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Scattering of electromagnetic waves by counter-rotating vortex streets in plasmas

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The scattering of electromagnetic waves from counter-rotating vortex streets associated with nonlinear convective cells in uniform plasmas has been considered. The vortex street solution of the Navier–Stokes or the Hasegawa–Mima (and of the "sinh-Poisson") equation is adopted as a scatterer. Assuming arbitrary polarization and profile function for the incident electromagnetic field, a compact expression for the scattering cross section has been obtained. Specific results for the differential cross section are obtained for the case in which the incident beam has a Gaussian profile and propagates as an ordinary mode. The results show that when the characteristic wavelength of the vortex street ($\lambda_v = 2\pi/a$) is larger than that of the incident electromagnetic wave ($\lambda_i = 2\pi/k_i$), the differential cross section $d\sigma/d\Omega$ has a very well-defined angular periodicity; in fact, it is a collection of Gaussians varying as $\exp[-f(k_i w)^2]$, where w is the waist and f is a function expressing a kind of "Bragg condition." On the other hand, for $\lambda_i > \lambda_v$ the incident electromagnetic beam is unable to distinguish the periodic structure of the vortex street. The effects of the vortex street as well as the incident beam parameters on the scattering cross section are examined. © *1996 American Institute of Physics.* [S1070-664X(96)01203-4]

I. INTRODUCTION

Recently, considerable interest has been devoted to the study of drift vortex streets[1–3] in the framework of a more general concern for nonlinear structures in plasma turbulence. Vortex streets are exact stationary solutions of the pseudo-three-dimensional Hasegawa–Mima equation[4] and have the distinctive feature of being periodic in one of the dimensions. These periodic solutions are also present in the theory of vortices in fluids,[5] in the problem of acoustic-gravity waves[3] in the atmosphere, and in the dynamics of drift waves in dusty plasmas.[6] The vortex street that is often encountered in the literature[1–3,6] is represented by the so-called "Kelvin–Stuart's cat's eyes;"[1] but the one we found more appropriate for our purposes here is the "breather" solution.[5]

Drift-like vortex streets may be present in inertial confinement fusion and in tokamak plasmas, where they may play a role as one of the several components of the turbulent drift wave spectrum. We recall that the convective cell and drift wave turbulence are one of the possibilities for explaining the anomalous transport in fusion plasma experiments.[7–9] They can also be present in space plasmas, where they could act as an effective scatterer for radio waves in the Earth's lower ionosphere.

In this paper, we consider scattering of electromagnetic waves from counter-rotating vortex streets in plasmas, in order to assess whether the presence of convective cell vortex streets could be detected by this method. The main goal of this work is, therefore, to obtain analytical expressions for the scattering cross section associated with counter-rotating vortex streets. We closely follow the method of a recent work by Dendy and Mendonça,[10] where scattering of electromagnetic waves by *drift dipolar vortices* was considered.[10,11] The calculation is made in the first-order Born approximation and for an arbitrary polarization of the incident electromagnetic wave. Generalizing this method, we consider the possibility of an arbitrary wave profile and then specialize to the case of a Gaussian beam.

In Sec. II, we describe the geometry of the model and the nature of the incident electromagnetic wave and discuss the choice of the vortex street solution and the main assumptions involved in our model. In Sec. III, we derive an analytical expression for the scattering cross section involving arbitrary density perturbations. In Sec. IV, we present a simple case of a periodic (yet arbitrary) density perturbation and take advantage of the simplification it can introduce in the calculation. In Sec. V, we study a more specific situation, i.e., the propagation of a Gaussian beam that is polarized in the ordinary mode. Finally, Sec. VI contains the conclusions of our investigation.

II. THE MODEL

Let us assume an unbounded uniform plasma, embedded in an external static magnetic field $\mathbf{B}_0 = B_0\hat{\mathbf{z}}$, where $\hat{\mathbf{z}}$ is the unit vector along the z axis. The incident electromagnetic wave, with frequency ω_i and wave vector $\mathbf{k}_i$, propagates along an arbitrary direction to $\hat{\mathbf{z}}$, with an arbitrary polarization. We also assume that $\omega_i \gg \omega_{pe}, \omega_{ce}$, where ω_{pe} (ω_{ce}) is the electron plasma (gyro)frequency, so that we can neglect the effects of the density perturbation associated with electromagnetic waves. This condition is absolutely fulfilled by current laser experiments. We adopt the standard approach of scattering experiments that do not perturb the plasma because of the low power of the incident beam. Thus, the ponderomotive force effect, which is proportional to $(\omega_{pe}/\omega)^2 \ll 1$, is negligibly small, and we may assume that the vortex street structure is unaffected by the incident radiation.

