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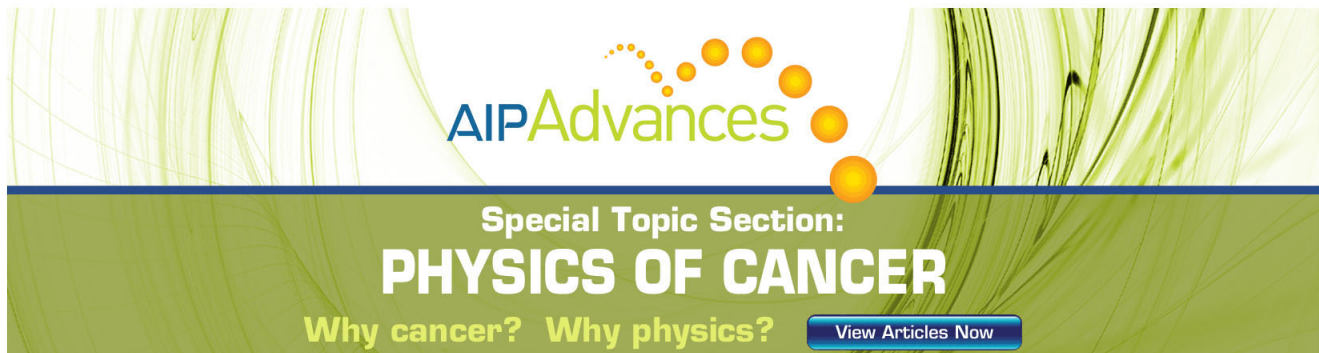
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Neutral particle and radiation effects on Pfirsch–Schlüter fluxes near the edge

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The edge plasma of a tokamak is affected by atomic physics processes and can have density and temperature variations along the magnetic field that strongly modify edge transport. A closed system of equations in the Pfirsch–Schlüter regime is presented that can be solved for the radial and poloidal variation of the plasma density, electron and ion temperatures, and the electrostatic potential in the presence of neutrals and a poloidally asymmetric energy radiation sink due to inelastic electron collisions. Neutrals have a large diffusivity so their viscosity and heat flux can become important even when their density is not high, in which case the neutral viscosity alters the electrostatic potential at the edge by introducing strong radial variation. The strong parallel gradient in the electron temperature that can arise in the presence of a localized radiation sink drives a convective flow of particles and heat across the field. This plasma transport mechanism can balance the neutral influx and is particularly strong if multifaceted asymmetric radiation from the edge (MARFE) occurs, since the electron temperature then varies substantially over the flux surface.

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I. INTRODUCTION

Tokamak performance appears to be sensitive to the edge plasma region just inside the last closed flux surface. In the work presented here we investigate how its behavior can be affected by interactions with neutral particles and poloidal asymmetry in the radiation energy loss due to inelastic electron collisions. These processes are usually unimportant in the plasma core and are therefore normally neglected in plasma transport equations. However, because their diffusivity is large, neutrals can enhance energy and momentum loss and be responsible for the radial variation of the electrostatic potential, even though the neutral density is typically much smaller than the plasma density. In addition to directly modifying the energy balance, poloidally asymmetric radiation loss also creates strong poloidal variation in the electron temperature and thereby drives strong convective fluxes. Convective electron fluxes are particularly strong in the presence of multifaceted asymmetric radiation from the edge (MARFE).¹ To illustrate the effects of neutrals and radiation loss we generalize the conventional Pfirsch–Schlüter regime treatment of tokamak transport² to include charge exchange in the short mean-free path limit and an electron energy sink, both of which may involve significant poloidal asymmetries. Our model differs from the collisional model of Hinton and Kim³ by retaining neutrals and a radiation loss term, by considering the weak plasma flow limit, and by neglecting anomalous effects and the region beyond the separatrix. Moreover, it differs from the collisional model implemented

by Rognlien and Ryutov,⁴ which treats the magnetic field as constant, enhances classical transport to model anomalous effects, ignores neutrals and radiation, and, like Hinton and Kim, works in the large flow limit of Braginskii.⁵ The large flow Braginskii expressions for the ion viscosity do not reduce to the small flow form of Hazeltine² because of the different orderings employed.

The interaction of neutrals with ions via charge exchange influences the electric field by introducing a flux surface averaged neutral toroidal angular momentum flux that can compete with or even dominate over that of the ions. In the absence of neutrals the radial electric field is found to be the square of the inverse aspect ratio smaller than the radial temperature gradient.² As a result, neutral effects on the electric field are expected to be more pronounced in conventional than spherical tokamaks. The neutrals can be responsible for strong radial variation of the electric field and thereby be responsible for large shear in the $\mathbf{E} \times \mathbf{B}$, poloidal, and parallel ion flows, which may have an influence on turbulence.⁶ The neutrals also introduce a heat flux that can compete with the radial ion heat flux and thereby enhance heat transport losses.

To substantially simplify the algebra and illustrate the effects of neutrals in the most transparent way possible, we ignore elastic ion–neutral interactions and assume the charge exchange rate constant is speed independent. Since elastic ion–neutral collisions are thought to be weaker than charge exchange collisions for energies greater than about 10 eV,⁷

they are not expected to quantitatively alter our results. Consequently, we may quite reasonably account for the collisionality between the ions and neutrals by simply employing the deuterium charge exchange cross section σ_x and using the estimate $\sigma_x = 6 \times 10^{-15} \text{ cm}^2$. The constant charge exchange rate approximation⁸ does not appreciably alter the transport coefficients.⁹

To estimate the size of neutral diffusivity effects we can compare the radial neutral and ion heat fluxes, $\vec{q}_n \sim (v_i^2/N_i \langle \sigma v \rangle_x) N_n \partial T_i / \partial r$ and $\vec{q}_i \sim (q^2 \rho_i^2 / \tau_i) N_i \partial T_i / \partial r$, where N_n and N_i are the neutral and ion densities, $v_i = (T_i/M)^{1/2}$ the ion thermal speed with T_i the ion temperature, ρ_i the ion gyroradius, τ_i the ion-ion collision frequency, $\langle \sigma v \rangle_x$ the charge exchange rate constant, q the safety factor, and r the minor radius. For $T_i = 100 \text{ eV}$, $N_i = 3 \times 10^{14} \text{ cm}^{-3}$, $B = 5 \text{ T}$, and $q = 3$, we obtain

$$\frac{\langle \vec{q}_n \cdot \nabla \psi \rangle}{\langle \vec{q}_i \cdot \nabla \psi \rangle} \sim \frac{N_n v_i^2 / N_i \langle \sigma v \rangle_x}{N_i q^2 \rho_i^2 / \tau_i} \sim \frac{N_n}{N_i} \times 10^4,$$

with ψ the poloidal flux function and $\langle \dots \rangle$ denoting a flux surface average. Here $\sigma_x = 6 \times 10^{-15} \text{ cm}^2$ is used for the charge exchange cross section, and the neutral mean-free path is $1/N_i \sigma_x \sim 0.5 \text{ cm}$. The neutral heat flux is large because the neutral diffusivity, $v_i/N_i \sigma_x$, is extremely large, of order $5 \times 10^6 \text{ cm}^2/\text{s}$ for the preceding numbers (while $q^2 \rho_i^2 / \tau_i \sim 400 \text{ cm}^2/\text{s}$). Consequently, even at neutral densities a thousandth of the plasma density a sizable neutral effect occurs, and the effect is larger at lower N_i and higher B and T_i . Moreover, by comparing viscosities instead of heat fluxes we will find that neutral densities smaller by the square of an inverse aspect ratio than the preceding estimate can alter the electrostatic potential dramatically.

