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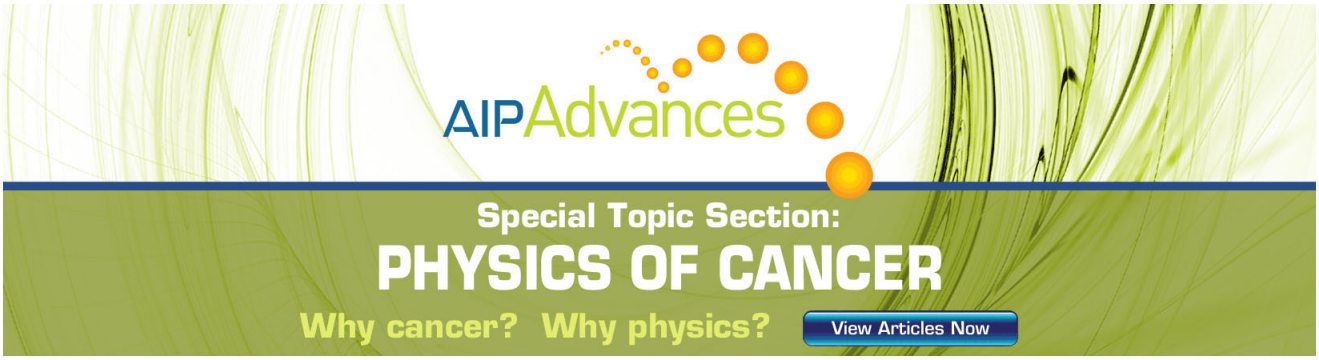

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Ion transport in a partially ionized impure edge plasma

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The kinetic theory of transport in an impure, partially ionized, edge plasma is developed. It is found that the thermal force between bulk ions and impurities is enhanced by the presence of neutral atoms, but the dynamical friction is not affected by the neutrals. When the neutral viscosity is large, an additional force on the impurities also arises. This force is parallel to the magnetic field, and is proportional to the shear of the parallel plasma velocity and the perpendicular ion density and temperature gradients. [S1070-664X(97)01612-1]

I. INTRODUCTION

Neutral atoms play an important role in edge plasmas. In the interaction with the wall bounding a laboratory plasma, ions are mostly recycled as neutrals, and the edge plasma is therefore necessarily incompletely ionized. It is the purpose of the present paper to examine the kinetic theory of ion transport in a partially ionized edge plasma. In particular, we focus on the problem of impurity transport, which is an area of prime importance for present and future fusion experiments. Whereas the classical transport theory of a fully ionized, multispecies plasma is well developed,[1–3] the influence of neutral atoms has mostly been neglected in the past. Conversely, the literature dealing with transport in partially ionized plasmas usually ignores the presence of impurities.[4–6]

Neutral atoms are particularly abundant in detached divertor plasmas. It is desirable to create such plasmas in burning fusion experiments in order to avoid excessive heat and particle loads on the first wall. Detachment can be induced by introducing impurities into the plasma edge, which gives rise to radiative cooling of the divertor plasma and can lower the temperature to the range of 1 eV or less, where recombination occurs. The neutral atom density then rises in the divertor, and the plasma loses parallel momentum and energy when interacting with the recycling neutrals by charge exchange. The plasma pulls off from the plates, and a self-sustained gas target is thus established.[7,8] A key difficulty with this scenario is the problem of impurity transport. The burning core plasma must necessarily be quite clean to avoid bremsstrahlung losses, while enough impurities should be present in the edge to facilitate detachment. It is well known that the thermal force associated with the parallel temperature gradient tends to drive impurities up from the divertor toward the core. To some extent, this effect is counteracted by the friction of ions flowing into the divertor, but this effect may not be strong enough to prevent core plasma contamination.

In the analysis of the present paper, we find that the presence of neutrals affects the kinetics of impurity transport in two ways. First, the thermal force between the main ions and the impurities is enhanced by a factor $(n_i+n_n)/n_i$, where n_i and n_n are the ion and neutral atom densities. Second, whenever the neutral viscosity plays a significant role in removing parallel plasma momentum—a key ingredient in the standard picture of detachment[7,9]–an additional force on the impurities arises. This new force on the impurities, which acts in addition to the usual friction and thermal force, is parallel to the magnetic field, and is proportional to the shear of the parallel plasma velocity and the perpendicular ion density and temperature gradients. In the Lorentz limit ($Z_{\mathrm{eff}} \gg 1$), it is equal to

$$R_n = -\tfrac{3}{5} m_i n_n \nabla_\perp \cdot (D_n \nabla_\perp V_\parallel) - m_i n_n D_n \nabla_\perp V_\parallel \cdot \left(\tfrac{6}{5}\nabla_\perp \ln n_n + \tfrac{27}{5}\nabla_\perp \ln T_i\right),$$

where n_n is the neutral density, m_i and T_i the ion mass and temperature, and $V_\parallel$ the parallel impurity flow velocity. $D_n = T_i/m_i n_i K_x$ is the neutral diffusion coefficient due to charge exchange with the main hydrogenic ion species, with the rate constant K_x.

The analysis is carried out under the assumption of short mean-free path with respect to Coulomb collisions and charge exchange, so that both the plasma and the neutrals can be treated as a fluid. This assumption is normally satisfied in devices such as Alcator C-Mod[10] and the International Thermonuclear Experimental Reactor,[11] which operate at high edge density, and may fail in lower density regions of other experiments.

The remainder of the paper is organized as follows. In the next section, we discuss the relevant orderings for an edge plasma interacting with neutral atoms. Then, in Sec. III, the kinetic equations for neutrals and ions are solved, and the forces acting on impurities are calculated. Numerical estimates of these forces are presented in the following section, and our conclusions are summarized in the last section.

