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On transport barriers and low–high mode transitions

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Transport barriers and transitions between modes of low and high confinement in tokamak plasmas are often attributed to suppression of turbulence by a shear flow related to a plasma gradient, e.g., of density. However, such shear flow is also affected by the second derivative of density. When this is introduced there is no unique relation between flux and gradient—it depends on the source distribution within the plasma and on conditions at the plasma edge (e.g., imposed by the scrape-off layer). This edge gradient must lie within prescribed limits if a stationary plasma profile (which may include an improved confinement zone) is to exist.
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Two important features of plasma confinement in tokamaks are (i) the spontaneous formation of narrow ''transport barriers,'' where heat or particle transport is significantly less than elsewhere; and (ii) an abrupt transition from a configuration with poor overall confinement (the low confinement mode, L-mode) to one of significantly better confinement (as in various high confinement modes, H-modes) accompanied by a change in the overall density or temperature profiles. For a recent review of these phenomena, see Refs. 1 and 2.

Several authors[3–12] have suggested explanations for this behavior that involve the following elements: (i) In poorly confined plasmas there is a high level of small scale turbulence (e.g., due to drift-wave instabilities), which produces a large particle or energy transport. (ii) The reduced transport at barriers, or in the H-mode, is attributed to the suppression of this turbulence by $\mathbf{E}\times\mathbf{B}$ sheared (i.e., varying from one magnetic surface to another) plasma flow. (iii) This shear flow increases with the density, temperature or pressure gradient—as implied by the equilibrium condition $ne(\mathbf{E}+\mathbf{v}\times\mathbf{B})-\nabla p=0$ when the plasma momentum is small.

In this picture the diffusion equation is nonlinear. In particular an increase in gradient leads to a decrease in transport coefficient, tending to increase the gradient further. Consequently, there is a built-in potential for a bifurcation ''catastrophe'' between high and low transport regimes. This bifurcation can be illustrated using a model diffusion coefficient, similar to that of Refs. 1, 5, 7, such as

$$D_1=D_0\{1+\beta/[1+\alpha(dn/dx)^{\gamma}]\}, \tag{1}$$

where the first term represents the underlying quiescent diffusion and the second represents a turbulent diffusion inhibited by a plasma flow that increases with density gradient. Then, when $\beta>4\gamma/(\gamma-1)^2$, the flux $\Gamma=-D\,dn/dx$ exhibits a classical catastrophe and hysteresis as in Fig. 1. This produces an abrupt rise in density gradient (corresponding to improved plasma confinement) as the flux increases, and an abrupt fall in gradient, occurring at a lower flux level, as the flux decreases.

The attraction of this model is that it provides a clear, intuitive picture for transport barriers and $L-H$ mode transitions. However, while it is certainly appealing, the picture could be somewhat misleading. This is because it is plasma velocity that is most closely correlated with density gradient, whereas turbulence is inhibited only by velocity shear. Consequently, second derivatives (curvature) of density or temperature should be involved in the suppression of turbulence. Therefore, instead of Eq. (1), a more instructive model for the diffusion coefficient might be

$$D_1=D_0\left(1+\frac{\beta}{1+\alpha_1(dn/dx)^{\gamma_1}+\alpha_2(d^2n/dx^2)^{\gamma_2}}\right). \tag{2}$$

This change has profound consequences. The relation between flux and gradient can no longer be described locally; it depends on the global source distribution and (because the transport equation is now of higher order) on an *additional* boundary condition, which, as we will see, plays a crucial role.

In this note we explore the effect of changing from a ''first-order'' transport coefficient such as (1) to a ''second-order'' coefficient such as (2). Specifically, we compare the qualitative behavior of a first-order flux

$$Q_1=[1+\beta/(1+g^2)]g, \tag{3}$$

where $g=-dn/dx$, with that of a second-order flux

$$Q_2=\{1+\beta/[1+g^2+\alpha(dg/dx)^2]\}g. \tag{4}$$

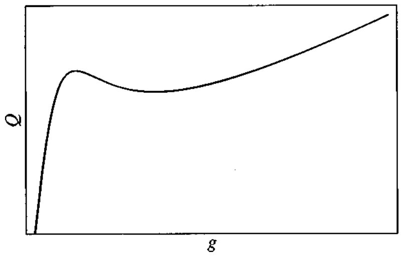

FIG. 1. Universal flux-gradient relation for first-order transport, showing bifurcation and hysteresis [from Eq. (3), $\beta=15$].

