Rotational stabilization of the resistive wall mode by coupling to a dissipative rational surface

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Abstract. Fusion power from a tokamak increases like β^2, (β is the ratio of the plasma and magnetic field pressures) and so the mitigation of instabilities such as the resistive wall mode (RWM), that can prevent high β operation, is important. Stabilization of the RWM with a plasma rotation frequency of below 1% Ω_A, where Ω_A is the Alfvénic rotation frequency, has been observed in a number of tokamaks. An analytical model for this stabilization, in a cylindrical plasma with a resonant layer, is discussed here. The layer theory of Porcelli (Porcelli 1987 *Phys. Fluids* **30** 1734) is used to provide a model of the physics within the resonant layer. A dispersion relation connecting the plasma equilibrium to the layer physics in a rotating plasma is developed. Two mechanisms for RWM stabilization are investigated. The first includes viscosity in the resonant layer. The second assumes that stabilization occurs in the transition from one layer response to another. These models indicate *a priori* that there is a large parameter space where stabilization of the RWM by rotation is possible. However, if experimentally realistic timescales and rotation, namely $O(1)$% Ω_A, is considered then only a small window for stabilization exists. It is therefore unlikely that these mechanisms explain the observed experimental RWM stabilization.

PACS numbers: 52.35.Fa, 52.55.Tn

Submitted to: *Plasma Phys. Control. Fusion*

1. Introduction

1.1. Background

The resistive wall mode (RWM) in a tokamak is a mode which is stable with a perfectly conducting wall at the plasma edge, but is unstable in a non-rotating plasma with no wall. If the wall is not perfectly conducting but the discharge is sufficiently short compared to the wall time, τ_w (the vertical field diffusion time), then the wall will look approximately ideally conducting and the mode will be stable. However, if the discharge duration is long compared to τ_w then the mode will be unstable and it will grow at a rate comparable to τ_w^{-1}. The long pulse lengths required in a potential power plant mean that RWMs may be present if the β is high enough to drive the mode, where $\beta = 2\mu_0 p/B^2$ is the ratio of the plasma pressure, p, to the magnetic pressure. Mitigating the RWM will improve the efficiency that can be achieved in a tokamak power plant since fusion power increases like β^2 [1].

Experimental results, which control plasma rotation using magnetic braking [2, 3] and using neutral beam injection [4, 5] on DIII-D and other machines, have indicated that the RWMs may be stabilized by the rotation of the plasma. If the tokamak has major radius R_0 and plasma density ρ_0, then this stabilization occurs at plasma rotation frequencies of $O(1)\%\ \Omega_A$ and lower, where $\Omega_A = B/(2\pi R_0\sqrt{\rho_0\mu_0})$ is the Alfvén rotational frequency. To explain this result the following theoretical models have been proposed: RWM coupling to a dissipative rational surface [6, 7]; Alfvén continuum damping [8]; sound wave damping [8, 9]; ion Landau damping [10]; and precessional drift resonance [11]. However, none of these models yet satisfactorily explains all the facets of the experimental results.

The RWM coupling to a dissipative rational surface model will be investigated further here. The model is based on a cylindrical plasma, which is unstable without a wall and contains a resonant layer, as analyzed in [7]. This provides a qualitative model of a tokamak plasma in a toroidal geometry. The plasma was assumed to be governed by ideal MHD outside the resonant layer, but by resistivity inside the layer (the 'tearing' response of Furth, Killeen and Rosenbluth [12]). It was shown in [7] that a small window in parameter space exists where stabilization of the RWM is possible, but it is so operationally small that it seems unlikely that it could be exploited experimentally.

This paper is motivated by investigating two mechanisms that may provide an explanation of the experimental results, as suggested in [13]. These were deduced by considering (8) below, taken from Porcelli [14], which models the plasma inside the resonant layer. The physical effects that are included in the resonant layer equation depend, in part, on Ω_p, the plasma rotation frequency. If $\Omega_p \sim \Omega_A$ then only the inertial terms are required. However, if the plasma rotation were reduced other physical effects, such as viscosity and resistivity, would become important and the terms relating to these effects should be included in the layer equation. It is possible to determine the point at which these terms should be included either by balancing terms in (8) or by considering table I in [14].