Representing the radiation losses by a sink, S , in the electron energy balance equation is a sensible approximation since any non-Maxwellian features in the electron distribution function due to inelastic electron collisions are negligible.¹⁰ We illustrate the effects of radiation loss by assuming that ion-impurity collisions are negligible to simplify the presentation. To estimate the impurity density necessary to make a poloidally asymmetric energy sink result in stronger poloidal electron temperature variation than the usual Pfirsch-Schlüter terms, we note that $S \sim N_I E_I N_e v_e \sigma_I$, where N_e and N_I are the electron and impurity densities, σ_I is the excitation cross section for a typical energy loss E_I , and $v_e = (T_e/m)^{1/2}$ is the electron thermal speed. The poloidal variation of the electron temperature is estimated by balancing the sink S with the parallel electron heat variation, $\vec{n} \cdot \nabla q_{\parallel e}$, where $q_{\parallel e} \sim N_e v_e \lambda \vec{n} \cdot \nabla T_e$. Here λ is the Coulomb mean-free path and $\vec{n} = \vec{B}/B$ the unit vector in the direction of the magnetic field \vec{B} . Normalizing the poloidal variation of the electron temperature by the poloidal variation of the magnetic field magnitude gives

$$\frac{\vec{n} \cdot \nabla \ln T_e}{\vec{n} \cdot \nabla \ln B} \sim \frac{qR}{\epsilon \lambda} \frac{E_I}{T_e} N_I \sigma_I q R,$$

where ϵ is the inverse aspect ratio and within a Pfirsch-Schlüter treatment this ratio must be small compared to unity. For a cool, dense edge the poloidal variation due to

radiation losses is much larger than that due to the usual Pfirsch-Schlüter electron transport,¹¹ which is proportional to the electron gyroradius ρ_e ,

$$\frac{\vec{n} \cdot \nabla \ln T_e}{\vec{n} \cdot \nabla \ln B} \sim \frac{qR}{\epsilon \lambda} \frac{q \rho_e}{w},$$

with w the characteristic edge scale length, which is of the order of the penetration depth $1/N_i (\sigma_x \sigma_z)^{1/2}$, where σ_z is the ionization cross section. For example, taking $w = 2 \text{ cm}$, $T_e = 100 \text{ eV}$, $N_e = 3 \times 10^{14} \text{ cm}^{-3}$, $B = 5 \text{ T}$, $q = 3$, $R = 100 \text{ cm}$, $\sigma_I = 5 \times 10^{-16} \text{ cm}^2$, and $E_I = 25 \text{ eV}$, gives $qR N_e \sigma_I E_I / T_e \sim 10$, while $q \rho_e / w \sim 10^{-3}$ and $qR/\lambda \sim 3$. Consequently, localized impurity to plasma density ratios $N_I/N_e \sim 10^{-3} - 10^{-2}$ are sufficient to give large radiation sink effects and strong poloidal variation in the electron temperature. Very high impurity densities would give poloidal variations comparable to that of the magnetic field, but would complicate the analysis by requiring us to keep ion-impurity collisions and to treat the poloidal electron temperature variation as the same order as that of the poloidal variation of the magnetic field.

In Sec. II we consider the flux-averaged description for the plasma density, ion and electron temperatures, and electrostatic potential, and present the Pfirsch-Schlüter fluxes with the neutral contributions to the heat and toroidal angular momentum fluxes. The neutral viscosity is free of the aspect ratio factors that make the radial variation of the electrostatic potential weak in the conventional Pfirsch-Schlüter treatment,² so that in a large aspect ratio tokamak the strongest impact of the neutrals is on the electrostatic potential. The equations governing the poloidal variation of the plasma density, ion and electron temperatures, and electrostatic potential are obtained in Sec. III to complete our ‘‘four-field’’ model. The strong poloidal variation of the electron temperature due to poloidal variation in the radiation sink is the source of the large particle and electron heat fluxes found in Sec. II. These sink-driven convective fluxes can easily dominate the usual Pfirsch-Schlüter particle and electron heat fluxes that are small in the electron gyroradius. A comparison of the poloidally asymmetric sink-driven convective flux to typical gyro-Bohm transport is given in Sec. IV, along with estimates for the neutral density, which confirm the consistency of our orderings. In Sec. V we present a summarizing discussion.

II. FLUX SURFACE AVERAGED DESCRIPTION

The plasma density $N_e = N_i$, electron T_e and ion T_i temperatures, and electrostatic potential Φ are determined by the flux surface averaged equations for the conservation of number, electron and ion energies, and total toroidal angular momentum, and are flux functions to lowest order in the gyroradius. In the presence of neutrals and a radiation sink S to account for electron energy loss due to inelastic scattering and ionization with rate constant $\langle \sigma v \rangle_z$; the four conservation equations involving the flux surface averaged plasma particle flux $= \langle \vec{\Gamma}_e \cdot \nabla \psi \rangle = \langle \vec{\Gamma}_i \cdot \nabla \psi \rangle$, neutral particle flux $= \langle \vec{\Gamma}_n \cdot \nabla \psi \rangle$, electron heat flux $= \langle \vec{q}_e \cdot \nabla \psi \rangle$, ion heat flux $= \langle \vec{q}_i \cdot \nabla \psi \rangle$, neutral heat flux $= \langle \vec{q}_n \cdot \nabla \psi \rangle$, toroidal ion angu-

lar momentum flux $= \langle R \vec{\zeta} \cdot \vec{\pi}_i \cdot \nabla \psi \rangle$, and toroidal neutral angular momentum flux $= \langle R \vec{\zeta} \cdot \vec{\pi}_n \cdot \nabla \psi \rangle$, are modified to become