(Here $\alpha^{1/2}$ is a scale length, β is the ratio of the maximum turbulent flux to the quiescent flux; other parameters have been absorbed into g, Q, and x.)

The behavior of Q_1 is already summarized by Fig. 1. To describe the behavior of Q_2 it is convenient to regard the gradient g as a function of the local (outward) flux $Q(x)$ due to fixed sources. (This flux is assumed to vanish in the plasma interior and increase monotonically to the plasma edge.) Then

$$\alpha(dQ/dx)^2(dg/dQ)^2=R(g,Q)/(Q-g), \tag{5}$$

where

$$R(g,Q)=g^3-Qg^2+(1+\beta)g-Q. \tag{6}$$

The general form of solutions of Eq. (5) can be illustrated by two simple examples. In the first, the prescribed source flux is linear, $Q=Q_0x/a$; then

$$\epsilon(dg/dQ)^2=R(g,Q)/(Q-g), \tag{7}$$

where $\epsilon=\alpha Q_0^2/a^2$. Real solutions $g(Q)$ of this equation lie between $R(g,Q)=0$ (the original "bifurcated flux" curve), and the line $(Q-g)=0$ (see Fig. 2). They emerge from the origin (corresponding to the plasma interior) tangential to the line $Q=\lambda g$, where

$$\lambda^3-\lambda^2(1+\beta)+\epsilon\lambda-\epsilon=0. \tag{8}$$

Solutions that emerge above the line $Q=\lambda g$ terminate on the curve $R(g,Q)=0$ with infinite slope dQ/dg. Those that emerge below the line terminate on $(Q-\lambda g)=0$ with zero slope.

In the second example, the prescribed flux decreases exponentially with distance into the plasma, $Q=Q_0\exp(-\mu x/a)$. Then

$$\alpha(\mu^2/a^2)(dg/dQ)^2=(1/Q^2)[R(g,Q)/(Q-g)]. \tag{9}$$

In this case, all solutions emerge from the origin tangential to the line $Q=(1+\beta)g$. As before, those that lie above a critical curve terminate on $R(g,Q)=0$ with infinite slope, while those that lie below it terminate on $(Q-g)=0$ with zero slope (see Fig. 3).

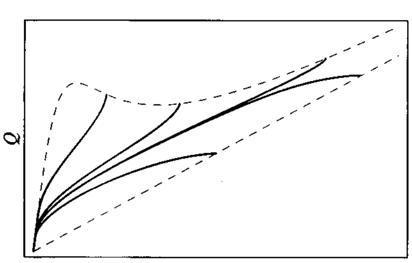

FIG. 2. Flux-gradient relations for second-order transport and linear flux [Eq. (7), $\beta=15$, $\epsilon=10$].

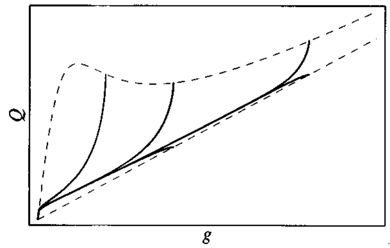

FIG. 3. Flux-gradient relations for second-order transport and exponential flux [Eq. (9), $\beta=15$, $\alpha\mu^2/a^2=10$].