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II. ORDERINGS

The parallel momentum equation summed over ions and neutrals is

$$m_i n(\mathbf{V}\cdot\nabla)V_\parallel+\nabla_\parallel p+(\nabla\cdot\boldsymbol{\pi})_\parallel=n_i eE_\parallel, \tag{1}$$

with $E_\parallel$ the parallel electric field, $p=p_i+p_n$ the total (ion+neutral) pressure, $n=n_i+n_n$ the total density, and $\boldsymbol{\pi}=\boldsymbol{\pi}_i+\boldsymbol{\pi}_n$ the total viscosity. The latter is usually dominated by the neutrals in a partially ionized plasma. In the short-mean-free-path ordering, all species have approximately the same flow velocity, which to the requisite order can be taken to be equal to that of the impurities, $V_\parallel$, in the present equation. Pressure detachment of the divertor plasma occurs when the neutral viscosity is large enough to cause a significant loss of parallel plasma momentum to the divertor side walls. This happens when the last two terms on the left side of (1) are large and comparable.

A balance between the pressure gradient and viscosity is commonplace in fluid dynamics; it is, for instance, realized in any fluid flowing in a pipe. Nevertheless, it has unexpected consequences for kinetic transport theory since the viscous force is proportional to the mean-free path λ whereas the pressure gradient is not. The ratio between these forces is

$$\frac{(\nabla\cdot\boldsymbol{\pi}_n)_\parallel}{\nabla_\parallel p}\sim\frac{\lambda L_\parallel}{L_\perp^2}, \tag{2}$$

where $L_\parallel$ and $L_\perp$ are the parallel and perpendicular scale lengths. Transport theory normally assumes that the fluid is nearly homogeneous on the length scale of the mean-free path, i.e.,

$$\lambda\ll L_\perp, \quad \lambda\ll L_\parallel, \tag{3}$$

and the theory is carried out to first order in $\lambda/L_\perp$ and $\lambda/L_\parallel$. However, if the pressure gradient and the viscous force are in balance, Eq. (2) shows that the perpendicular scale length must be much shorter than the parallel one,

$$\frac{L_\perp}{L_\parallel}\sim\frac{\lambda}{L_\perp}\ll 1.$$

This requirement indicates that *two* perpendicular gradients balance *one* parallel gradient. The expansion must therefore be carried through to second order in $\lambda/L_\perp$.

It is known that this circumstance affects the diffusion of a gas mixture flowing in a pipe.[12,13] As we shall see, the situation in a plasma is similar. Conventional fluid equations, for instance as developed by Braginskii,[1] are incomplete when applied to problems of this type, and should be supplemented with additional terms reflecting second-order perpendicular transport. The point is that if the viscosity is significant then other second-order transport phenomena are likely to be important as well. A similar argument has recently been put forward in connection with classical[14] and anomalous[15] transport in the scrape-off layer.

Based on these considerations, we adopt the ordering

$$\delta\sim\frac{v_T/L_\perp,\nu_z,\nu_r}{\Omega,\nu_{ii},\nu_x}\sim\frac{L_\perp}{L_\parallel}\ll 1 \tag{4}$$

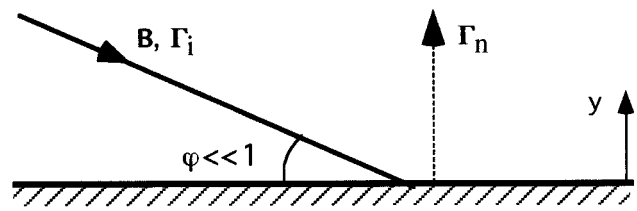

FIG. 1. Recycling at the wall. The neutrals diffuse vertically into the plasma while the ions stream along the field to the wall.

to develop the kinetic theory of transport in a detached, partially ionized, impure plasma. Here $\Omega\equiv eB/m_i$ is the ion gyrofrequency, $\nu_x=n_iK_x$ the charge exchange frequency of neutrals, $v_T\equiv(2T_i/m_i)^{1/2}$ the ion thermal velocity, ν_{ii} the ion collision frequency, and ν_z and ν_r the ionization and recombination rates of the neutrals and ions, respectively. Apart from the distinction between the parallel and perpendicular length scales, this ordering is identical to that used in Ref. 6 to derive neutral fluid equations. In the analysis that follows, it is convenient to eventually relax the ordering by taking

$$\Omega\gg\nu_x\sim\nu_{ii}, \tag{5}$$

which is usually satisfied with some margin in experiments. This subsidiary ordering reflects the circumstance that the neutrals diffuse faster across the magnetic field than do the ions.

There is also another, independent, reason for adopting the ordering $L_\perp/L_\parallel\sim\delta$ in a high recycling plasma, whether it detaches or not. In a magnetized edge plasma, the ions flow along the magnetic field toward the wall at the sound speed, and the neutrals must diffuse upstream against this flow as they are recycled from the wall. However, the flow of neutrals is diffusive and thus subsonic, with a velocity of the order $\lambda v_T/L_\perp=\delta v_T$. The neutrals are nevertheless able to penetrate into the plasma since the field lines usually intersect the wall at a shallow angle $\varphi\ll 1$, see Fig. 1. If the flux of neutrals into the plasma,

$$\Gamma_{ny}\sim -D_n\frac{dn_n}{dy}\sim\delta n_n v_T,$$

is to balance the outflux of ions,

$$\Gamma_{ni}\sim -n_i v_T\sin\varphi,$$

the scale lengths in the wall plasma must adjust to satisfy $L_\perp/L_\parallel\sim\delta\sim\varphi$.

We shall consider the plasma to be in a steady state, so the electric field is $\mathbf{E}=-\nabla\Phi$. Since we expect the electrostatic potential to be of the order $e\Phi\sim T_i$, the assumption of small Larmor radius inherent in (4) forces the $\mathbf{E}\times\mathbf{B}$ drift across the field to be slow,

$$\frac{\mathbf{E}\times\mathbf{B}}{B^2}\sim\delta v_T,$$

while the plasma flow along the field is sonic, as required by the boundary conditions at the divertor plates. The scale length of magnetic field variation is assumed to be $L_\parallel$ in the perpendicular as well as the parallel directions.