The first proposed mechanism is that viscosity is needed in the layer to provide the dissipation required. The viscosity terms in the layer equation become important when $\Omega_p \simeq 1\%\Omega_A$, if realistic timescales are used in the calculation. The second proposed mechanism is based not on the absolute layer response, but on the transition from an inertial to a resistive response or from an inertial to a viscous-inertial response. Again, by balancing terms in (8) and using realistic timescales it can be deduced that these transitions occur at $\Omega_p \simeq 1\%\Omega_A$.

1.2. Overview

This paper seeks to extend the previous work of reference [7]. The two mechanisms suggested above will be investigated. The effect of viscosity in the resonant layer will be established first. Following this two models of the transition of the physical response of the resonant layer will be investigated.

A cylindrical model which contains one rational surface will be developed in section 2 using force balance and ideal MHD. Special consideration is required at the wall and at the resonant layer where non-ideal effects are required. The resistive wall will be assumed to be electromagnetically 'thin', i.e. the width of the wall is very much less than the skin depth. Section 3 deals with how the non-ideal effects will be modelled inside the resonant surface using work by Porcelli [14]. There are several different plasma responses in the layer depending on the level of plasma rotation and these will be catalogued. The layer responses that model the transition from the so-called visco-ideal to ideal and from tearing to ideal responses are discussed. Rotational stabilization of the RWM with viscosity included in the resonant layer will be considered in section 4. Characteristic timescales from experiments will be calculated in section 5. These timescales will then be used to assess the viscosity result from the previous chapter and to numerically investigate plasma rotation with the visco-ideal to ideal and tearing to ideal layer responses. In particular, the parameter space where stabilization of the RWM with rotation is possible will be investigated. Finally, conclusions and discussion are given in section 6.

2. Cylindrical model

In work on the rotational stabilization of the RWM by coupling to a dissipative rational surface Finn [6, 15] and Bondeson and Xie [16] used a cylindrical model with a resistive wall. This model aimed to qualitatively model the toroidal plasma so the cylindrical plasma contains a resonant layer. The formulation that Bondeson and Xie [16] used for the magnetic perturbation outside of the resistive wall and resonant layer will be used here.

A cylindrical plasma is modelled with a single mode rational surface. A rational surface occurs at the radius at which $m = nq(r)$, where n, m are the toroidal and poloidal mode numbers and $q(r)$ is the safety factor profile. The plasma is modelled with ideal MHD outside of the resonant layer, but non-ideal effects, such as resistivity and viscosity, may be included at the rational surface $r = r_s$ and at the wall $r = r_w$. Bondeson and Xie [16] use the marginal force balance equation to model the perturbed poloidal magnetic flux $\psi(r)$ sufficiently far from the rational surface yielding

$$\frac{1}{r}\frac{\mathrm{d}}{\mathrm{d}r}\left(r\frac{\mathrm{d}\psi}{\mathrm{d}r}\right) - \frac{m^2}{r^2}\psi - \frac{mj'}{rF}\psi = 0, \tag{1}$$

where $F = (B_\theta/r)(m - nq)$, $'$ denotes the radial derivative, j is the equilibrium current density, and B and B_θ are the unperturbed toroidal and poloidal magnetic field. The solutions of this equation are matched at the resonant surface and also at the resistive wall $r = r_w$. Bondeson and Xie [16] define two solutions of (1): $\psi_0(r)$, the solution with no wall; and $\psi_\infty(r)$ the solution with a perfectly conducting wall at r_w. These solutions are normalized so that $\psi_0(r_w) = 1$ and $\psi'_\infty(r_w^-) = -1/r_w$. The solution to the resistive wall problem is a linear combination of these two solutions

$$\psi = a\psi_0 + b\psi_\infty, \tag{2}$$

with a and b constants.

The boundary condition at the resistive wall can be calculated using the pre-Maxwell equations and a thin wall approximation to be

$$\Delta'_w = \frac{[\psi']_{r_w}}{\psi(r_w)} = p\tau_w, \tag{3}$$

where p is the growth rate of the mode and

$$[f]_r = \lim_{\epsilon\to 0}\left(f(r+\epsilon) - f(r-\epsilon)\right). \tag{4}$$

