Physics of Plasmas

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Citation: Phys. Plasmas **5**, 1209 (1998); doi: 10.1063/1.872629
View online: http://dx.doi.org/10.1063/1.872629
View Table of Contents: http://pop.aip.org/resource/1/PHPAEN/v5/i4
Published by the American Institute of Physics.

Neoclassical transport in a rotating impure plasma

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(Received 7 October 1997; accepted 29 December 1997)

In a toroidally rotating tokamak plasma, heavy impurity ions accumulate on the outside of each flux surface under the action of the centrifugal force. The collision frequency therefore varies over the flux surface. This circumstance is shown to enhance the neoclassical transport processes, including the bootstrap current, by making collisions occur preferentially where the magnetic field is weak. © *1998 American Institute of Physics.* [S1070-664X(98)01104-5]

Neoclassical theory[1,2] has attracted renewed attention after the discovery of enhanced confinement in tokamaks with negative, or nearly constant, magnetic shear.[3–7] In these experiments, ion transport in the core region is comparable to neoclassical values. To enable an accurate comparison with experimental results, it is important to address the remaining issues in neoclassical theory.

The region of enhanced confinement is frequently observed to experience relatively rapid toroidal rotation, typically with a squared Mach number of the order $M^2 \equiv m_i \omega^2 R^2/(T_e+T_i) \sim 0.1$, where ω is the angular frequency of toroidal rotation, m_i the ion mass, R the major radius, and T_e and T_i the electron and ion temperatures. Note that, since the latter tend to be constant along the magnetic field, the Mach number varies over a flux surface. The original neoclassical theory[1,2] assumed $M \ll 1$, but was later extended to include the case $M = O(1)$ by Hinton and Wong,[8] and Catto, Bernstein, and Tessarotto.[9] More recently, the subject has been revisited by Sugama and Horton.[10] In these works, the modifications to neoclassical transport by finite Mach number were found to be of the order of M^2, and are therefore typically not very large in most experiments. It is the purpose of the present Brief Communication to point out a new and simple way in which toroidal rotation increases neoclassical transport. This effect is caused by the impurity ions generally present in a tokamak, and arises because their poloidal distribution is nonuniform in a spinning plasma. As shown here, even if $M \ll 1$ this circumstance can increase the transport coefficients, including the bootstrap current, by up to 45% in a standard large-aspect-ratio tokamak.

The distribution of heavy ions is nonuniform over a flux surface because, in a rotating plasma, the centrifugal force pushes heavy particles to the outside of the torus. This was noted in the papers already cited and in Ref. 11, and has recently been re-emphasized by Wesson.[12] If the magnetic field is $\mathbf{B} = I(\psi)\nabla\varphi + \nabla\varphi \times \nabla\psi$, the density of each species (a) varies over the flux surface ψ with the poloidal angle θ as

$$n_a(\psi,\theta) = N_a(\psi) \exp\left(\frac{m_a \omega^2 (R^2 - R_0^2)}{2T_a} - \frac{e_a \Phi}{T_a} \right). \tag{1}$$

Here e_a is the charge, and $R = R_0$ at some point on the flux surface where the electro-static potential Φ is chosen to be zero. The latter is determined by quasineutrality,

$$\sum_a e_a n_a(\psi,\theta) = 0, \tag{2}$$

at each point. Equations (1) and (2) were solved in a number of illustrative examples by Wesson. Here, we only consider the simplest case of a single, heavy impurity species ($z \gg 1$) in a hydrogenic plasma with relatively low density, $n_z \ll n_e/z$. The generalization to several species is straightforward. The electrostatic potential obtained by solving Eqs. (1) and (2) is then simply given by $e\Phi/T_e = \frac{1}{2}(M^2 - M_0^2)$, where $M_0 = M(R_0)$, and the impurity density becomes

$$n_z(\psi,\theta) = N_z(\psi) \exp\left[\frac{z(M^2 - M_0^2)}{2} \left(\frac{T_e + T_i}{T_z} \frac{m_z}{z m_i} - \frac{T_e}{T_z} \right) \right].$$