We now turn our attention to the discussion of the counter-rotating vortex street, which is a stationary solution of the pseudo-three-dimensional Hasegawa–Mima equation,[4]

$$\frac{\partial}{\partial t}\,(\phi-\nabla_\perp^2\phi)-J(\phi,\nabla_\perp^2\phi)=0, \tag{1}$$

where the convective cell potential ϕ is in the unit of T_e/e, the time and space variables are normalized by ω_{ci}^{-1} and $\rho_s=c_s/\omega_{ci}$, respectively, c_s is the ion sound speed, and $J(f,g)=(\partial_a f)(\partial_y g)-(\partial_y f)(\partial_x g)$ is the Jacobian. Furthermore, the last term on the left-hand side of (1) is the usual Jacobian nonlinearity arising from the nonlinear ion polarization drift.

In order to obtain the stationary solution of (1), we set $\partial_t=0$ and note that $J(\phi,\nabla_\perp^2\phi)=0$. The latter is satisfied by $\nabla_\perp^2\phi=F(\phi)$, where F is any well-behaved function of ϕ. On choosing $F(\phi)=-(\phi_0/4)(a^2-b^2)\sinh(4\phi/\phi_0)$, $b\leqslant a$, where a, b, and ϕ_0 are arbitrary constants, we find that the "sinh-Poisson" equation has an exact solution,[5]

$$\phi(x,y)=\phi_0\left(\frac{b\,\cos(ay)}{a\,\cosh(bx)}\right),\quad b\leqslant a, \tag{2}$$

which represents a row of counter-rotating vortices.

We note that the "breather solution" (2) is bounded. On the other hand, if we choose $F(\phi)=(1-\epsilon^2)\exp(-2\phi)$, then the Liouville's equation $\nabla_\perp^2\phi=(1-\epsilon^2)\exp(-2\phi)$, where $|\epsilon|<1$, admits an infinite row of identical Stuart's vortices, given by $\phi(x,y)=\log(\cosh y-\epsilon\cos x)$; the latter then exhibits $\phi(x,\infty)\to\infty$. In the following we shall, therefore, use the solution (2) for our purposes. It is also interesting to note that on using $\operatorname{arctanh}(x)=(\frac{1}{2})\log[(1+x)/(1-x)]$ (for $|x|<1$, which is the case), ϕ can be written in the form

$$\phi(x,y)=\frac{\phi_0}{2}\{\log[a\,\cosh(bx)+b\,\cos(ay)] \\ -\log[a\,\cosh(bx)-b\,\cos(ay)]\}, \tag{3}$$

which is actually a subtraction of two Kelvin–Stuart's cat's eyes solutions.

Finally, we define the incident electromagnetic wave by writing the electric field vector $\mathbf{E}_i$ as

$$\mathbf{E}_i(\mathbf{r},t)=\mathrm{Re}\left(\frac{E_0}{2}\,\hat{\mathbf{a}}_i p(\mathbf{r})\exp[i(\mathbf{k}_i\cdot\mathbf{r}-\omega_i t)]\right), \tag{4}$$

where $\hat{\mathbf{a}}_i$ is the unit polarization vector and $p(\mathbf{r})$ is a dimensionless profile function.

III. THE SCATTERING CROSS SECTION

The wave equation for the scattered field is obtained[12] by combining the plasma hydrodynamic equations with Maxwell's equations. Assuming that the scattered field is polarized with a given unit vector $\hat{\mathbf{a}}_s$, we obtain

$$\left(\mathbf{k}\times\mathbf{k}\times\hat{\mathbf{a}}_s+\frac{\omega^2}{c^2}\,\boldsymbol{\epsilon}\cdot\hat{\mathbf{a}}_s\right)E_s(\mathbf{k},\omega)=-i\omega\mu_0\mathbf{J}_{\mathrm{nl}}(\mathbf{k},\omega), \tag{5}$$

