear evidence of the intensity falling off rapidly with harmonic number, as implied by the linear growth rates in Table I. Our model does, however, demonstrate the essential point that obliquely propagating fast Alfvén waves can be destabilized by a sub-Alfvénic population of fusion products at frequencies close to their low cyclotron harmonics. As we noted in the Introduction, strictly perpendicular propagating fast Alfvén waves cannot be destabilized in the absence of super-Alfvénic ions. The inclusion of finite $k_\parallel$ draws in additional physics,[15] which, in combination with velocity space anisotropy, makes instability possible in the sub-Alfvénic régime. The importance of anisotropy in related electromagnetic instabilities has been known for some time (see, for example, Refs. 31 and 32, and references therein).

It is possible that the observed emission at harmonics $l \geqslant 3$ is the result of nonlinear wave–wave interactions, or, alternatively, that the inability of our model to predict instability at high l is simply due to the limitations of Eq. (7) and Eq. (13) as representations of fusion ion distributions in the TFTR edge plasma. In the next section we prove, however, that f_α must at least be anisotropic in order for sub-Alfvénic ions to drive the magnetoacoustic cyclotron instability at all.

V. STABILITY OF ISOTROPIC FUSION IONS IN TFTR

The perturbative analysis used to derive Eq. (8) can be generalized in a straightforward way to the case of a gyrotropic but otherwise arbitrary energetic ion distribution. Following the method outlined in Ref. 15, we obtain

$$\gamma = \frac{\omega_{p\alpha}^2}{\omega_{pi}^2} \frac{\Omega_i^4}{[\Omega_i + (\omega - \Omega_i) N_\parallel^2][\Omega_i - (\omega + \Omega_i) N_\parallel^2]} \frac{\pi^2}{\omega k_\parallel} \times \left[\left(\frac{\omega^2 - \Omega_i^2}{\Omega_i^2} N^2 + 1 \right) T_{xx} + \left(\frac{\omega^2 - \Omega_i^2}{\Omega_i^2} N_\parallel^2 + 1 \right) T_{yy} - 2 \frac{\omega}{\Omega_i} T_{xy} \right]. \tag{14}$$

Here $N^2 = N_\parallel^2 + N_\perp^2$ and the matrix elements T_{ij} are given by

$$T_{ij}\left[v_\parallel \equiv \frac{(\omega - l\Omega_\alpha)}{k_\parallel}, v_\perp \right] = \int_0^\infty v_\perp^2 \, dv_\perp \, F_{ij}(v_\perp) \left((\omega - k_\parallel v_\parallel) \frac{\partial f_\alpha}{\partial v_\perp} + k_\parallel v_\perp \frac{\partial f_\alpha}{\partial v_\parallel} \right), \tag{15}$$

where

$$F_{ij}(v_\perp) = \begin{pmatrix} (lJ_l/\zeta)^2 & lJ_l J_l'/\zeta \\ -lJ_l J_l'/\zeta & (J_l')^2 \end{pmatrix}, \tag{16}$$

the argument of J_l being $\zeta \equiv k_\perp v_\perp / \Omega_\alpha$. Equations (15) and (16) can be derived using the expressions for the dielectric tensor elements given, for example, in Ref. 33. We have taken the limit of small γ, in which case one may use the Landau prescription to evaluate the imaginary parts of the dielectric tensor elements.[33]

The integrals in Eq. (15) can be readily evaluated for the case of an isotropic infinitely thin shell distribution of energetic ions,

$$f_\alpha = \frac{1}{4\pi u^2} \delta(v - u). \tag{17}$$

Distributions of this type were used in Refs. 12–14 to study the magnetoacoustic cyclotron instability in the case of strictly perpendicular propagation ($k_\parallel = 0$). In Ref. 14, f_α was taken to be an isotropic shell of finite thickness. It is clear from these studies that instability cannot occur if $k_\parallel = 0$ and $u < c_A$. Substituting Eq. (17) into Eq. (15), we obtain, for $|\omega - l\Omega_\alpha| \neq k_\parallel u$,

$$\gamma = -2\pi \frac{\omega_{p\alpha}^2}{\omega_{pi}^2} \frac{\Omega_i^4 H(k_\parallel u - |\omega - l\Omega_\alpha|)}{[\Omega_i + (\omega - \Omega_i) N_\parallel^2][\Omega_i - (\omega + \Omega_i) N_\parallel^2][k_\parallel^2 u^2 - (\omega - l\Omega_\alpha)^2]^{1/2}} \left[\left(\frac{\omega^2 - \Omega_i^2}{\Omega_i^2} N^2 + 1 \right) \frac{l^2 J_l J_l'}{z_\alpha} + \left(\frac{\omega^2 - \Omega_i^2}{\Omega_i^2} N_\parallel^2 + 1 \right) \frac{(l^2 - \zeta_0^2) J_l J_l'}{z_\alpha} + \frac{l\zeta_0}{z_\alpha} \frac{\omega}{\Omega_i} \left((J_l')^2 + \frac{l^2 - \zeta_0^2}{\zeta_0^2} J_l^2 \right) \right], \tag{18}$$

where the Bessel function argument is now

$$\zeta_0 = \left(z_\alpha^2 - \frac{\tan^2\theta (\omega - l\Omega_\alpha)^2}{\Omega_\alpha^2} \right)^{1/2} \leqslant z_\alpha. \tag{19}$$