$$\frac{\partial N_e}{\partial t} + \frac{1}{V'} \frac{\partial}{\partial \psi} (V' \langle \vec{\Gamma}_e \cdot \nabla \psi \rangle) = \langle \sigma v \rangle_z \langle N_n \rangle N_e, \quad (1)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{3}{2} N_e T_e \right) + \frac{1}{V'} \frac{\partial}{\partial \psi} \left[V' \left(\langle \vec{q}_e \cdot \nabla \psi \rangle + \frac{5}{2} \langle \vec{\Gamma}_e \cdot \nabla \psi \rangle \right) \right] \\ = - \langle S \rangle + \frac{3mN_e(T_i - T_e)}{M\tau_{ei}}, \end{aligned} \quad (2)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{3}{2} N_i T_i \right) + \frac{1}{V'} \frac{\partial}{\partial \psi} \left\{ V' \left[\langle (\vec{q}_i + \vec{q}_n) \cdot \nabla \psi \rangle \right. \right. \\ \left. \left. + \frac{5}{2} \langle (\vec{\Gamma}_i + \vec{\Gamma}_n) \cdot \nabla \psi \rangle \right] \right\} = - \frac{3mN_e(T_i - T_e)}{M\tau_{ei}}, \end{aligned} \quad (3)$$

and

$$\begin{aligned} MN_i \frac{\partial}{\partial t} \langle R \vec{\zeta} \cdot \vec{V}_i \rangle + \frac{1}{V'} \frac{\partial}{\partial \psi} [V' \langle R \vec{\zeta} \cdot (\vec{\pi}_i + \vec{\pi}_n) \cdot \nabla \psi \rangle] \\ = \frac{1}{c} \langle \vec{J} \cdot \nabla \psi \rangle, \end{aligned} \quad (4)$$

where we assume the neutral density to be smaller than the plasma density. In the preceding equations \vec{V}_i is the mean velocity of the ions, M denotes the ion and neutral mass, m is the electron mass, the electron-ion collision time is $\tau_{ei} = 3m^{1/2}T_e^{3/2}/4(2\pi)^{1/2}e^4N_e \ln \Lambda$, with $\ln \Lambda$ the Coulomb logarithm, ζ is the toroidal angle variable with $\vec{\zeta}$ the corresponding unit vector, the radial variable is the poloidal flux function ψ , the magnetic field is written as $\vec{B} = I\nabla\zeta + \nabla\zeta \times \nabla\psi$, with $I = I(\psi) = RB_T$ and $B = |\vec{B}|$, R is the major radius and B_T the toroidal magnetic field, the current density is \vec{J} , and the flux surface average is defined as

$$\langle \cdots \rangle = \frac{1}{V'} \int \frac{d\vartheta(\cdots)}{\vec{B} \cdot \nabla \vartheta},$$

with $V' = \int d\vartheta \vec{B} \cdot \nabla \vartheta$ and ϑ the poloidal angle variable. Using $\vec{\zeta} \cdot \vec{V}_i \sim (q\rho_i/\epsilon w)v_i$ to estimate the Pfirsch-Schlüter flow and $4\pi \langle \vec{J} \cdot \nabla \psi \rangle = - \langle (\partial E/\partial t) \cdot \nabla \psi \rangle \sim (RB_p/e w) \partial T_e / \partial t$, we see that the $\vec{J} \times \vec{B}$ force term on the right side of Eq. (4) is of order $(\epsilon v_A/qc)^2$ compared to the time derivative on the left, where v_A is the Alfvén speed. Often, $v_A/c \ll 1$, so the toroidal $\vec{J} \times \vec{B}$ force on the right of Eq. (4) can be neglected.

The terms that arise from the neutrals enter Eqs. (3) and (4), while those due to the sink S alter the plasma particle and electron heat fluxes in Eqs. (1) and (2). We consider neutral effects first.

A. Neutral and ion contributions to Pfirsch-Schlüter fluxes

To describe the neutrals we employ the neutral kinetic equation with charge exchange collisions¹² and ionization retained, namely

$$\partial f_n / \partial t + \vec{v} \cdot \nabla f_n = \langle \sigma v \rangle_x (N_n f_i - N_i f_n) - \langle \sigma v \rangle_z N_e f_n, \quad (5)$$

where f_n and f_i are the neutral and ion distribution functions, and the moments of species k are defined by

$$\begin{aligned} N_k &= \int d^3v f_k, \quad N_k \vec{V}_k = \int d^3v \vec{v} f_k, \\ p_k &= N_k T_k = \frac{M}{3} \int d^3v v^2 f_k, \quad \vec{Q}_n = \frac{1}{2} M \int d^3v v^2 \vec{v} f_n, \\ \vec{q}_k &= \vec{Q}_k - \frac{5}{2} p_k \vec{V}_k = \frac{1}{2} \int d^3v (Mv^2 - 5T_k) \vec{v} f_k, \end{aligned}$$

and

$$\vec{\pi}_n = M \int d^3v \left(\vec{v} \vec{v} - \frac{1}{3} v^2 \vec{I} \right) f_n, \quad (6)$$

with \vec{I} the unit dyad.

In the short neutral mean-free path limit, the lowest-order neutral distribution function f_0 may be taken as $f_0 \equiv N_n f_i / N_i$, where to lowest order f_i is the stationary Maxwellian f_M . We may then use a Grad moment procedure¹³ in Eq. (5) to write the neutral moments in terms of the ion moments by adopting the ordering

$$\langle \sigma v \rangle_x N_i f_n \sim \langle \sigma v \rangle_x N_n f_i \gg \vec{v} \cdot \nabla f_n \sim \langle \sigma v \rangle_z N_e f_n \gg \partial f_n / \partial t. \quad (7)$$

Neglecting the time derivative term in the $M[\vec{v}\vec{v} - (v^2/3)\vec{I}]$ and $M\vec{v}v^2/2$ moments, we find

$$\begin{aligned} \vec{\pi}_n &= \frac{N_n \langle \sigma v \rangle_x}{N_i (\langle \sigma v \rangle_x + \langle \sigma v \rangle_z)} \vec{\pi}_i + \frac{2\tau}{3} \vec{I} \nabla \cdot \vec{Q}_n \\ &\quad - \tau \nabla \cdot \left(M \int d^3v \vec{v} \vec{v} \vec{v} f_0 \right), \end{aligned} \quad (8)$$

$$\vec{Q}_n = \frac{N_n \langle \sigma v \rangle_x}{N_i (\langle \sigma v \rangle_x + \langle \sigma v \rangle_z)} \vec{Q}_i - \frac{\tau}{2} \nabla \cdot \left(M \int d^3v v^2 \vec{v} \vec{v} f_0 \right), \quad (9)$$

where $\tau \equiv 1/N_i (\langle \sigma v \rangle_x + \langle \sigma v \rangle_z) \approx 1/N_i \langle \sigma v \rangle_x$.