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III. KINETIC THEORY

A. Kinetic equations

We consider an impure, partially ionized, edge plasma, where the (hydrogenic) main ions interact with neutral atoms by charge exchange, ionization and recombination. The kinetic equations for ions and neutrals are[4–6]

$$\mathbf{v}\cdot\nabla f+\frac{e}{m_i}(\mathbf{E}+\mathbf{v}\times\mathbf{B})\cdot\frac{\partial f}{\partial\mathbf{v}}=C(f)-X(f,g)+\nu_z g-\nu_r f, \tag{6}$$

$$\mathbf{v}\cdot\nabla g=X(f,g)-\nu_z g+\nu_r f, \tag{7}$$

with f and g the ion and neutral distribution functions, $\mathbf{v}$ the velocity vector, and $\mathbf{B}$ the magnetic field. The ion collision operator describes collisions among the main ions (i) and collisions with heavy impurities (z), i.e., $C=C_{ii}+C_{iz}$. As a matter of convenience, we shall eventually neglect C_{ii} to enable an analytical solution of the transport equations. The ions also experience collisions with the electrons,

$$C_{ie}(f)=-\frac{1}{m_i n_i}\mathbf{R}_{ie}\cdot\frac{\partial f}{\partial\mathbf{v}}.$$

Mathematically, these collisions are most easily accounted for by incorporating the force $\mathbf{R}_{ie}$ from the electrons in the electric field by writing $\mathbf{E}_*\equiv\mathbf{E}+\mathbf{R}_{ie}/e$. Self-collisions among the neutral particles are neglected.

In the charge exchange operator,

$$X(f,g)=\int\sigma_x|\mathbf{v}-\mathbf{v}'|[f(\mathbf{v})g(\mathbf{v}')-f(\mathbf{v}')g(\mathbf{v})]d^3v',$$

it is customary, and fairly accurate, to take the charge exchange cross section σ_x to be inversely proportional to the relative velocity, $\sigma_x|\mathbf{v}-\mathbf{v}'|\equiv K_x=\text{const}$. The charge exchange frequency of neutrals then becomes $\nu_x=n_iK_x$, and the charge exchange operator simplifies to

$$X(f,g)=K_x(n_nf-n_ig).$$

Rather than solving the kinetic equations as they stand, it is convenient to solve the sum equation (6)+(7) and the neutral equation (7) since the atomic processes do not enter in the former equation.[5] It is also convenient to transform the kinetic equations to a frame moving with the plasma. To this end, we introduce the velocity

$$\mathbf{V}\equiv\frac{\mathbf{E}_*\times\mathbf{B}}{B^2}+\mathbf{V}_\parallel,$$

where, again, $\mathbf{V}_\parallel$ is the parallel impurity flow velocity. In terms of the relative velocity variable $\mathbf{u}\equiv\mathbf{v}-\mathbf{V}$, we can then write the kinetic equations (6)+(7) and (7) as

$$(\mathbf{u}+\mathbf{V})\cdot\left(\nabla-\nabla\mathbf{V}\cdot\frac{\partial}{\partial\mathbf{u}}\right)(f+g)+\frac{e}{m_i}(\mathbf{E}_{\parallel *}+\mathbf{u}\times\mathbf{B})\cdot\frac{\partial f}{\partial\mathbf{u}}=C(f),$$

$$(\mathbf{u}+\mathbf{V})\cdot\left(\nabla-\nabla\mathbf{V}\cdot\frac{\partial}{\partial\mathbf{u}}\right)g=X(f,g)-\nu_z g+\nu_r f. \tag{8}$$

These equations are now solved order by order in δ.

If we denote the gyroangle by α, we have in zeroth order the equations

$$\Omega\frac{\partial f_0}{\partial\alpha}=C(f_0),$$

$$X(f_0,g_0)=0,$$

implying Maxwellian distribution functions in this order,

$$\frac{f_0}{n_i}=\frac{g_0}{n_n}=\left(\frac{m_i}{2\pi T_i}\right)^{3/2}e^{-m_iu^2/2T_i}.$$

In first order, we have

$$\mathbf{u}_\perp\cdot\left(\nabla-\nabla V_\parallel\frac{\partial}{\partial u_\parallel}\right)(f_0+g_0)+\Omega\frac{\partial f_1}{\partial\alpha}=C(f_1), \tag{9}$$

$$\mathbf{u}_\perp\cdot\left(\nabla-\nabla V_\parallel\frac{\partial}{\partial u_\parallel}\right)g_0=X(f_0,h_1)-\nu_z g_0+\nu_r f_0, \tag{10}$$

where we have introduced h, defined by

$$g\equiv\frac{n_n}{n_i}f+h,$$

so that $X(f,g)=X(f,h)=-\nu_x h$. The solution to (10) is

$$\begin{aligned}h_1&=-\nu_x^{-1}\left[\mathbf{u}_\perp\cdot\left(\nabla-\nabla V_\parallel\frac{\partial}{\partial u_\parallel}\right)+\nu_z-(n_i/n_n)\nu_r\right]g_0\\&=-\frac{g_0}{\nu_x}\left\{\mathbf{u}_\perp\cdot\left[\nabla\ln n_n+\left(\frac{m_iu^2}{2T_i}-\frac{3}{2}\right)\nabla\ln T_i+\frac{m_iu_\parallel}{T_i}\nabla V_\parallel\right]\right.\\&\quad\left.+\nu_z-(n_i/n_n)\nu_r\right\}.\end{aligned} \tag{11}$$