If $\epsilon = r/R$ is the inverse aspect ratio of the torus and $R = R_0$ on the magnetic axis, then $M^2 - M_0^2$ is of the order of $2\epsilon M^2$. It follows that most impurity ions are localized on the outside of the torus if $\epsilon z M^2 \gg 1$. In a collisional plasma, the impurity density can also be up–down asymmetric,[13] but we shall not be concerned with this more subtle effect here. Substantial poloidal asymmetry of the impurity ion distribution in spinning plasmas has been observed in the Axisymmetric Divertor Experiment (ASDEX)[14] and the Joint European Torus (JET).[15]

We now consider neoclassical, banana-regime transport in such a plasma. To focus on the effect of poloidal impurity localization, we take $M^2 - M_0^2$ to be much smaller than $\epsilon = r/R$, which is allowed to be finite. This ordering conveniently excludes other effects already considered in the literature: since $M \ll 1$ the conventional effects of finite Mach number are negligible, and since $e\Phi/T_e \ll \epsilon$ electrostatic trapping[16,17] is unimportant. In addition to making these simplifications, we restrict our attention to the particularly simple limit $Z_{\rm eff} \equiv 1 + n_z z^2/n_e \gg 1$, so that collisions among the electrons and bulk ions can be neglected, being less frequent than their collisions with impurties. Since the latter are mostly localized on the outside of the torus, an electron or an ion is most likely to experience collisions there. This changes the kinetics of the system, and hence the transport.

To quantify this effect, consider the first-order drift kinetic equation[1,2]

$$v_\parallel \nabla_\parallel \left(f_{a1} + \frac{I v_\parallel}{\Omega_a} \frac{\partial f_{a0}}{\partial \psi} \right) - \frac{e_a v_\parallel E_\parallel}{T_a} f_{a0} = C_a(f_{a1}), \qquad (3)$$

where f_{a0} is the Maxwellian distribution function, $E_\parallel$ the parallel induced electric field, and $\Omega_a = e_a B/m_a$ the gyrofrequency. The species index a refers to either the electrons or the bulk ions, and the collision operator for the species in question with heavy impurity ions is $C_a(f_{a1}) = \nu_{az}\mathscr{L}(f_{a1})$, $\nu_{az} = (3\pi^{1/2}/4\tau_{az})(v_{Ta}/v)^3$, where $v_{Ta} = (2T_a/m_a)^{1/2}$, $\tau_{az} = 3(2\pi)^{3/2}\epsilon_0^2 m_a^{1/2} T_a^{3/2}/n_z z^2 e^2 e_a^2 \ln \Lambda$, and $\mathscr{L} = \frac{1}{2}(\partial/\partial\xi)(1-\xi^2)(\partial/\partial\xi)$ is the Lorentz operator with $\xi = v_\parallel/v$. The only difference between the drift kinetic equation (3) and its analogue in conventional neoclassical theory is that the collision frequency ν_{az} varies strongly over the flux surface. In the banana regime, the solution is

$$f_{a1} = -\frac{I}{\Omega_a}\left(v_\parallel - \frac{H(\lambda_c - \lambda) V_\parallel B}{B_0} \right) \frac{\partial f_{a0}}{\partial \psi} + \frac{e_a}{T_a} \frac{\langle E_\parallel B\rangle}{B_0} \frac{H(\lambda_c-\lambda) V_\parallel}{\langle \nu_{az}\rangle} f_{a0}, \qquad (4)$$

where $\langle \cdots \rangle$ is the usual flux-surface average, $\lambda \equiv (1-\xi^2)/B$ is a constant of motion, and $\lambda_c = 1/B_{\max}$ is its value at the trapped/passing boundary. In the distribution function (4) only the quantity $V_\parallel$, defined by

$$V_\parallel(v,\lambda,\psi) \equiv \langle \nu_{az}\rangle \frac{vB_0}{2} \int_\lambda^{\lambda_c} \frac{d\lambda}{\langle \nu_{az}\xi\rangle},$$

differs from the usual neoclassical theory,[18] reflecting the nonuniformity of the impurity density n_z.