Equations (8)–(9) and the number, momentum, and energy moments of Eq. (5) provide a complete description of the neutrals, provided we know the ion distribution function. To lowest order, conservation of momentum and energy give relations between the neutral and ion temperatures and mean velocities:

$$T_n = \frac{\langle \sigma v \rangle_x}{\langle \sigma v \rangle_x + \langle \sigma v \rangle_z} T_i - \frac{2\tau}{3N_n} \nabla \cdot \vec{Q}_n \approx T_i, \quad (10)$$

and

$$\vec{V}_n = \frac{\langle \sigma v \rangle_x}{\langle \sigma v \rangle_x + \langle \sigma v \rangle_z} \vec{V}_i - \frac{\tau}{MN_n} \nabla p_n \approx \vec{V}_i. \quad (11)$$

Using a lowest-order Maxwellian ($f_0 \equiv N_n f_M / N_i$) in Eq. (9), we find

$$\begin{aligned} \vec{Q}_n &= \vec{q}_n - \frac{5}{2} p_n \vec{V}_n = \frac{N_n \langle \sigma v \rangle_x}{N_i (\langle \sigma v \rangle_x + \langle \sigma v \rangle_z)} \vec{Q}_i \\ &\quad - \frac{5\tau}{2M} \nabla (N_n T_n^2), \end{aligned} \quad (12)$$

or, upon using Eq. (11) and neglecting short mean-free path corrections, the alternate form,

$$\vec{q}_n = -\frac{5\pi p_n}{2M} \nabla T_n + \frac{N_n \langle \sigma v \rangle_x}{N_i (\langle \sigma v \rangle_x + \langle \sigma v \rangle_z)} \vec{q}_i. \quad (13)$$

Next, we consider the final moment of interest. To evaluate the last term in Eq. (8) we need the leading corrections to the Maxwellian that are odd in \vec{v} and to simplify the algebra we note that we need only evaluate $R\vec{\zeta} \cdot \vec{\pi}_k \cdot \nabla \psi$. To evaluate this quantity we extract the required higher-order terms in the ion distribution function f_i from Hazeltine,² correct the numerical coefficients of temperature gradient terms,¹⁴ and write the result in terms of the ion flow velocity \vec{V}_i and ion heat flux \vec{q}_i given by

$$\vec{V}_i = V_{\parallel} \vec{n} + \frac{cT_i}{eB} \left(\frac{1}{p_i} \frac{\partial p_i}{\partial \psi} + \frac{e}{T_i} \frac{\partial \Phi}{\partial \psi} \right) \vec{n} \times \nabla \psi \quad (14a)$$

and

$$\vec{q}_i = q_{\parallel} \vec{n} + \frac{5cp_i}{2eB} \frac{\partial T_i}{\partial \psi} \vec{n} \times \nabla \psi, \quad (14b)$$

where $\vec{n} = \vec{B}/B$, $\vec{n} \times \nabla \psi = I\vec{n} - RB\vec{\zeta}$, $I = RB_T$,

$$V_{\parallel} = \vec{n} \cdot \vec{V}_i = -\frac{cIT_i}{eB} \left[\frac{1}{p_i} \frac{\partial p_i}{\partial \psi} + \frac{e}{T_i} \frac{\partial \Phi}{\partial \psi} + \left(\frac{9}{5\langle B^2 \rangle} + \frac{\langle (\vec{n} \cdot \nabla \ln B)^2 \rangle}{20\langle (\vec{n} \cdot \nabla B)^2 \rangle} \right) \frac{B^2}{T_i} \frac{\partial T_i}{\partial \psi} \right], \quad (15a)$$

and

$$q_{\parallel} = \vec{n} \cdot \vec{q}_i = -\frac{5cIp_i}{2eB} \left(1 - \frac{B^2}{\langle B^2 \rangle} \right) \frac{\partial T_i}{\partial \psi}. \quad (15b)$$

Then, the resulting expression for f_i may be written conveniently as

$$f_i = f_M + \frac{M}{T_i} \left[\vec{V}_i \cdot \vec{v} + \left(\frac{Mv^2}{2T_i} - \frac{5}{2} \right) \frac{2}{5p_i} \vec{q}_i \cdot \vec{v} + \frac{8q_{\parallel} v_{\parallel}}{75p_i} L_2^{(3/2)}(Mv^2/2T_i) \right] f_M + \dots, \quad (16)$$

where $f_M = N_i (M/2\pi T_i)^{3/2} \exp(-Mv^2/2T_i)$ and $L_2^{(3/2)}(x^2) = [x^4 - 7x^2 + (35/4)]/2$ is a Sonine or generalized Laguerre polynomial. In writing Eqs. (15)–(16) we assume that the neutral density is small enough not to affect the usual Pfirsch–Schlüter results.

We may then use Eq. (16) to evaluate the last term of Eq. (8) by first noting that upon using $\nabla_v f_M = -(M/T_i) \vec{v} f_M$ to integrate by parts,

$$\int d^3v \vec{v} \vec{v} \vec{v} \vec{v} \left(\frac{Mv^2}{2T_i} - \frac{5}{2} \right) f_M = \int d^3v \vec{v} \vec{v} \vec{v} \vec{v} f_M, \\ \int d^3v v_{\alpha} v_{\beta} v_{\sigma} v_{\gamma} f_M = N_i \left(\frac{T_i}{M} \right)^2 [\delta_{\alpha\beta} \delta_{\sigma\gamma} + \delta_{\alpha\gamma} \delta_{\sigma\beta} + \delta_{\alpha\sigma} \delta_{\beta\gamma}],$$

and

$$\int d^3v \vec{v} \vec{v} \vec{v} \vec{v} L_2^{(3/2)}(Mv^2/2T_i) f_M \\ = \int d^3v \vec{v} \vec{v} \vec{v} \vec{v} \left(\frac{Mv^2}{2T_i} - \frac{7}{2} \right) f_M = 0,$$

where the δ_{ij} are Kronecker delta functions and we have used the orthogonality of $L_2^{(3/2)}$ to $L_1^{(3/2)}$ and $L_0^{(3/2)}$. As a result, the last term shown in Eq. (16) does not contribute and we obtain

$$R\vec{\zeta} \cdot \left[\nabla \cdot \left(M \int d^3v \vec{v} \vec{v} \vec{v} f_0 \right) \right] \cdot \nabla \psi \\ = \nabla \psi \cdot \nabla \vec{W} \cdot \vec{\zeta} R + R\vec{\zeta} \cdot \nabla \vec{W} \cdot \nabla \psi \\ = \nabla \psi \cdot \nabla (\vec{W} \cdot \vec{\zeta} R) - \vec{W} \cdot \nabla \nabla \psi \cdot \vec{\zeta} R - R\vec{\zeta} \cdot \nabla \nabla \psi \cdot \vec{W} \\ \approx \nabla \psi \cdot \nabla (\vec{W} \cdot \vec{\zeta} R),$$

where $\vec{W} \equiv N_n T_i [\vec{V}_i + (2/5p_i) \vec{q}_i]$ and $\nabla(R\vec{\zeta}) = (\nabla R) \vec{\zeta} - \vec{\zeta} \nabla R$. Fortunately, all terms involving $\nabla \nabla \psi$ may be neglected as small since the radial and/or poloidal variation of N_n , T_i , Φ , $\vec{\zeta} \cdot \vec{V}_i$, and $\vec{\zeta} \cdot \vec{q}_i$ in the plasma edge is much stronger than that associated with the poloidal flux function. As a result, when we gather up the preceding expressions and neglect the N_n/N_i correction to the $\vec{\pi}_i$ term, we find that we may write