In second order, the sum equation (8) becomes

$$\begin{aligned}&(\mathbf{u}_\parallel+\mathbf{V})\cdot\left(\nabla-\nabla\mathbf{V}\cdot\frac{\partial}{\partial\mathbf{u}}\right)(f_0+g_0)-\mathbf{u}_\perp\cdot\nabla\mathbf{V}_\perp\cdot\frac{\partial(f_0+g_0)}{\partial\mathbf{u}}\\&\quad+\frac{eE_{\parallel *}}{m_i}\frac{\partial f_0}{\partial u_\parallel}+\mathbf{u}_\perp\cdot\left(\nabla-\nabla V_\parallel\frac{\partial}{\partial u_\parallel}\right)(f_1+g_1)+\Omega\frac{\partial f_2}{\partial\alpha}\\&\quad=C_{iz}(f_2),\end{aligned} \tag{12}$$

where, for simplicity, we have neglected collisions among the main ions. This approximation is appropriate in the Lorentz limit, $n_zZ^2\gg n_i$, and greatly simplifies the analysis of the transport problem we obtain by taking the gyroaverage,

$$\langle\cdots\rangle\equiv\frac{1}{2\pi}\int_0^{2\pi}(\cdots)d\alpha,$$

of (12). The detailed calculation of the gyroaverage of the various terms in (12) can be found in the Appendix. Here, we merely note that cross-field diffusion is described by the terms involving f_1 and g_1. The ion contribution is small because of the assumption (5), which implies that the classical ion diffusion coefficient $\nu_{ii}(u_\perp/\Omega)^2$ is smaller than the diffusion coefficient of neutral particles

$$D(\mathbf{u})\equiv\frac{u_\perp^2}{2\nu_x}.$$

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In our kinetic problem this leaves neutral diffusion by charge exchange to compete with parallel transport, as desired. Only the part of f_2 that is odd in $u_\parallel$ contributes to the transport. Denoting it by F, we find

$$C_{iz}(F)=u_\parallel\left[A_1+\left(\frac{m_iu^2}{2T_i}-\frac{5}{2}\right)A_2\right]f_0, \tag{13}$$

where the thermodynamic forces A_1 and A_2 are defined by

$$A_1\equiv\frac{n}{n_i}\left(\nabla_\parallel \ln p+\frac{m_i\mathbf{V}\cdot\nabla V_\parallel}{T_i}\right)-\frac{eE_{\parallel *}}{T_i}-\frac{n_n}{n_i}\frac{m_i}{T_i}$$
$$\times[\nabla_\perp\cdot(D\nabla_\perp V_\parallel)+2D\nabla_\perp V_\parallel\cdot\nabla_\perp \ln n_n],$$

$$A_2\equiv\frac{n}{n_i}\nabla_\parallel \ln T_i-\frac{n_n}{n_i}\frac{2m_iD}{T_i}\nabla_\perp V_\parallel\cdot\nabla_\perp \ln T_i.$$

Equation (13) is the ''Spitzer problem'' for ion transport in the situation we are considering. It has the same structure as Eq. (92) in Ref. 14 and Eqs. (6)–(8) in Ref. 15. It differs from the usual Spitzer problem[16] in that the thermodynamic forces are modified by the presence of neutrals and their cross-field transport.

B. Fluxes and forces

We now proceed to solve (13) in the Lorentz limit, where

$$C\simeq C_{iz}=\frac{\nu_{iz}(u)}{2}\frac{\partial}{\partial\xi}(1-\xi^2)\frac{\partial}{\partial\xi},$$

$$\nu_{iz}(u)=\frac{3\pi^{1/2}}{4\tau_{iz}}\left(\frac{v_T}{u}\right)^3,$$

with $\xi\equiv u_\parallel/u$, and $\tau_{iz}=3(2\pi)^{3/2}\epsilon_0^2m_i^{1/2}T_i^{3/2}/n_zZ^2e^4\ln\Lambda$ the ion-impurity collision time.

It is convenient to write A_1 and A_2 as

$$A_1=a_1+\frac{m_iu_\perp^2}{2T_i}a_3,$$

$$A_2=a_2+\frac{m_iu_\perp^2}{2T_i}a_4,$$

with

$$a_1\equiv\frac{n}{n_i}\left(\nabla_\parallel \ln p+\frac{m_i\mathbf{V}\cdot\nabla V_\parallel}{T_i}\right)-\frac{eE_{\parallel *}}{T_i},$$

$$a_2\equiv\frac{n}{n_i}\nabla_\parallel \ln T_i,$$

$$a_3\equiv-\frac{n_n}{n_i}\left[\nabla_\perp\cdot\left(\frac{\nabla_\perp V_\parallel}{\nu_x}\right)+\frac{2}{\nu_x}\nabla_\perp V_\parallel\cdot\nabla_\perp \ln n_n\right],$$

$$a_4\equiv-\frac{2n_n}{n_i\nu_x}\nabla_\perp V_\parallel\cdot\nabla_\perp \ln T_i.$$

Thus a_1 and a_2 are the thermodynamic forces appearing in the classical Spitzer problem enhanced by the factor n/n_i (except in the electric field term). The reason for this enhancement is that the neutrals through charge exchange reactions carry information about the ion kinetics, as explained in detail in Ref. 6. The appearance of the new thermodynamic forces a_3 and a_4 is due to the diffusion of neutrals across the magnetic field, which thus competes on an equal footing with conventional parallel transport.

To solve our Spitzer problem we note that

$$u_\parallel=uP_1(\xi),\quad u_\parallel u_\perp^2=(2u^3/5)[P_1(\xi)-P_3(\xi)],$$

where P_1 and P_3 are Legendre polynomials, and

$$C_{iz}(P_n)=-\nu_{iz}\frac{n(n+1)}{2}P_n.$$

Hence it is not difficult to derive

$$F=-\frac{uf_0}{\nu_{iz}}\left[a_1P_1+\left(\frac{m_iu^2}{2T_i}-\frac{5}{2}\right)a_2P_1\right.$$
$$\left.+\frac{m_iu^2}{5T_i}\left(a_3+\left(\frac{m_iu^2}{2T_i}-\frac{5}{2}\right)a_4\right)\left(P_1-\frac{1}{6}P_3\right)\right].$$

It is now straightforward to calculate the particle and heat fluxes of the main ions relative to the impurities,

$$\Gamma_\parallel=n_i(V_{i\parallel}-V_{z\parallel})\equiv\int u_\parallel F\,d^3u,$$

$$q_\parallel\equiv\int u_\parallel F\left(\frac{m_iu^2}{2T_i}-\frac{5}{2}\right)d^3u,$$

where $V_{z\parallel}=V_\parallel$ and $V_{i\parallel}$ is the parallel ion velocity. Only the P_1 component of F contributes, and we find

$$\Gamma_\parallel=-\frac{32}{3\pi}\frac{n_iT_i\tau_{iz}}{m_i}\left(a_1+\frac{3}{2}a_2+\frac{8}{5}a_3+4a_4\right),$$

$$q_\parallel=-\frac{32}{3\pi}\frac{n_iT_i\tau_{iz}}{m_i}\left(\frac{3}{2}a_1+\frac{25}{4}a_2+4a_3+18a_4\right).$$

Note that these parallel flows are driven not only by parallel gradients, but also by perpendicular gradients in a_3 and a_4.