where c is the speed of light, $\boldsymbol{\epsilon}$ is the dielectric tensor, and $\mathbf{J}_{\mathrm{nl}}$ is the nonlinear current density arising owing to the interaction of the vortex perturbation with the incident electromagnetic wave. Multiplying both sides of (5) by $\hat{\mathbf{a}}_s^*$ and rearranging terms, we have

$$E_s(\mathbf{k},\omega)=-\frac{i}{\epsilon_0\omega}\,\frac{\hat{\mathbf{a}}_s^*\cdot\mathbf{J}_{\mathrm{nl}}(\mathbf{k},\omega)}{D(\mathbf{k},\omega)}, \tag{6}$$

where

$$D(\mathbf{k},\omega)=(\mathbf{k}^2-|\mathbf{k}\cdot\hat{\mathbf{a}}_s|^2)\,\frac{c^2}{\omega^2}-\hat{\mathbf{a}}_s^*\cdot\boldsymbol{\epsilon}\cdot\hat{\mathbf{a}}_s \tag{7}$$

is the linear dispersion relation. Neglecting the ion contribution to the high-frequency electromagnetic waves, we retain only the terms due to the electrons in the nonlinear current density and write

$$\partial_t\mathbf{J}_{\mathrm{nl}}(\mathbf{r},t)=-e\,\partial_t(n_v v_i+n_i v_v)+\frac{n_0 e^2}{m}\left(\mathbf{v}_v\times\mathbf{B}_i \right. \\ \left. +\frac{m}{e}\,[(\mathbf{v}_i\cdot\boldsymbol{\nabla})\mathbf{v}_v+(\mathbf{v}_v\cdot\boldsymbol{\nabla})\mathbf{v}_i]\right), \tag{8}$$

where the subscripts v and i denote the corresponding quantities associated with the vortex and the incident wave, respectively, e is the magnitude of the electron charge, and m is the electron mass. Specifically, we have $n_v=n_0 e\phi/T_e$, where n_0 is the unperturbed plasma number density, $\mathbf{v}_v=(1/B_0)\hat{\mathbf{z}}\times\boldsymbol{\nabla}\phi$ is the vortex velocity vector, and $\mathbf{v}_i$ is obtained from $(\partial_t-\omega_{ce}\hat{\mathbf{z}}\times)\mathbf{v}_i=-(e/m)\mathbf{E}_i$. The electromagnetic fields are related by $\mathbf{B}_i=(\mathbf{k}_i/\omega_i)\times\mathbf{E}_i$. To obtain an estimate of the relative magnitude of various terms in (8), we linearize the differential operators and the hydrodynamic equations. We can neglect the term $-en_i\mathbf{v}_v$ in comparison with the term $-en_v\mathbf{v}_i$, because we are taking $\omega\gg\omega_{pe},\omega_{ce}$. Furthermore, the nonlinear Lorentz force and the advective convection terms are relatively smaller than $-e\partial_t(n_v\mathbf{v}_i)$, because we are assuming that the wave vector k_v and the frequency ω_v associated with the vortex perturbation satisfy $\omega_v/k_v=v_v<c$, which indeed is the case. Thus, we retain only the dominant nonlinear term $-e\partial_t(n_v\mathbf{v}_i)$, and write $\mathbf{J}_{\mathrm{nl}}(\omega,\mathbf{k})$ as

$$\mathbf{J}_{\rm nl}(\omega,\mathbf{k})=-e\pi\int \frac{d\mathbf{q}}{(2\pi)^3}\, n_v(\mathbf{k}-\mathbf{q})\boldsymbol{\mu}(\omega,\mathbf{q}) \times\left(\frac{E_0}{2}\,\hat{\mathbf{a}}_i p(\mathbf{q}-\mathbf{k}_i)\,\delta(\omega-\omega_i) + \frac{E_0^*}{2}\,\hat{\mathbf{a}}_i^* p(\mathbf{q}+\mathbf{k}_i)\,\delta(\omega+\omega_i)\right), \tag{9}$$

where $\boldsymbol{\mu}$ is the electron mobility tensor.