$$\langle R\vec{\zeta} \cdot \vec{\pi}_n \cdot \nabla \psi \rangle \approx -\tau \left\langle \nabla \psi \cdot \nabla \left(N_n T_i R \vec{\zeta} \cdot \vec{V}_i + \frac{2N_n}{5N_i} R \vec{\zeta} \cdot \vec{q}_i \right) \right\rangle \\ = -\tau \left\langle \nabla \psi \cdot \nabla \left(\frac{2N_n}{5N_i} R \vec{\zeta} \cdot \vec{Q}_i \right) \right\rangle, \quad (17)$$

with \vec{Q}_i the energy flux defined in Eq. (6),

$$\vec{\zeta} \cdot \vec{V}_i = \vec{\zeta} \cdot \vec{V}_n \\ = -\frac{cRT_i}{e} \left[\frac{1}{p_i} \frac{\partial p_i}{\partial \psi} + \frac{e}{T_i} \frac{\partial \Phi}{\partial \psi} + \left(\frac{9}{5\langle B^2 \rangle} + \frac{\langle (\vec{n} \cdot \nabla \ln B)^2 \rangle}{20\langle (\vec{n} \cdot \nabla B)^2 \rangle} \right) \frac{B_T^2}{T_i} \frac{\partial T_i}{\partial \psi} \right], \quad (18)$$

and

$$\vec{\zeta} \cdot \vec{q}_i = -\frac{5cRp_i}{2e} \left(1 - \frac{B_T^2}{\langle B^2 \rangle} \right) \frac{\partial T_i}{\partial \psi}. \quad (19)$$

In expressions (17)–(19) we may use $T_i = T_n$, and the definitions of moments are as in Eqs. (6). Recall, also that B_T is the toroidal magnetic field.

If we also neglect the N_n/N_i correction to \vec{q}_i , then Eq. (13) gives the neutral heat flux to be

$$\langle \vec{q}_n \cdot \nabla \psi \rangle \approx -\left\langle \frac{5\pi p_n}{2M} \nabla \psi \cdot \nabla T_n \right\rangle. \quad (20)$$

To complete the neutral description we use Eq. (11) and neglect the N_n/N_i correction to \vec{V}_i to obtain the neutral particle flux,

$$\langle \vec{\Gamma}_n \cdot \nabla \psi \rangle \approx - \left\langle \frac{\tau}{M} \nabla \psi \cdot \nabla (N_n T_n) \right\rangle. \quad (21)$$

The ion fluxes in Eqs. (3) and (4) are the standard Pfirsch–Schlüter results:^{2,11,14}

$$\langle \vec{q}_i \cdot \nabla \psi \rangle = - \frac{8Mc^2 I^2 p_i}{5e^2 \tau_i} \left(\left\langle \frac{1}{B^2} \right\rangle - \frac{1}{\langle B^2 \rangle} \right) \frac{\partial T_i}{\partial \psi} \quad (22)$$

and

$$\begin{aligned} \langle R \vec{\zeta} \cdot \vec{\pi}_i \cdot \nabla \psi \rangle \approx & - \frac{16M^2 c^3 I^4 p_i}{25e^3 \tau_i} \frac{\partial T_i}{\partial \psi} \left[\frac{e}{T_i} \frac{\partial \Phi}{\partial \psi} \left(\left\langle \frac{1}{B^4} \right\rangle \right. \right. \\ & - \frac{\langle B^{-2} \rangle}{\langle B^2 \rangle} + \frac{47}{50 T_i} \frac{\partial T_i}{\partial \psi} \left(\left\langle \frac{1}{B^4} \right\rangle \right. \\ & \left. \left. - 3 \frac{\langle B^{-2} \rangle}{\langle B^2 \rangle} + \frac{2}{\langle B^2 \rangle^2} \right) \right], \quad (23) \end{aligned}$$

where $\tau_i = 3M^{1/2} T_i^{3/2} / 4\pi^{1/2} e^4 N_e \ln \Lambda$. For aspect ratios of order unity, the classical contributions,⁵ which we have neglected for simplicity, could be added to Eqs. (22) and (23).

Hazeltine² considered the case without neutrals and noted that for small inverse aspect ratio,

$$\left\langle \frac{1}{B^4} \right\rangle - \frac{\langle B^{-2} \rangle}{\langle B^2 \rangle} \sim \frac{\epsilon^2}{B^4},$$

while

$$\left\langle \frac{1}{B^4} \right\rangle - 3 \frac{\langle B^{-2} \rangle}{\langle B^2 \rangle} + \frac{2}{\langle B^2 \rangle^2} \sim \frac{\epsilon^4}{B^4},$$

so that in the steady state $\langle R \vec{\zeta} \cdot \vec{\pi}_i \cdot \nabla \psi \rangle = 0$ gave the variation of the electrostatic potential to be weak (order ϵ^2) compared to that of the ion temperature. However, the neutral viscosity, of course, is free of these aspect ratio factors so that in a large aspect ratio tokamak the strongest impact of the neutrals is on the electrostatic potential! In the steady state when the neutrals dominate over the ions the radial variation of the electrostatic potential is simply given by $\langle R \vec{\zeta} \cdot \vec{\pi}_n \cdot \nabla \psi \rangle = 0$, so that $e \partial \Phi / \partial \psi \sim \partial T_i / \partial \psi \sim N_i^{-1} \partial p_i / \partial \psi$.

B. Sink and electron contributions to Pfirsch–Schlüter fluxes

The usual Pfirsch–Schlüter treatment of the electron particle and heat flows is modified by the presence of an energy sink S due to radiation losses. We consider the edge region inside the separatrix so the only sink appears in the electron heat balance equation. Our energy sink treatment corresponds to the weak equilibration limit of Wong,¹⁵ except that we retain induced electric field terms.

When the diamagnetic electron heat flux,

$$\vec{q}_{\perp e} = (5c p_e / 2e B^2) \vec{B} \times \nabla T_e, \quad (24)$$

is inserted into the lowest-order electron heat balance equation,

$$\nabla \cdot \vec{q}_e = -S, \quad (25)$$

the resulting equation for the poloidal variation is

$$\vec{B} \cdot \nabla \left(\frac{q_{\parallel e}}{B} - \frac{5c I p_e}{2e B^2} \frac{\partial T_e}{\partial \psi} \right) = -(S - \langle S \rangle),$$

where, as usual, poloidal derivatives of density and temperature are neglected compared to derivatives of magnetic field. Integrating from a convenient angle χ to ϑ gives

$$\frac{q_{\parallel e}}{B} = \frac{5c I p_e}{2e B^2} \frac{\partial T_e}{\partial \psi} - \int_{\chi}^{\vartheta} \frac{d\vartheta (S - \langle S \rangle)}{\vec{B} \cdot \nabla \vartheta} + L(\psi),$$

where the flux function L is determined by employing the parallel heat conduction expression