Finally, we calculate the force exerted upon the impurities by the bulk ions,

$$R_{zi}=-R_{iz}=-\int m_iu_\parallel C_{iz}(F)d^3u=-n_iT(a_1+a_3+a_4).$$

Comparing this relation with the expression for $\Gamma_\parallel$ gives

$$R_{zi}=\frac{3\pi}{32}\frac{m_in_i(V_{i\parallel}-V_{z\parallel})}{\tau_{iz}}+\frac{3}{2}(n_i+n_n)\nabla_\parallel T_i+R_n,$$

$$R_n=-\frac{3}{5}n_nT_i\nabla_\perp\cdot\left(\frac{\nabla_\perp V_{z\parallel}}{\nu_x}\right)-\frac{6T_i}{5\nu_x}\nabla_\perp V_{z\parallel}\cdot\nabla_\perp n_n$$
$$-\frac{6n_n}{\nu_x}\nabla_\perp V_{z\parallel}\cdot\nabla_\perp T_i. \tag{14}$$

The first term in R_{zi} is the usual friction force and is not modified by the presence of neutrals. The next term, which is the thermal force, is enhanced by a factor $(n_i+n_n)/n_i$ over the classical expression, and the third term, R_n, is caused entirely by the neutrals. It arises because of the diffusion of neutrals across the magnetic field, but is, of course, brought about by ion-impurity collisions.

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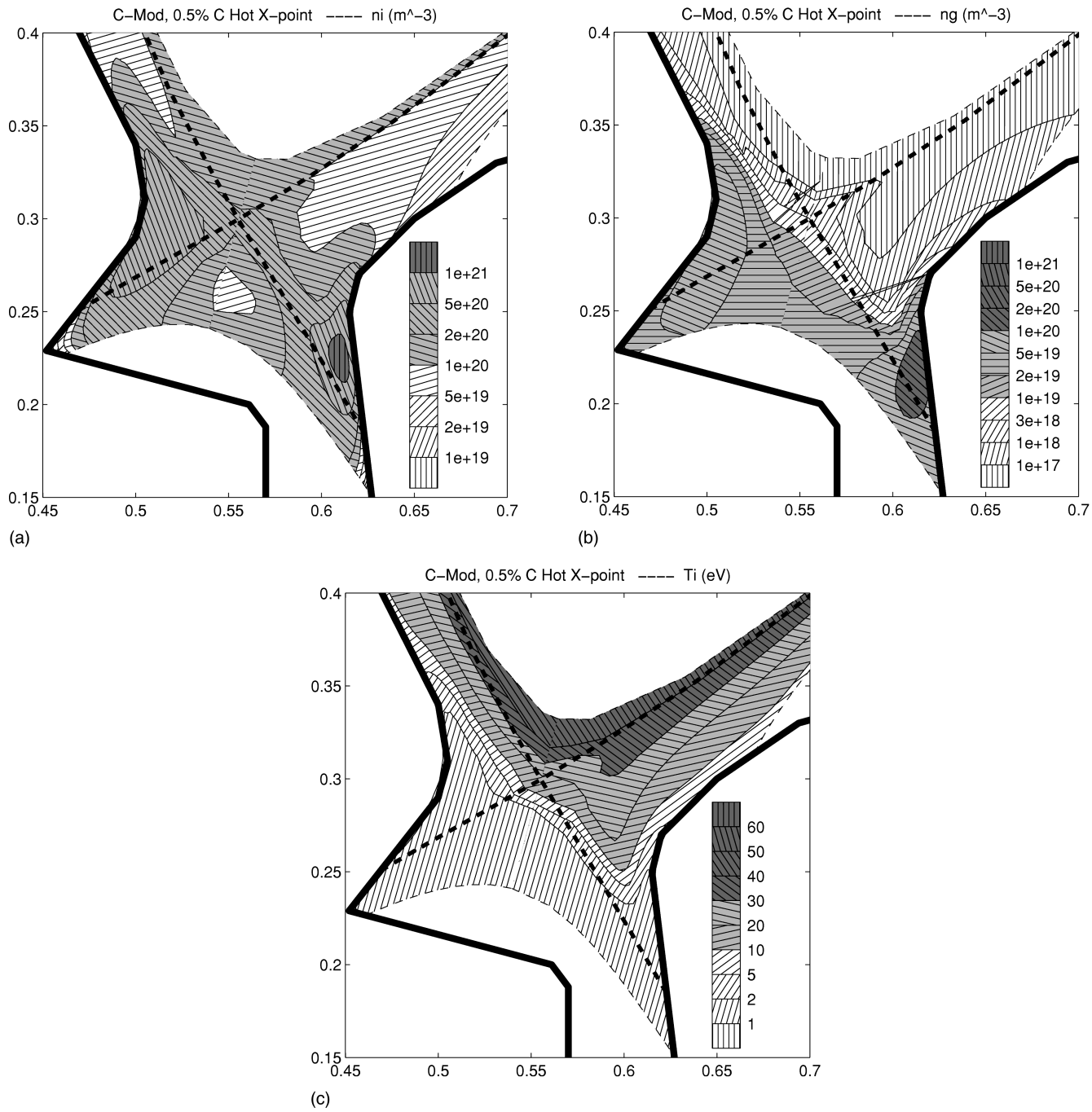

FIG. 2. (a) The ion density n_i; (b) the neutral density n_n (both in m^{-3}); and (c) the ion temperature T_i (in eV) in a simulation (Ref. 18) of detachment in the Alcator C-Mod divertor with 0.5% carbon impurities.