We now proceed to the calculation of the scattered power, by employing the usual definition,

$$P(t)=-\int \mathbf{J}_{\rm nl}(\mathbf{r},t)\cdot\mathbf{E}_s(\mathbf{r},t)d\mathbf{r}, \tag{10}$$

and averaging (10) over a time T long compared with $1/\omega_i$. On using (6) and (9) one can then express (10) as

$$\langle P\rangle=-\frac{ie^2|E_0|^2}{16\epsilon_0\omega_i}\int \frac{d\mathbf{k}}{(2\pi)^3}\frac{d\mathbf{q}}{(2\pi)^3}\frac{d\mathbf{q}'}{(2\pi)^3} \times n_v(\mathbf{k}-\mathbf{q})n_v^*(\mathbf{k}-\mathbf{q}')p(\mathbf{q}-\mathbf{k}_i)p^*(\mathbf{q}'-\mathbf{k}_i) \times\left(\frac{\mu^{*00}(\omega_i,\mathbf{q})\mu^{0**}(\omega_i,\mathbf{q}')}{D^*(\omega_i,\mathbf{k})} - \frac{\mu^{000}(\omega_i,\mathbf{q})\mu^{***}(\omega_i,\mathbf{q}')}{D^*(-\omega_i,-\mathbf{k})}\right), \tag{11}$$

where $\mu^{*00}(\omega_i,\mathbf{q})=\hat{\mathbf{a}}_s^*\cdot\boldsymbol{\mu}(\omega_i,\mathbf{q})\cdot\hat{\mathbf{a}}_i$ and $\mu^{0**}(\omega_i,\mathbf{q}') =\hat{\mathbf{a}}_s\cdot\boldsymbol{\mu}^*(\omega_i,\mathbf{q}')\cdot\hat{\mathbf{a}}_i^*$, and in the same way for μ^{000} and μ^{***}. The expression for the scattering cross section is obtained from (11) through

$$\sigma_v=Z_p\,\frac{\mathrm{Re}(P)}{|E_0|^2}. \tag{12}$$

On using the relation[13]

$$\mathrm{Re}\left(\frac{i}{D^*(\pm\omega,\pm\mathbf{k})}\right)=-\pi\,\frac{\delta[\pm\omega-\omega(\pm\mathbf{k})]}{\partial D/\partial\omega|_{\omega=\pm\omega(\pm\mathbf{k})}}, \tag{13}$$

where $\omega(\mathbf{k})$ is the solution of the dispersion relation $D(\omega,\mathbf{k})=0$, we obtain, for the scattering cross section,

$$\sigma_v=Z_p\,\frac{e^2\pi}{16\epsilon_0\omega_i}\int \frac{d\mathbf{k}}{(2\pi)^3}\frac{d\mathbf{q}}{(2\pi)^3}\frac{d\mathbf{q}'}{(2\pi)^3}\,n_v(\mathbf{k}-\mathbf{q}) \times n_v^*(\mathbf{k}-\mathbf{q}')p(\mathbf{q}-\mathbf{k}_i)p(\mathbf{q}'-\mathbf{k}_i) \times\left(\frac{\mu^{*00}(\omega_i,\mathbf{q})\mu^{0**}(\omega_i,\mathbf{q}')\,\delta[\omega_i-\omega(\mathbf{k})]}{\partial D/\partial\omega|_{\omega=\omega(\mathbf{k})}} - \frac{\mu^{000}(\omega_i,\mathbf{q})\mu^{***}(\omega_i,\mathbf{q}')\,\delta[\omega_i+\omega(-\mathbf{k})]}{\partial D/\partial\omega|_{\omega=-\omega(-\mathbf{k})}}\right). \tag{14}$$

Equation (14) is general and it can be applied to any type of electrostatic density perturbations. In the next section, we calculate the Fourier transform of the density perturbation associated with the vortex street (2) in order to specialize (14) for the case of periodic perturbations.

IV. FOURIER TRANSFORM OF THE DENSITY

Let us first derive an expression for the Fourier transform of a perturbation that is periodic in the y direction, $\phi(x,y)=\phi(x,y+2\pi/a)$, having periodicity wave number a. We also assume that ϕ is even in the y variable, which is the case for the perturbation (2). It follows that ϕ can be written as a Fourier series as

$$\phi(x,y)=\sum_{n=0}^{+\infty}\int_0^{2\pi}\frac{dt}{\pi}\,\phi\left(x,\frac{t}{a}\right)\cos(nt)\cos(nay), \tag{15}$$

and, consequently, the Fourier transform has the general form

$$n_v(\mathbf{k})=2\pi n_0\delta(k_z)\sum_{n=0}^{+\infty}[\delta(k_y-na) + \delta(k_y+na)]F_n(k_x), \tag{16}$$

with

$$F_n(k_x)=\int_{-\infty}^{+\infty}dx\left[\int_0^{2\pi}dt\;\phi\left(x,\frac{t}{a}\right)\cos(nt)\right]\exp(ik_xx), \tag{17}$$

where we have made use of the fact that for small perturbations $n_v(\mathbf{r})=n_0\phi(\mathbf{r})$.