$$\begin{aligned} q_{\parallel e} = & - \frac{p_e T_e \tau_{ei}}{m} [\kappa_{21} (\vec{n} \cdot \nabla \ln p_e + e E_{\parallel} / T_e) \\ & + \kappa_{22} \vec{n} \cdot \nabla \ln T_e], \quad (26) \end{aligned}$$

where, for $Z=1$, $\kappa_{21}=1.4$, and $\kappa_{22}=4.1$ (see Ref. 5 or 13, for example). Using the constraint

$$\langle q_{\parallel e} B \rangle = -(p_e T_e \tau_{ei} / m) \kappa_{21} e \langle E_{\parallel} B \rangle$$

to determine L , gives

$$\begin{aligned} q_{\parallel e} = & -B \int_{\chi}^{\vartheta} \frac{d\vartheta (S - \langle S \rangle)}{\vec{B} \cdot \nabla \vartheta} \\ & + \frac{B}{\langle B^2 \rangle} \left\langle B^2 \int_{\chi}^{\vartheta} \frac{d\vartheta (S - \langle S \rangle)}{\vec{B} \cdot \nabla \vartheta} \right\rangle \\ & - \frac{5c I p_e}{2e} \left(\frac{B}{\langle B^2 \rangle} - \frac{1}{B} \right) \frac{\partial T_e}{\partial \psi} - \frac{p_e \tau_{ei} \kappa_{21} e B \langle B E_{\parallel} \rangle}{m \langle B^2 \rangle}. \quad (27) \end{aligned}$$

For large, poloidally varying radiation losses, the new terms involving the sink S can be much larger than the usual Pfirsch–Schlüter terms.

To find expressions for the radial electron particle and heat fluxes, we also need the usual forms for the parallel current:^{11,13}

$$\begin{aligned} J_{\parallel} = & \frac{e p_e \tau_{ei}}{m} [\kappa_{11} (\vec{n} \cdot \nabla \ln p_e + e E_{\parallel} / T_e) + \kappa_{12} \vec{n} \cdot \nabla \ln T_e] \\ = & c I \left(\frac{B}{\langle B^2 \rangle} - \frac{1}{B} \right) \frac{\partial p}{\partial \psi} + \frac{p_e \tau_{ei} \kappa_{11} e^2 B \langle B E_{\parallel} \rangle}{m T_e \langle B^2 \rangle}, \quad (28) \end{aligned}$$

where the only novelty is that the total pressure $p = p_e + p_i + p_n$ contains the neutral pressure p_n , which for our purposes is negligible. In Eq. (28), $\kappa_{11}=1.9$ and $\kappa_{12}=\kappa_{21}$ for $Z=1$.^{5,13} From Eqs. (26)–(28) we can obtain $\vec{n} \cdot \nabla T_e$ and $\vec{n} \cdot \nabla p_e + e N_e E_{\parallel}$, which allow us to determine the radial electron fluxes:

$$\begin{aligned}
\langle \vec{q}_e \cdot \nabla \psi \rangle &= \frac{5cIp_e}{2e} \left\langle \frac{\vec{B} \cdot \nabla T_e}{B^2} \right\rangle \\
&= -\frac{5cm\kappa_{11}I}{2e\kappa\tau_{ei}} \left(\left\langle \int_{\chi}^{\vartheta} \frac{d\vartheta(S-\langle S \rangle)}{\vec{B} \cdot \nabla \vartheta} \right\rangle \right. \\
&\quad \left. - \frac{1}{\langle B^2 \rangle} \left\langle B^2 \int_{\chi}^{\vartheta} \frac{d\vartheta(S-\langle S \rangle)}{\vec{B} \cdot \nabla \vartheta} \right\rangle \right) \\
&\quad - \frac{5c^2mI^2T_e}{2e^2\kappa\tau_{ei}} \left(\left\langle \frac{1}{B^2} \right\rangle - \frac{1}{\langle B^2 \rangle} \right) \\
&\quad \times \left(\frac{5\kappa_{11}N_e}{2} \frac{\partial T_e}{\partial \psi} - \kappa_{21} \frac{\partial p}{\partial \psi} \right) \quad (29)
\end{aligned}$$

and

$$\begin{aligned}
\langle \vec{\Gamma}_e \cdot \nabla \psi \rangle &= \langle \vec{\Gamma}_i \cdot \nabla \psi \rangle \\
&= \frac{cI}{e} \left\langle \frac{1}{B^2} (\vec{B} \cdot \nabla p_e + eN_e B E_{\parallel}) \right\rangle - cN_e \langle R \vec{\zeta} \cdot \vec{E} \rangle \\
&= -\frac{cm\kappa_{12}I}{e\kappa\tau_{ei}T_e} \left(\left\langle \int_{\chi}^{\vartheta} \frac{d\vartheta(S-\langle S \rangle)}{\vec{B} \cdot \nabla \vartheta} \right\rangle \right. \\
&\quad \left. - \frac{1}{\langle B^2 \rangle} \left\langle B^2 \int_{\chi}^{\vartheta} \frac{d\vartheta(S-\langle S \rangle)}{\vec{B} \cdot \nabla \vartheta} \right\rangle \right) \\
&\quad + cIN_e \left[\frac{\langle B E_{\parallel} \rangle}{\langle B^2 \rangle} - \left\langle \frac{\vec{\zeta} \cdot \vec{E}}{B_T} \right\rangle \right] - \frac{c^2mI^2}{e^2\kappa\tau_{ei}} \left(\left\langle \frac{1}{B^2} \right\rangle \right. \\
&\quad \left. - \frac{1}{\langle B^2 \rangle} \right) \left(\kappa_{22} \frac{\partial p}{\partial \psi} - \frac{5\kappa_{12}N_e}{2} \frac{\partial T_e}{\partial \psi} \right), \quad (30)
\end{aligned}$$

with $\kappa = \kappa_{11}\kappa_{22} - \kappa_{12}\kappa_{21} = 5.8$.

The convective contributions due to the radiation sink S in the particle and electron heat fluxes of Eqs. (29) and (30) vanish if S is a flux function. Consequently, poloidal variation in S is necessary to drive convective fluxes and, as we shall see in the next section, is responsible for the strong poloidal variation of the electron temperature that results in the convection.

Equations (17), (20)–(23), (29), and (30) are the fluxes to be inserted in Eqs. (1)–(4), with $\vec{\zeta} \cdot \vec{V}_i = \vec{\zeta} \cdot \vec{V}_n$ and $\vec{\zeta} \cdot \vec{q}_i$ given by Eqs. (18) and (19).