IV. NUMERICAL RESULTS

The kinetic theory of the previous section is quite general, and applies to any partially ionized plasma satisfying the orderings. In this section we assess the importance of these results for the Alcator C-Mod divertor[10] by using numerical results from the UEDGE simulation code.[17]

A complete simulation of the impurity dynamics should take into account all the different forces and charge states associated with the impurities, which is far beyond the scope of the present paper. Our goal is limited to simple order-of-magnitude estimates as to whether the new force R_n is able to compete with the classical thermal and frictional forces. To this end, we simulate only the main ion and neutral dynamics, assuming a fixed fraction of stationary impurities which affect the plasma only through radiation losses in the energy equation. We then evaluate what the various forces on the impurities would be in this situation. Thus the simulation is not self-consistent; the expressions for the impurity forces are merely applied as a ''diagnostic'' *a posteriori* to give an idea of their approximate order of magnitude.

The general methodology and results of the detachment simulations in C-Mod we refer to are described in Ref. 18. The ion and neutral continuity and momentum equations and the energy equation are solved on a nonorthogonal grid to

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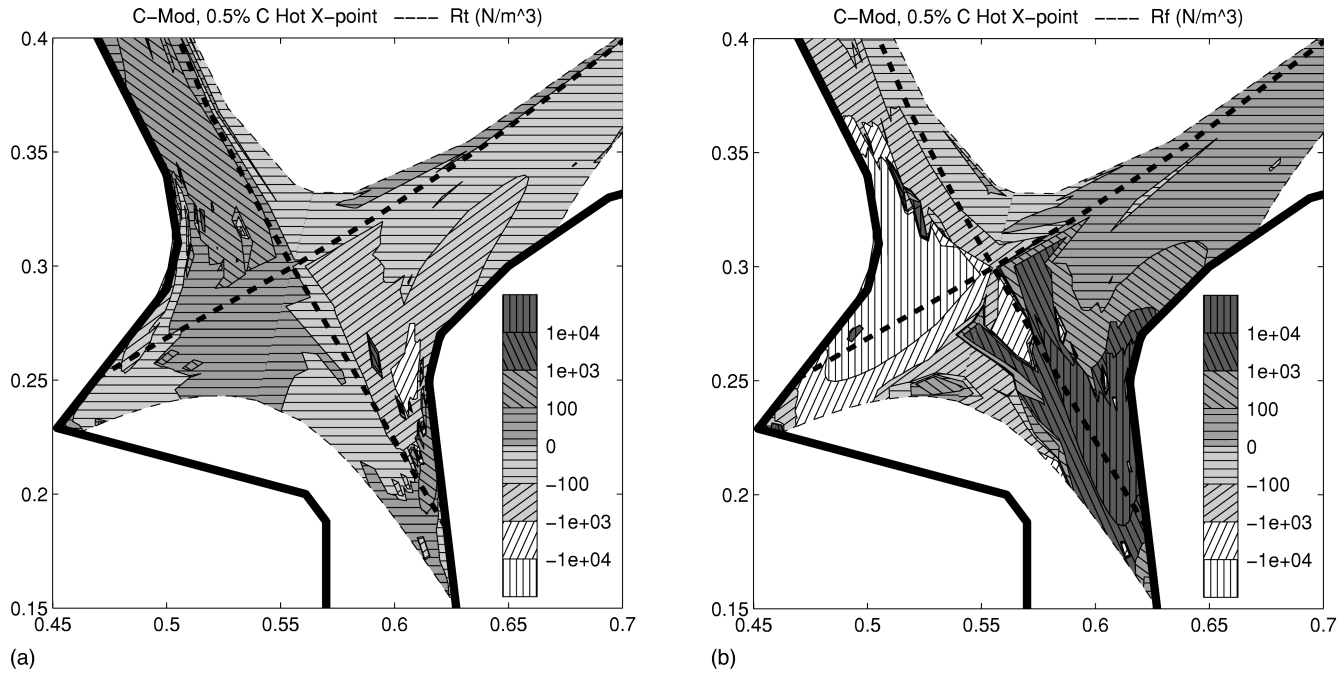

FIG. 3. (a) The thermal force; and (b) the friction force (both in N/m^3) acting on the impurities.

accommodate the complex geometry of the C-Mod divertor. The relevant atomic processes are incorporated in each of the equations. A charge-exchange diffusive flow of neutrals across the magnetic field enters in the neutral continuity equation, and the momentum equations are coupled through ionization, recombination and charge exchange giving rise to a friction force between the bulk ions and the neutrals. The energy equation accounts for perpendicular heat conduction of neutrals by charge exchange, and models impurity radiation by a noncoronal radiation function. Detachment is induced by introducing a fixed fraction of carbon, and results in a substantial decrease of the heat and particle fluxes to the targets with a simultaneous drop in electron temperature and increased volume recombination, as described in more detail in Ref. 18.

Figure 2 shows the ion and neutral densities, and the ion temperature in a simulation with 0.5% carbon impurities, and appear to be in broad agreement with typical experimental values in C-Mod.

The inferred classical thermal force on the impurities is shown in Fig. 3(a). A positive force is parallel and a negative force is antiparallel to the magnetic field, which encircles the plasma core clockwise. Accordingly, the thermal force, being positive on the left and negative on the right, is mostly directed toward the core. The reverse holds for the estimated dynamical friction shown in Fig. 3(b). Since the simulation does not calculate the impurity dynamics, the relative parallel velocity $V_{z\|}-V_{i\|}$ is unknown, and the dynamical friction force cannot be predicted confidently. The result in Fig. 3(b) is obtained by setting $V_{z\|}$ to zero. In other words, this figure shows what the friction force would have been if the impurities were stationary. In reality, the short-mean-free-path treatment considered implies that the impurities tend to follow the bulk ions to lowest order, $V_{z\|}\simeq V_{i\|}$, resulting in a smaller friction force than shown in the figure.

In accordance with the approximate nature of the simulation, we use the Lorentz approximation in evaluating the forces, although the limit $Z_{\mathrm{eff}}\gg 1$ is not strictly realized in C-Mod. Typically, counting all the impurities species, $Z_{\mathrm{eff}}\simeq 2$–3, making the actual friction coefficients numerically different from those obtained in the Lorentz limit. As a result, the numerical results for the thermal force R_t, friction force R_f (with $V_{z\|}=0$), and the new thermal force R_n presented in Figs. 3 and 4 are intended to suggestive rather than conclusive.