Using this distribution function, it is now straightforward to evaluate the transport. The general expressions for the neoclassical cross-field particle and heat fluxes are[1,2]

$$\langle \boldsymbol{\Gamma}_a \cdot \nabla \psi \rangle = -\left\langle \frac{I}{e_a B} (R_{a\parallel} + n_a e_a E_\parallel) \right\rangle = \left\langle \frac{I}{e_a B} \int m_a v_\parallel \left(\nu_{az} f_{a1} - \frac{e_a E_\parallel v_\parallel}{T_a} f_{a0} \right) d^3 v \right\rangle, \qquad (5)$$

$$\langle \mathbf{q}_a \cdot \nabla \psi \rangle = -\left\langle \frac{I H_{a\parallel}}{e_a B} \right\rangle = \left\langle \frac{I T_a}{e_a B} \int m_a v_\parallel \nu_{az} f_{a1} \left(\frac{m_a v^2}{2T_a} - \frac{5}{2} \right) d^3 v \right\rangle,$$

where $R_{a\parallel}$ is the parallel friction force acting on the species a, and $H_{a\parallel}$ is the corresponding "heat friction." Evaluating these integrals with the distribution function (4) gives the fluxes

$$\langle \boldsymbol{\Gamma}_a \cdot \nabla \psi \rangle = -\frac{p_a I^2}{m_a} \left\langle \frac{1}{\Omega_a^2 \tau_{az}} \left(1 - \frac{B^2}{B_0^2} f_c \right) \right\rangle \left(\frac{d \ln p_a}{d\psi} - \frac{3}{2} \frac{d \ln T_a}{d\psi} \right) - n_a I \left(\left\langle \frac{E_\parallel}{B} \right\rangle - f_c \frac{\langle E_\parallel B \rangle}{B_0^2} \right), \qquad (6)$$

$$\langle \mathbf{q}_a \cdot \nabla \psi \rangle = -\frac{p_a T_a I^2}{m_a} \left\langle \frac{1}{\Omega_a^2 \tau_{az}} \left(1 - \frac{B^2}{B_0^2} f_c \right) \right\rangle \times \left(-\frac{3}{2} \frac{d \ln p_a}{d\psi} + \frac{13}{4} \frac{d \ln T_a}{d\psi} \right), \qquad (7)$$

and the parallel current becomes

$$j_\parallel = \sum_a \int e_a v_\parallel f_{a0} d^3 v = -\frac{I}{B}\left(1 - \frac{B^2}{B_0^2} f_c \right) \frac{dp}{d\psi} + f_c \sigma \frac{B \langle E_\parallel B \rangle}{B_0^2}, \qquad (8)$$

where $B_0 \equiv \langle B^2 \rangle^{1/2}$, $p_a \equiv n_a T_a$, $p \equiv \Sigma_a p_a$, $\sigma \equiv (32/3\pi)\langle \tau_{ez}\rangle$ is the electric conductivity, and

$$f_c = \frac{3B_0^2}{4} \langle n_z \rangle \int_0^{\lambda_c} \frac{\lambda\, d\lambda}{\langle n_z \sqrt{1-\lambda B}\rangle} \qquad (9)$$

is the effective fraction of circulating particles.

If the impurity density n_z is uniform over the flux surface, this definition reduces to the conventional one,[2] and Eqs. (6)–(8) coincide with the conventional transport laws for a Lorentz plasma.[19] However, if n_z is peaked on the outside of the torus f_c becomes smaller than usual, making the transport larger. Take for instance the following average of the bootstrap current, $\langle (j_\parallel - f_c \sigma E_\parallel) B \rangle = -f_t I\, (dp/d\psi)$. In a torus with large aspect ratio and circular cross section, the effective fraction of trapped particles, $f_t \equiv 1 - f_c$, is equal to $f_t \simeq 1.46\epsilon^{1/2}$ if n_z is constant over the flux surface. In contrast, if $\epsilon z M^2 \gg 1$ so that all the impurities are concentrated in the outer midplane, the effective trapped particle fraction is 45% larger since then $f_t = 1 - (B_0^2/B_{\min}^2)(1 - \frac{3}{2}x^{1/2} + \frac{1}{2}x^{3/2}) \simeq 3(\epsilon/2)^{1/2}$, with $x = 1 - B_{\min}/B_{\max} \simeq 2\epsilon$. The particle and heat fluxes, Eqs. (6) and (7), are enhanced by the same factor at large aspect ratio. Thus the poloidal localization of impurities on the outside of the torus enhances neoclassical transport by increasing the effective number of trapped particles, Eq. (9). This is a consequence of the fact that most collisions take place in the outer midplane rather than anywhere along the orbit. Note that the *effective* number of trapped particles is a collisional average of the effects of the trapped population, and this average increases when the collisions are poloidally localized. The *actual* number of trapped electrons and bulk ions is not affected since our orderings rule out electrostatic trapping.