III. POLOIDAL VARIATION

In the edge region just inside the separatrix, strong poloidal variation is observed and expected because of the presence of neutrals and radiation. Within the framework of a Pfirsch–Schlüter treatment, the poloidal variation of the plasma density, ion temperature, and potential must be assumed weak compared to that of the magnetic field. As can be verified *a posteriori*, for a high aspect ratio ($\epsilon = r/R \ll 1$) tokamak, this assumption requires

$$q\rho_i/w \ll \epsilon\lambda/qR, \quad (31)$$

where again w and R are the radial scale length of the edge region and the major radius, and $\lambda = v_i\tau_i$ is the Coulomb mean-free path. Inequality (31) follows because the poloidally varying portion of the ion temperature is small compared to its flux surface averaged value by $(q\rho_i/w)(qR/\lambda) \ll 1$, while the poloidal variation of B is of order ϵ . Not surprisingly, inequality (31) is more restrictive by only aspect ratio factors than the requirement that the radial ion heat diffusion time $w^2\tau_i/q^2\rho_i^2$ be larger than the parallel ion heat conduction time $(qR)^2/\lambda v_i$. If this latter condition is not satisfied, the transport along and across the field occurs on similar time scales, making the problem two dimensional. In the absence of a sink, the poloidal variation of the electron temperature would be smaller than that of the ion temperature (and plasma density and electrostatic potential) by ρ_e/ρ_i .

The poloidal variation of the ion temperature is found by equating the usual expressions for the parallel ion heat conduction and its Pfirsch–Schlüter counterpart,

$$q_{\parallel i} = -\frac{125p_iT_i\tau_i}{32M} \vec{n} \cdot \nabla \ln T_i = \frac{5cIp_i}{2e} \left(\frac{B}{\langle B^2 \rangle} - \frac{1}{B} \right) \frac{\partial T_i}{\partial \psi}, \quad (32)$$

to obtain

$$\vec{n} \cdot \nabla T_i = \frac{16McI}{25eB\tau_i} \left(1 - \frac{B^2}{\langle B^2 \rangle} \right) \frac{\partial T_i}{\partial \psi}, \quad (33)$$

where $\vec{n} \cdot \nabla \ln T_i \sim q\rho_i/\lambda w$, as remarked earlier.

Total parallel pressure balance and the requirement that the total pressure be a lowest-order flux function then gives an equation for the poloidal variation of the plasma density,

$$\vec{n} \cdot \nabla p = \vec{n} \cdot \nabla [N_e(T_i + T_e)] = 0, \quad (34)$$

where the neutral pressure contribution is neglected as small.

The poloidal variations of the electrostatic potential and the electron temperature follow from Eqs. (26)–(28), which can be combined to obtain $\vec{n} \cdot \nabla p_e + eN_e E_{\parallel}$ and $\vec{n} \cdot \nabla T_e$, or

$$\begin{aligned}
\frac{e}{T_e} \vec{n} \cdot \nabla \Phi - \vec{n} \cdot \nabla \ln p_e \\
&= \frac{m\kappa_{12}B}{\kappa p_e T_e \tau_{ei}} \left(\int_{\chi}^{\vartheta} \frac{d\vartheta(S-\langle S \rangle)}{\vec{B} \cdot \nabla \vartheta} - \frac{1}{\langle B^2 \rangle} \right. \\
&\quad \left. \times \left\langle B^2 \int_{\chi}^{\vartheta} \frac{d\vartheta(S-\langle S \rangle)}{\vec{B} \cdot \nabla \vartheta} \right\rangle \right) + \frac{e}{T_e} [B E_{\parallel}^I - \langle B E_{\parallel}^I \rangle] \\
&\quad - \frac{mcI}{eB\kappa\tau_{ei}} \left(1 - \frac{B^2}{\langle B^2 \rangle} \right) \left(\frac{\kappa_{22}}{p_e} \frac{\partial p}{\partial \psi} - \frac{5\kappa_{12}}{2T_e} \frac{\partial T_e}{\partial \psi} \right) \quad (35)
\end{aligned}$$

and

$$\begin{aligned} \vec{n} \cdot \nabla T_e &= \frac{m \kappa_{11} B}{\kappa p_e \tau_{ei}} \left(\int_{\chi}^{\vartheta} \frac{d\vartheta (S - \langle S \rangle)}{\vec{B} \cdot \nabla \vartheta} - \frac{1}{\langle B^2 \rangle} \right) \\ &\times \left\langle B^2 \int_{\chi}^{\vartheta} \frac{d\vartheta (S - \langle S \rangle)}{\vec{B} \cdot \nabla \vartheta} \right\rangle + \frac{m c I}{e B \kappa T_e \tau_{ei}} \\ &\times \left(1 - \frac{B^2}{\langle B^2 \rangle} \right) \left(\frac{\kappa_{21}}{p_e} \frac{\partial p}{\partial \psi} - \frac{5 \kappa_{11}}{2 T_e} \frac{\partial T_e}{\partial \psi} \right), \end{aligned} \quad (36)$$

where the superscript *I* on E_{\parallel} denotes that only the induced electric field enters on the right side of Eq. (35). Notice that in the absence of a poloidally varying energy sink S that the poloidal variation of the electron temperature is weak and poloidal variation of the electrostatic potential is therefore a Maxwell–Boltzmann response to lowest order.

As noted in the Introduction, only poloidal asymmetry in the radiation sink S and the condition $q R N_I \sigma_I E_I / T_e \gg q \rho_e / w$ is required to make the radiation sink-driven poloidal variation stronger than the usual Pfirsch–Schlüter poloidal variation of the electron temperature.

Equations (33)–(36) are the four equations for the poloidal variation of the four unknowns T_i , N_e , Φ , and T_e . Notice that the neutrals do not influence the poloidal variation since their density is assumed low compared to that of the plasma.

IV. SINK-DRIVEN AND NEUTRAL FLUX ESTIMATES

To get a feel for the size of the fluxes driven by a poloidally asymmetric radiation sink, $S \sim N_I E_I N_e v_e \sigma_I$, we can compare it with gyro-Bohm heat transport in the following way. We use our estimate from the Introduction to define F as

$$F \equiv \frac{\vec{n} \cdot \nabla \ln T_e}{\vec{n} \cdot \nabla \ln B} \sim \frac{q R E_I}{\epsilon \lambda T_e} N_I \sigma_I q R,$$

and then note from Eq. (29) that poloidal variation in B is needed to generate a radial heat flux so that $\langle B^{-2} \vec{B} \cdot \nabla T_e \rangle \sim \epsilon T_e F / q R B$. As a result, the radial electron heat flux driven by a poloidally asymmetric radiation sink, q_s , is of order

$$q_s \sim D_B F \frac{P_e}{R},$$

where $D_B \equiv c T_e / e B$ denotes the Bohm diffusion coefficient. A gyro-Bohm heat flux q_g in the edge is of order

$$q_g \sim D_B \frac{\rho_i P_e}{w^2},$$

giving

$$\frac{q_s}{q_g} \sim \frac{w^2 F}{\rho_i R}.$$

For the parameters listed in the Introduction, $\rho_i R / w^2 \sim 1$, while our results require $F \ll 1$. However, for stronger radiation losses than our treatment allows, $F > 1$ is permissible and sink-driven fluxes larger than gyro-Bohm fluxes can occur. In particular, the sink-driven transport should be particu-

larly strong if a MARFE is formed, since the electron temperature then varies substantially over the flux surface, making $F \gg 1$.