We now calculate the function $F_n(k_x)$ for the breather vortex, given by (2). To do this, we write it in the form (3) and perform the t integration by employing

$$\int_0^{2\pi}\log(c\pm\cos x)\cos(nx)dx =-\frac{2\pi}{n}(\mp c\pm\sqrt{c^2-1})^n,\quad |c|>1,\quad n>0.$$

For $n=0$ the result is $2\pi\log[(c+\sqrt{c^2-1})/2]$ for both cases, and these results imply that $F_n(k_x)=0$ if $n=$even. On the other hand, when n is odd, we find that

$$F_n(k_x)=\frac{2\pi\phi_0}{n}\int_{-\infty}^{+\infty}\left[\frac{a}{b}\cosh(bx) - \sqrt{\left(\frac{a}{b}\cosh(bx)\right)^2-1}\right]^n\cos(k_xx)dx. \tag{18}$$

The x integration in (18) can be readily carried out,[14] yielding

$$F_n(k_x)=\frac{\pi\phi_0}{2b}\frac{(b/a)^n}{n!}\Gamma\left(\frac{n}{2}+i\frac{k_x}{2b}\right)\Gamma\left(\frac{n}{2}-i\frac{k_x}{2b}\right),\quad \text{when}\quad n>0\quad\text{is odd} \tag{19}$$

and

$$F_n(k_x)=0,\quad\text{when}\quad n>0\quad\text{is even}, \tag{20}$$

where we have used

$$\left[\frac{a}{b}\cosh(bx)-\sqrt{\left(\frac{a}{b}\cosh(bx)\right)^2-1}\right]^n \simeq\frac{1}{2^n}\left(\frac{b}{a}\right)^n\frac{1}{\cosh^n(bx)}. \tag{21}$$

With the general form (16) for $n_v(\mathbf{k})$, we can further simplify the expression (14). Thus, the final expression for the scattering cross section for our purposes reads as

$$\sigma_v=\frac{Z_p\epsilon^2 n_0^2}{64\pi\epsilon_0\omega_i}\int \frac{d\mathbf{k}\,dq_x\,dq_x'}{(2\pi)^5}\sum_{n,m=1}^{+\infty}F_n(k_x-q_x)$$
$$\times F_m^*(k_x-q_x')\sum_{\alpha,\beta=1,2}p(\mathbf{q}_\alpha-\mathbf{k}_i)p(\mathbf{q}_\beta'-\mathbf{k}_i)$$
$$\times\left(\mu^{*00}(\omega_i,\mathbf{q}_\alpha)\mu^{0**}(\omega_i,\mathbf{q}_\beta')\frac{\delta[\omega_i-\omega(\mathbf{k})]}{\partial_\omega D|_{\omega=\omega(\mathbf{k})}}\right.$$
$$\left.-\mu^{000}(\omega_i,\mathbf{q}_\alpha)\mu^{***}(\omega_i,\mathbf{q}_\beta')\frac{\delta[\omega_i+\omega(-\mathbf{k})]}{\partial_\omega D|_{\omega=-\omega(-\mathbf{k})}}\right), \tag{22}$$

where we have used the notations $\partial_\omega=\partial/\partial\omega$, $\mathbf{q}_1=(q_x,k_y-na,k_z)$, $\mathbf{q}_2=(q_x,k_y+na,k_z)$, $\mathbf{q}_1'=(q_x',k_y-ma,k_z)$, and $\mathbf{q}_2'=(q_x',k_y+ma,k_z)$.