These examples demonstrate the important distinctions between first-order and second-order transport. With first-order transport there is a universal, local, relation between flux and gradient, incorporating a potential bifurcation between high and low flux regimes. With second-order transport there is no such local or universal relation, only a global one that depends on the distribution of sources within the plasma. More significantly, even with a given source distribution the relation between flux and gradient is not unique. In order to select a unique solution from the family of solutions in Figs. 2 and 3 one must impose some form of boundary condition on the gradient, e.g., by specifying its value g_b at the plasma boundary (where $Q=Q_b$). [Of course, once the appropriate solution $g(Q)$ is selected, the full plasma density profile can be constructed using another boundary condition such as $n_b=0$, just as it is for first-order transport.] The fact that the relation between flux and gradient is not unique, but is determined by a boundary condition, emphasizes the important role played by the plasma edge in controlling global plasma profiles and transport.

It should be emphasized that the boundary condition required to select the appropriate solution of Eq. (7) or (9) is in addition to any condition required with first-order transport. Remarkably, although it must originate outside the present theory, e.g., from the properties of a scrape-off layer, it must place g_b within a restricted region if a solution of the transport equation is to be found.

In order to make contact with earlier results based on first-order transport, it is interesting to examine the situation when $\alpha\rightarrow 0$. In this limit, in addition to a boundary layer, such as CD in Fig. 4, there may also be an internal transition layer AB across which there is an abrupt change in plasma gradient—and hence in the effective transport coefficient Q/g. This is shown in Fig. 5 and gives rise to a zone of

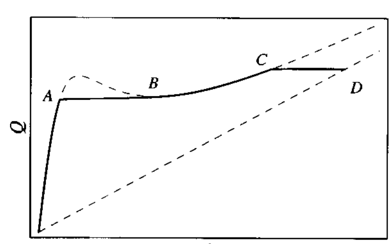

FIG. 4. Flux and gradient for second-order transport and linear flux in the limit $\epsilon\rightarrow 0$.

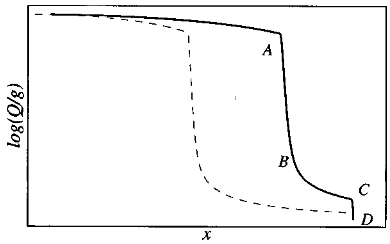

FIG. 5. Variation of the effective transport coefficient Q/g with the radius in the limit $\epsilon \to 0$. The broken curve corresponds to a higher flux at the plasma boundary.

improved confinement, relative to the core, in the outer plasma. The thickness of this zone increases with the flux at the plasma edge. This behavior is similar to that with first-order transport.[7,11] However, there is no hysteresis and the transition layer is located at the minimum of the curve $R(Q,g)=0$, which differs from the location given by the ''Maxwell construction.''

For more general source distributions, $Q(x)$, the behavior of $g(Q)$ will be similar to the examples above. For each source distribution there will be a family of solutions lying between $R(g,Q)=0$ and $(Q-g)=0$. The appropriate solution must be selected by a condition prescribed at the plasma edge, and this condition itself must lie within restricted limits.

In conclusion, our simple model shows that there are significant differences between first-order plasma transport (related only to the gradient of density or temperature) and second-order transport (involving also the curvature of density or temperature). With first-order transport the flux-gradient relation is unique; it can be defined locally and has a built-in potential for bifurcation between high and low transport regimes. With second-order transport the flux-gradient relation cannot be described locally; it depends on the global distribution of sources. More importantly, even with a given source distribution, the relation between flux and plasma gradients is not unique; there is a family of possible flux-gradient relations. The appropriate one depends on a boundary condition at the plasma edge (such as might emerge from a model of the scrape-off layer). This boundary condition plays a crucial role in determining the overall plasma profile and demonstrates the dominant effect of plasma edge on tokomak profiles and on confinement. It is a condition that is not required, indeed is not admissible, with first-order transport, where it would overdetermine the problem. However, if the transport equation is to have a stationary solution it must place the edge gradient within prescribed limits. When a stationary solution does exist, it may involve both a boundary layer and a transition across which the effective transport coefficient changes abruptly. This gives rise to a zone of improved confinement in the outer plasma. On the other hand, if the edge gradient imposed by the scrape-off layer lies outside the prescribed limits, then there is no stationary plasma profile and relaxation oscillations would presumably occur.

ACKNOWLEDGMENT

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