Figure 4 shows the new force R_n, in the approximation $V_{z\|}=V_{i\|}$, and its ratio to the thermal force and the dynamical friction. Notice that R_n exceeds, or is comparable to, the thermal force throughout the lower part of the divertor and the private flux region, and in regions inside the separatrix, as shown by Fig. 4(b). In these regions R_n is of comparable importance in determining impurity transport relative to the flow of bulk ions. Even though the Lorentz approximation overestimates all three forces in this situation, the relative magnitudes of R_n and the thermal force are in error by $O(1)$ at most. Recall from the structure of the kinetic equation solved in the previous section that the ratio between R_n and the thermal force is always of the order a_4/a_2 or a_3/a_2, regardless of the value of Z_{eff}.

The comparison between R_n and the usual thermal force is not the whole story, however, since we can see from Fig. 4(c) that the friction force tends to be larger than both R_n and the thermal force. Moreover, the friction force is underestimated compared with the other forces by the Lorentz limit. In reality, however, the friction force is reduced because $V_{z\|}$ is approximately equal to $V_{i\|}$ as already mentioned. More properly, we should insert Eq. (14) in the impurity momentum conservation equation and view the result as an equation

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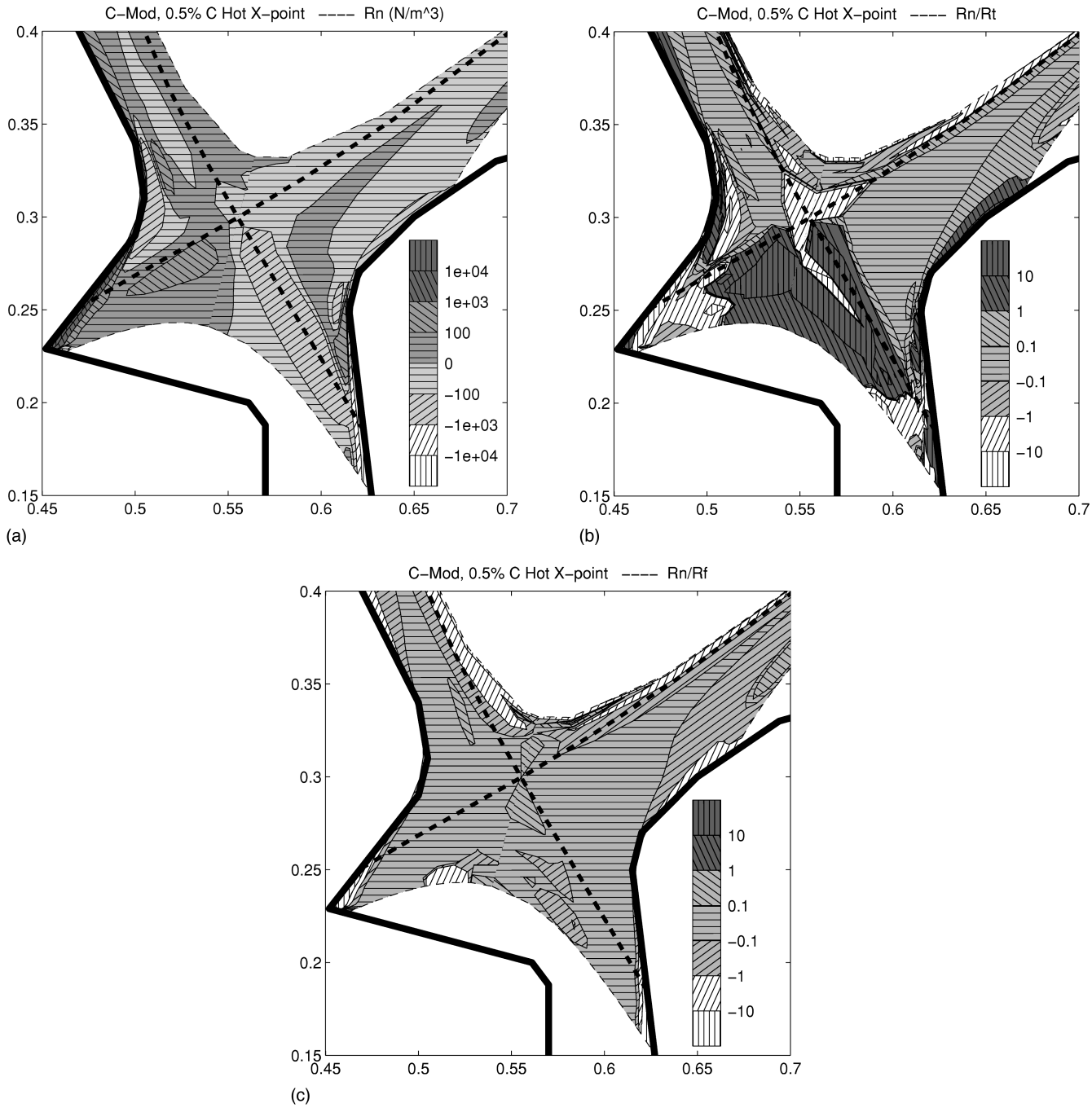

FIG. 4. (a) The force R_n (in N/m^3); and its ratio to the (b) thermal force, and (c) the friction force.

for $V_{z\|}$ with the thermal force and R_n responsible for most of the typically small difference between it and $V_{i\|}$. As a result, R_n is expected to strongly alter the behavior of the impurities in regions where it competes with *both* the classical thermal force *and* the $V_{z\|}=0$ portion of the friction force. In the simulation considered here such regions may be present near the strike points and just inside the separatrix, where, however, the requirement of short ion mean-free path does not hold uniformly. (The neutral mean-free path is still short.)