V. APPLICATION: ORDINARY WAVE AND GAUSSIAN BEAM

We now present some specific results for the scattering cross section by assuming that (i) the incident wave propagates along the x direction and is polarized in the ordinary mode. Thus $\hat{\mathbf{a}}_i=\hat{\mathbf{z}}$ and $\mathbf{k}_i=k_i\hat{\mathbf{x}}$; (ii) the radiation phase velocity is much larger than the electron thermal velocity, so that one can use the cold plasma dispersion relation $D(\omega,\mathbf{k})=1-(k^2c^2+\omega_{pe}^2)/\omega^2$, which is valid for $\omega_i\gg\omega_{pe},\omega_{ce}$. Using the cold plasma mobility tensor, it is easy to see that the scattered wave is also polarized in the ordinary mode, viz. $\hat{\mathbf{a}}_s=\hat{\mathbf{z}}$. This means that in this case there is no mode coupling; (iii) the incident beam is Gaussian and that its divergence is negligible. This means that we have $p(\mathbf{r})=\exp[-(y^2+z^2)/w^2]$, where w is the beam waist, and thus $p(\mathbf{k})=2\pi\delta(k_x)p(k_y,k_z)$.

With these assumptions in mind, we can now derive a much more simple expression for the scattering cross section. The cold plasma mobility tensor gives $|\mu^{*00}(\omega_i,\mathbf{q}_{\alpha,\beta})|=e/(m_e\omega_i)$, and the same for the other three terms in (22). Using spherical coordinates in Eq. (22), one obtains

$$\sigma_v=\frac{3}{16}n_0^2\sigma_T\int_0^{2\pi}d\varphi\sum_{n,m=1}^{+\infty}F_n[k_i(\cos\varphi-1)]$$
$$\times F_m^*[k_i(\cos\varphi-1)]\sum_{\alpha,\beta=-1,+1}p(k_i\sin\varphi$$
$$+\alpha na,0)p^*(k_i\sin\varphi+\beta ma,0). \tag{23}$$

Now, $p(k_y,k_z)=\pi w^2\exp[-(k_y^2+k_z^2)w^2/4]$ and thus we consider that the product of the two profile functions in (23) contributes to the integral only in the case $\alpha=\beta$ and if $m=n$. Then, by writing

$$\sigma_v=\int d\varphi\,\frac{d\sigma_v}{d\varphi}, \tag{24}$$

the differential cross section is defined by

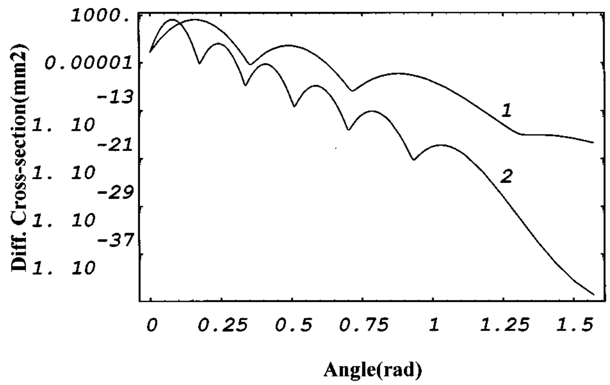

FIG. 1. Differential cross section (on the logarithmic scale) versus φ (in radians) for two different values of k_i ($k_i=2\pi$ mm^{-1} in curve 1 and $k_i=4\pi$ mm^{-1} in curve 2). We note that sharper peaks in the cross section arise with decreasing incident wavelength.

$$\frac{d\sigma_v}{d\varphi}=\frac{3\pi^2}{16}n_0^2w^4\sigma_T\sum_{n=-\infty}^{+\infty}|F_{|n|}[k_i(\cos\varphi-1)]|^2$$
$$\times\exp\left(-(k_i\sin\varphi-na)^2\frac{w^2}{2}\right). \tag{25}$$

Here n can take only odd values. Equation (25) displays, as expected, a kind of Bragg's diffraction law. This is because the vortex street acts like a diffraction grating. The intensity maxima are given by the condition

$$\sin\varphi=n\frac{a}{k_i}. \tag{26}$$

If $d=2\pi/a$ is the spacing between two adjacent vortices, one obtains the familiar equation for the maximum intensity lines of a diffraction grating,

$$d\sin\varphi=n\lambda_i. \tag{27}$$

The analogy with the classical Bragg diffraction is slightly broken because the Bragg peaks are modulated by the function $|F_{|n|}[k_i(\cos\varphi-1)]|^2$, which is a decreasing function of its argument. This results in a decrease of the intensity of the peaks between zero and $\pi/2$.