The Heaviside function in Eq. (18) has the effect of making γ identically zero outside a range of frequencies given by $\omega = l\Omega_\alpha \pm k_\parallel u$. The relevant value of z_α is again lu/c_A. If $u < c_A$, both J_l and its first derivative are then positive, and it is clear that every term in Eq. (18) makes a negative contribution to γ, so that fast Alfvén waves are damped. Since Eq. (17) represents the most singular (and presumably therefore the least stable) form of isotropic distribution, it follows from this that fast Alfvén waves cannot be destabilized by an isotropic distribution of sub-Alfvénic energetic ions. Our model for the emission mechanism thus implies that the distribution of fusion products in the edge plasma of TFTR cannot be isotropic. One can easily find values of $u/c_A > 1$, on the other hand, which yield $\gamma > 0$ in Eq. (18). In contrast to the sub-Alfvénic case, isotropic distributions of super-Alfvénic ions can thus destabilize both perpendicular propagating waves[12–14] and obliquely propagating waves.

VI. CONCLUSIONS

Confined fusion products created in the center of the TFTR plasma can undergo large excursion orbits to the plasma edge, where, during the early stages of beam-heated supershot discharges, they are likely to have a narrow range of sub-Alfvénic speeds and a highly anisotropic pitch angle distribution. In those circumstances the magnetoacoustic cyclotron instability can occur locally, giving rise to the excitation of obliquely propagating waves at frequencies lying close to fusion product cyclotron harmonics: such waves may be observed as ion cyclotron emission (ICE). The monotonically decreasing character of the fusion product distribution deeper inside the plasma[30] means that ICE is unlikely to be generated there; in any case, waves generated deep inside the plasma are likely to be strongly damped before reaching the detectors, because of propagation effects.[15] The location of the detectors, at poloidal angles of $\pm 90°$ relative to the midplane, implies that the waves propagate poloidally from the source region: poloidal propagation appears to be essential, in any case, in order for waves excited via the magnetoacoustic cyclotron instability to be amplified to the observed levels. If the fusion ions have an isotropic but otherwise arbitrary distribution, the instability cannot occur in the sub-Alfvénic regime, either for oblique or perpendicular propagation, although it can still operate in the super-Alfvénic regime characteristic of the JET H-mode outer midplane. Observations of fusion ion-driven ion cyclotron emission (ICE) in beam-heated TFTR discharges, if attributable to the magnetoacoustic cyclotron instability, can thus be used to set constraints on the velocity–space distribution of fusion ions in the edge plasma.

Further diagnostic information has been obtained by studying the time evolution of ICE spectral structure. Using a simple analytical model due to Sigmar,[23] one can compute the width in velocity space of the alpha particle population in the plasma center as a function of time. This model, combined with calculations of magnetoacoustic cyclotron instability growth rates for oblique propagation, indicates that the observed disappearance of fusion-driven ICE during the course of beam-heated supershot discharges in TFTR may be due to collisional broadening of the fusion ion population occurring more rapidly than the replenishment of the distribution by newly created ions. The time evolution of the computed maximum growth rate closely follows the observed time evolution of ICE amplitude, averaged over several TFTR supershot discharges: in order to fully understand the reasons for this correlation, it may be necessary to consider nonlinear processes, such as wave–wave coupling, which lie beyond the scope of the present paper. In the case of JET, the edge plasma is accessible to fusion ions with a narrower range of energies and pitch angles, and the magnetoacoustic cyclotron instability is consequently stronger. Furthermore, because fusion ions in JET are super-Alfvénic, the instability is, in any case, much less vulnerable to stabilization due to velocity space broadening. Recent studies of TFTR L modes provide further confirmation that the Alfvén speed in the outer midplane plays a crucial role in determining both the strength and the longevity of fusion product-driven ICE.[29] We thus have a convincing explanation of the fact that whereas fusion product-driven ICE in JET is continuous, the corresponding process in TFTR supershots only persists for about 100 ms after the onset of neutron emission. ICE thus provides a unique diagnostic of the early slowing-down evolution of the fusion alpha particle population in TFTR.

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