We can also estimate the size of the neutral heat flux. To do so, we first estimate the neutral density by assuming that the plasma edge inside the separatrix is fully recycling. The convective, poloidally asymmetric radiation sink-driven outward particle flux,

$$\Gamma_s \sim D_B F \frac{N_e}{R},$$

must equal the inward neutral flux,

$$\Gamma_n \sim \frac{v_i^2 N_n}{N_i \langle \sigma v \rangle_{xw}},$$

for a fully recycling edge. Equating these two fluxes gives the following estimate for the neutral to plasma density ratio:

$$\frac{N_n}{N_e} \sim \frac{N_i \langle \sigma v \rangle_{xw} D_B F}{v_i^2 R}.$$

If this estimate is used to eliminate the neutral density in the ratio of neutral to ion heat flux given in the Introduction we obtain

$$\frac{\langle \vec{q}_n \cdot \nabla \psi \rangle}{\langle \vec{q}_i \cdot \nabla \psi \rangle} \sim F \frac{w}{R} \frac{\tau_i D_B}{q^2 \rho_i^2} \sim F \frac{\lambda}{q R} \frac{w}{q \rho_i} \gg \frac{F}{\epsilon},$$

where the Pfirsch–Schlüter validity inequality (31) is used to demonstrate the consistency of our orderings. Consequently, a poloidal radiation sink asymmetry resulting in a poloidal variation of the electron temperature of $F \sim \epsilon$ can (i) cause a sink-driven outward convective plasma particle flux that balances the incoming neutral particle flux; (ii) result in diffusive neutral and convective sink driven heat fluxes that are larger than the Pfirsch–Schlüter ion heat flux; and (iii) cause the neutrals to determine the radial behavior of the electrostatic potential.

V. DISCUSSION

We have derived a complete system of equations for the radial and poloidal variation of the plasma density $N_e = N_i$, electrostatic potential Φ , ion temperature T_i , and electron temperature T_e in the presence of neutrals and a poloidally asymmetric radiation sink. Equations (17), (20)–(23), (29), and (30) are the radial fluxes to be inserted in Eqs. (1)–(4), with toroidal velocity and heat flows given by Eqs. (18) and (19). Equations (33)–(36) are the four equations for the poloidal variation of the four unknowns T_i , N_e , Φ , and T_e . The neutral density can either be assumed to be specified or can be found from the neutral continuity equation,

$$\frac{\partial N_n}{\partial t} + \nabla \cdot (N_n \vec{V}_n) = - \langle \sigma v \rangle_z N_n N_i,$$

with the neutral velocity given by Eq. (11) with $p_n = N_n T_i$. As noted in the Introduction and at the end of Sec. II A, rather small neutral densities can result in large effects on the radial variation of the electrostatic potential and introduce neutral heat and angular momentum fluxes as large as, or

larger than, those associated with the usual Pfirsch–Schlüter ion fluxes. Since the neutrals are localized to the edge, strong shear in the $\mathbf{E} \times \mathbf{B}$, poloidal, and parallel flows can result, which may have an influence on the level of the turbulence.⁶

The effects of a poloidally asymmetric radiation sink are also retained in our system of equations. For a collisional edge, the poloidal variation in the electron temperature due to radiation losses can easily be much larger than that due to the usual Pfirsch–Schlüter electron transport and the resulting convective electron heat and particle fluxes can be comparable to gyro-Bohm fluxes. Consequently, edge transport descriptions retaining only diffusive fluxes are expected to be incomplete for many of the situations of experimental interest.

Note added in proof. When evaluating $\vec{\pi}_n$ we neglected departures of the neutral particle and heat flows from those of the ions [recall Eqs. (11) and (13)] because only the toroidal component of these flows appears in Eq. (17). To retain these departures the leading short mean free path correction to f_0 must be retained in Eqs. (8) and (9).

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- ¹B. Lipschultz, B. LaBombard, E. S. Marmor, M. M. Pickrell, J. L. Terry, R. Watterson, and S. M. Wolfe, *Nucl. Fusion* **24**, 977 (1984).
- ²R. D. Hazeltine, *Phys. Fluids* **17**, 961 (1974) with the following corrections: (i) 0.27 replaced by 0.05 in Eqs. (60) and (61); (ii) 3.1 replaced by 2.9 in Eq. (67); (iii) -2.1 replaced by -1.9 in the $\Delta \ll 1$ form of Eq. (69); (iv) the slash removed in S_1^0 between Eqs. (51) and (52) so it reads as $S_1^0 = x^3 + 2x^5$; (v) an extra power of mass removed in Eq. (77); and (vi) the last term in Eq. (82) should be $2.9 n \partial^2 T / \partial t \partial r$.
- ³F. L. Hinton and Y.-B. Kim, *Nucl. Fusion* **34**, 899 (1994).
- ⁴T. D. Rognlien and D. D. Ryutov, *Contrib. Plasma Phys.* **38**, 152 (1998).
- ⁵S. I. Braginskii, *Zh. Eksp. Teor. Fiz.* **33**, 459 (1957) [*Sov. Phys. JETP* **6**, 358 (1958)]; and *Reviews of Plasma Physics* (Consultants Bureau, New York, 1965), Vol. 1, p. 205.
- ⁶K. H. Burrell, *Phys. Plasmas* **4**, 1499 (1997).
- ⁷D. R. Schultz, S. Yu. Ovchinnikov, and S. V. Passovets, in *Atomic and Molecular Processes in Fusion Edge Plasmas*, edited by R. K. Janev (Plenum, New York, 1995), p. 279.
- ⁸P. J. Catto, *Phys. Plasmas* **1**, 1936 (1994).
- ⁹P. Helander, S. I. Krasheninnikov, and P. J. Catto, *Phys. Plasmas* **1**, 3174 (1994).
- ¹⁰P. J. Catto, S. Krasheninnikov, and R. D. Hazeltine, *Phys. Plasmas* **3**, 461 (1996).
- ¹¹A. A. Galeev and R. Z. Sagdeev, *Zh. Eksp. Teor. Fiz.* **53**, 348 (1967) [*Sov. Phys. JETP* **29**, 475 (1969)], and R. D. Hazeltine and F. L. Hinton, *Phys. Fluids* **16**, 1883 (1973).
- ¹²D. N. Zubarev and V. N. Klimov, in *Plasma Physics and the Problem of Controlled Thermonuclear Reactions*, edited by M. A. Leontovich (Pergamon, Oxford, 1961), Vol. 1, p. 162.
- ¹³H. Grad, *Commun. Pure Appl. Math.* **2**, 331 (1949).
- ¹⁴S. P. Hirshman and D. J. Sigmar, *Nucl. Fusion* **21**, 1079 (1981).
- ¹⁵S. K. Wong, *Phys. Fluids* **21**, 299 (1978), where the matrix L is given by Eq. (45) of S. P. Hirshman, *ibid.* **20**, 589 (1977).