To close this section, we re-emphasize that in a consistent simulation all the forces will have to be included in the impurity dynamics. The value of the present analysis rests entirely in giving an estimate of the orders of magnitude involved. It appears that the force R_n can be comparable to the classical forces in some parts of the divertor and hence play a role in the transport of impurities. In the divertor of an ignited tokamak, complicated recirculating flow patterns of neutrals are often envisaged, which could make R_n even larger.

V. CONCLUSIONS

We have investigated the influence of neutral hydrogen atoms on impurity transport in an edge plasma. The neutrals are shown to increase the classical thermal force between the different ion species, and to give rise to a new type of force

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owing to the diffusion of neutrals across the field. The total parallel force between main ions and a heavy impurity is given by (14), where the first term on the right is the dynamical friction, the second term the thermal force, and the third term is the new force.

The usual thermal force is, of course, entirely due to ion-impurity collisions. It may therefore seem surprising that it should be enhanced by the presence of neutrals. The underlying reason is that the neutral and ion populations are coupled through charge exchange, and thus carry kinetic information about each other. Charge exchange does not randomize velocities, and the hydrogenic particles exist alternately as ions and neutrals. For reasons explained in detail in Ref. 6 (for a clean plasma) this leads to increases in the ion transport coefficients such as the one encountered here. On the other hand, we note that the dynamical friction force between ions and impurities is not affected by the presence of neutrals.

It has recently been pointed out that the competition between parallel and perpendicular transport inherent in edge plasmas can lead to new parallel transport phenomena caused by perpendicular diffusion.[14,15] The new force between ions and impurities found here is an example of this circumstance, and arises because of the cross-field diffusion of neutrals. Again, the neutrals do not collide directly with the impurities. Only the ions do, but they carry kinetic information about the neutrals through charge exchange. The new force involves the shear of the parallel velocity and perpendicular density and temperature gradients.

Simulations of the edge plasma in Alcator C-Mod suggest that the new thermal force can compete with the classical thermal force during detachment, and may influence impurity flow. More careful and comprehensive simulations are, however, required to establish the true importance of the new force for the problem of impurity transport.

Moreover, extensions of the theory presented here, and of the fluid description implemented in numerical simulations, are necessary to account more accurately for all the relevant atomic processes. In particular, we expect the retention of elastic ion-neutral collisions to reduce the force R_n due to the reduction of the neutral mean-free path caused by the additional randomization. These collisions are usually ignored in transport theory, but recent theoretical and computational work indicates they may have a larger cross section than formerly believed, up to 2–3 times larger than that for charge exchange for temperatures greater than 1 eV.[19] We have restricted ourselves to the simpler model presented here in order to focus on the origin and physical basis of the ion-impurity forces in the presence of neutrals, leaving an extended model (also retaining ion-ion collisions) to future work.

Note added in proof. As described in Ref. 18, the solution found in the divertor simulation is bifurcated, with one of the solutions having a hot X-point and the other displaying a Marfe-like structure below the X-point. Only the former solution was considered here, but the general conclusions hold for both.

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APPENDIX: GYROAVERAGE OF THE KINETIC EQUATION

The purpose of this appendix is to provide the necessary details to derive (13) from the gyroaverage of (12). The gyroaverage of the terms involving f_0 and g_0 in (12) are

$$\langle(\mathbf{u}_\parallel+\mathbf{V})\cdot\nabla(f_0+g_0)\rangle=(u_\parallel+V_\parallel)\nabla_\parallel(f_0+g_0), \tag{A1}$$

$$\left\langle -u_\parallel\nabla_\parallel\mathbf{V}\cdot\frac{\partial(f_0+g_0)}{\partial\mathbf{u}}\right\rangle=\frac{m_iu_\parallel^2}{T_i}(\nabla_\parallel V_\parallel)(f_0+g_0), \tag{A2}$$

$$\left\langle -\mathbf{V}\cdot\nabla\mathbf{V}\cdot\frac{\partial(f_0+g_0)}{\partial\mathbf{u}}\right\rangle=\frac{m_iu_\parallel}{T_i}(\mathbf{V}\cdot\nabla V_\parallel)(f_0+g_0), \tag{A3}$$

$$\left\langle -\mathbf{u}_\perp\cdot\nabla\mathbf{V}_\perp\cdot\frac{\partial(f_0+g_0)}{\partial\mathbf{u}}\right\rangle=\frac{m_iu_\perp^2}{2T_i}(\nabla_\perp\cdot\mathbf{V}_\perp)(f_0+g_0), \tag{A4}$$

$$\left\langle\frac{eE_\parallel}{m_i}\frac{\partial f_0}{\partial u_\parallel}\right\rangle=\frac{eE_{\parallel *}}{m_i}\frac{\partial f_0}{\partial u_\parallel}. \tag{A5}$$

These terms contain the usual parallel gradient driving forces appearing in the Spitzer problem,[16] and in addition the effects of cross-field convection by the $\mathbf{E}\times\mathbf{B}$ drift.

Cross-field diffusion is described by the terms in (12) involving f_1 and g_1. The ion contribution is small because of (5), leaving neutral diffusion to be due to the term h_1 and giving

$$\begin{aligned}\left\langle\mathbf{u}_\perp\cdot\left(\nabla-\nabla V_\parallel\frac{\partial}{\partial u_\parallel}\right)h_1\right\rangle &=-\left(\nabla_\perp-\nabla_\perp V_\parallel\frac{\partial}{\partial u_\parallel}\right)\cdot\left\{g_0D\left[\nabla_\perp\ln n_n\right.\right.\\ &\quad\left.\left.+\left(\frac{m_iu^2}{2T_i}-\frac{3}{2}\right)\nabla_\perp\ln T_i+\frac{m_iu_\parallel}{T_i}\nabla_\perp V_\parallel\right]\right\},\end{aligned} \tag{A6}$$

which competes with parallel transport. Here $D=u_\perp^2/\nu_x$. Thus we have

$$\langle C_{iz}(f_2)\rangle=(\mathrm{A1})+\cdots+(\mathrm{A6}).$$

Since we eventually solve the Spitzer problem only in the Lorentz limit, we have neglected self-collisions [e.g., a term $C_{ii}(f_1,f_1)$] already here. Collecting all terms odd in $u_\parallel$ finally gives (13).

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