We now turn to the graphical illustration of our results. A complete set of parameters have been chosen, namely k_i, a, b, w, and ϕ_0. Once we neglect the divergence effects on the Gaussian beam, k_i and w define the beam completely. The beam waist is naturally limited between the mm to cm ranges.

Figure 1 displays the variation of the differential cross section against φ for two values of k_i. The two plotted curves have a common set of parameters: $\phi_0=1$, $n_0=10^{10}$ mm^{-3}, $w=5$ mm, $b=0.5$ mm^{-1}, and $a=1$ mm^{-1}. For the lowest curve we have $k_i=4\pi$ mm^{-1}, whereas for the upper we have $k_i=2\pi$ mm^{-1}. From this figure, it is evident that increasing k_i leads to sharper peaks in the differential cross section. This can be explained if we keep the analogy with the diffraction grating: the angular spread of the lines is proportional to the wavelength of the incident wave. Of course, this could be immediately predicted from the exponential of

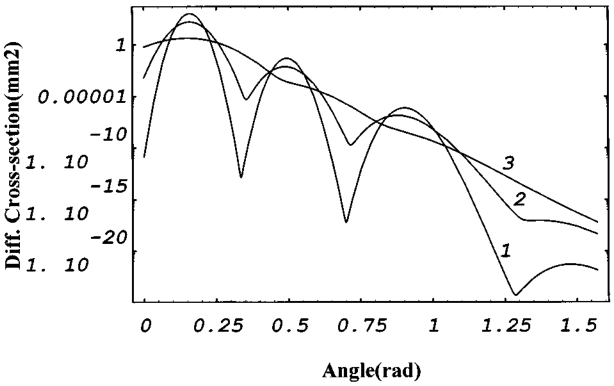

FIG. 2. Differential cross section versus φ for three different values of w ($w=8$, 5, and 2 mm for curves 1, 2, and 3, respectively). We note the loss of resolution with decreasing waist.

(25). Based on Fig. 1, we can state with generality that if we choose a laser beam such that $k_i \gg 1/w$, a slight angular deviation from the Bragg condition will turn into a very severe decrease of the cross section. This means that $d\sigma/d\varphi$ is so sharply peaked that, for any practical purposes, it is the sum of delta functions. This is the case for CO_2 laser, which has $k_i \simeq 593$ mm^{-1}, much greater than a typical value of $1/w$. If $k_i \simeq 1/w$, the form of the cross section changes from a sum of lines (delta functions) to a sum of Gaussian peaks. Again, on the basis of the figure, we see that, depending on the several parameters involved, the damping of the larger angles can be so strong that only the first peak is experimentally detectable. In this case, the proper identification of the vortex street is not possible.

In Fig. 2 we keep $k_i=2\pi$ mm^{-1} and $a=1$ mm^{-1}, but vary the value of the waist, w. We observe that for smaller w values the resolution of the peaks is lost. Mathematically, this can be easily understood from the Gaussian factor of (25): the exponent is proportional to w^2 and so the angular spread of the peaks is proportional to $1/w^2$. Thus, decreasing w will produce overlap of the peaks with a consequent loss of resolution. Physically, the interpretation is also simple: if $w<2\pi/a$, the beam is incapable of seeing the periodic structure, because it is "smaller" than its characteristic wavelength. We see that in the limit of $w\rightarrow\infty$ (plane wave) the peaks transform into lines.

VI. CONCLUSION

In this paper, we have studied the scattering of electromagnetic waves from a counter-rotating vortex street, which is described by a stationary solution of the Hasegawa–Mima equation. The calculation of the scattering cross section is made in the first-order Born approximation and is kept general, with arbitrary polarization and profile of the incident electromagnetic wave. As a simple but realistic application of our general results, we have studied the scattering of an electromagnetic wave that is polarized in the ordinary mode and has a Gaussian profile function. We obtain results that are closely linked to the effect of a diffraction grating. The difference from the classical Bragg diffraction lies in the fact that the peaks are weighted by a decreasing function of the scattering angle. This strong damping may imply that only the first peak can be seen and thus the vortex street cannot be properly identified. In conclusion, we stress that the results of our investigation should be useful in identifying coherent nonlinear structures as well as for plasma diagnostics in space and laboratory plasmas.

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