In the limit of unit aspect ratio, $\epsilon \to 1$, there are no circulating particles, $f_c = 0$, and the fluxes (6) and (7) are proportional to $\langle n_z/B^2 \rangle$. Clearly, this flux-surface average is increased if the impurity density n_z peaks on the outside of the torus where the magnetic field is weak. Physically, this reflects that the transport is enhanced if collisions predominantly take place where the Larmor radius, and hence the random walk step length, is large. Indeed, in the limits of uniform and peaked n_z, respectively, we have

$$\left\langle \frac{n_z}{B^2} \right\rangle = \begin{cases} \langle n_z \rangle \langle B^{-2} \rangle, & zM^2 \ll 1, \\ \langle n_z \rangle B_{\min}^{-2} & zM^2 \gg 1. \end{cases}$$

Thus at unit aspect ratio the poloidal localization of impurities can increase the particle and heat fluxes (for given $\langle n_z \rangle$) by a factor up to $1/B_{\min}^2 \langle B^{-2} \rangle$, which is around 2 in the edge of the Small Tight Aspect Ratio Tokamak (START) at Culham.[20] To understand the reason for this enhancement, we recall that the toroidal canonical momentum $p_\varphi = mR(B_\varphi/B)v_\parallel - Ze\psi$ is a constant of motion in the absence of collisions, and the banana tips of a trapped orbit are located on the flux surface $\psi_{tp} = -p_\varphi/Ze$. If a collision changes the parallel velocity by $\Delta v_\parallel$, the banana orbit is displaced radially by $\Delta\psi_{tp} = -(I\Delta v_\parallel/\Omega)$. The step size is thus inversely proportional to the magnetic field strength, and if most collisions take place in the outer midplane, the neoclassical diffusion coefficient, which scales as $(\Delta\psi_{tp})^2$, becomes proportional to $B_{\min}^{-2}$ rather than to $\langle B^{-2} \rangle$.

The total particle flux is always ambipolar because of parallel momentum conservation in Eq. (5). The heavy impurities flow in the opposite direction from the bulk ions, $\Sigma_a e_a \langle \mathbf{\Gamma}_a \cdot \nabla \psi \rangle = -e_z \langle \mathbf{\Gamma}_z \cdot \nabla \psi \rangle$, and the impurity transport is therefore enhanced by the same factor as the transport of bulk ions and electrons.

In summary, we have demonstrated that the bootstrap current and the neoclassical transport of all species are enhanced by the poloidally nonuniform distribution of impurity ions in a rotating tokamak plasma. The circumstance that most collisions with impurities occur on the outside of the torus enhances the neoclassical diffusion by increasing the effective fraction of trapped particles and increasing the step size taken in the collisional random walk. For simplicity we have restricted the analysis to the Lorentz limit of a plasma with highly charged impurity ions and $Z_{\text{eff}} \gg 1$. Since only collisions with impurities matter in this limit, and these particles have the most nonuniform distribution, the effect is then as large as possible. The maximum possible transport enhancement in the banana regime is 45% at large aspect ratio and about a factor of 2 in a typical equilibrium with tight aspect ratio.

ACKNOWLEDGMENTS

Helpful discussions with Patrick Carolan, Jack Connor, Jim Hastie, Michele Romanelli, and Colin Roach are gratefully acknowledged.

This work was funded jointly by the UK Department of Trade and Industry, and Euratom.

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