Imposing this boundary condition on (2) determines the ratio a/b so that the solution becomes

$$\psi = a\left(\psi_0 + p\tau_w\psi_\infty\right). \tag{5}$$

The outer solutions are continuous across the resonant layer, but experience a jump in their first derivatives. On defining the well-known quantity

$$\Delta'_s = \frac{[\psi']_{r_s}}{\psi(r_s)}, \tag{6}$$

where r_s is the location of the resonant surface, Bondeson and Xie [16] used this definition to deduce

$$\Delta'_s = \frac{\delta_0 + p\tau_w\delta_\infty}{\psi_0(r_s) + p\tau_w\psi_\infty(r_s)}, \tag{7}$$

where $\delta_0 = [\psi'_0]_{r_s}$ and $\delta_\infty = [\psi'_\infty]_{r_s}$. Bondeson and Xie [16] note that in the denominator of (7) $\psi_0(r_s) = 0$ and $\psi_\infty(r_s) = 0$ denote the marginal stability of the ideal kink mode without and with a perfectly conducting wall at r_w, respectively. If $\psi(r_s) < 0$ then the mode is ideally unstable since the perturbed magnetic flux has been normalized at the wall, so that $\psi(r_w) = 1$. This means that $\psi(r) = 0$ somewhere in the subinterval $r_s < r < r_w$ and so by Newcomb [17] Theorem 5 and its corollaries the plasma is unstable.

3. Layer physics

Equation (1) is used outside the resonant layer. The solutions to this equation couple the wall mode to the resonant layer. Inside the layer non-ideal effects may be required. Porcelli [14] has analyzed the effects of including resistivity, momentum and viscosity on the growth rate of the resonant layer and also gives the regimes in which they apply. The Fourier transform of the non-ideal MHD equations considered in a slab geometry produces

$$\frac{\mathrm{d}}{\mathrm{d}k}\left(\frac{k^2}{1+\delta_\eta^2 k^2}\frac{\mathrm{d}\xi(k)}{\mathrm{d}k}\right) - \delta_{in}^2 k^2 \xi(k) - \delta_\mu^4 k^4 \xi(k) = 0, \tag{8}$$

where $\xi(k)$ is the Fourier transformed displacement. The non-dimensionalized inertial, viscous and resistive quantities used in this formula are defined as

$$\delta_{in}^2 \equiv \lambda^2, \qquad \delta_\mu^4 \equiv \epsilon_\mu \lambda, \qquad \delta_\eta^2 \equiv \epsilon_\eta/\lambda, \tag{9}$$

where λ is the mode growth rate normalized by $\tau_A = \Omega_A^{-1}$ the Alfvén time, $\epsilon_\mu \equiv \nu_\mu \tau_A$ is the viscous Reynolds number, and $\epsilon_\eta \equiv \nu_\eta \tau_A$ is the resistive Reynolds number both normalized by the Alfvén speed. The unnormalized growth rate is p so that $\lambda = p\tau_A$. The rate of collisional transport of transverse ion momentum is ν_μ and the rate of collisional resistive diffusion of the magnetic field is ν_η [14]. An analytic solution to the full equation is not known, but analytic solutions are possible in certain limits.

The solution outside the layer is matched to the solution inside the layer by setting the $\Delta'_s = \Delta'_L(p)$, where Δ'_s is given by (6) and $\Delta'_L(p)$ is given by the solution of (8). The layer response that is appropriate depends on the value of Δ'_s. The results for the so-called tearing, visco-resistive, visco-ideal and ideal layer responses will be catalogued next, based on the solutions of (8). These results are obtained from Porcelli [14] but have been rescaled so that the full dependence on timescales can be seen. The resistive timescale is defined as $\tau_\eta = \nu_\eta^{-1}$ and the viscous timescale is defined as $\tau_\mu = \nu_\mu^{-1}$.

3.1. Tearing

The tearing response (or reconnecting mode [14]) is used when resistivity dominates inertia and viscosity in the layer. This layer response gives

$$\Delta'_L(p) = (p\tau_L)^{5/4} = p^{5/4}\tau_A^{2/4}\tau_\eta^{3/4}, \tag{10}$$

inside the layer, so that $\tau_L = \tau_A^{2/5}\tau_\eta^{3/5}$. This is the layer theory used by Gimblett and Hastie [7]. It applies whenever

$$\frac{\tau_\eta^{7/6}}{\tau_\mu^{5/6}\tau_A^{1/3}} < \Delta'_s < \frac{\tau_\eta^{1/3}}{\tau_A^{1/3}}. \tag{11}$$

3.2. Visco-resistive

The visco-resistive response (or visco-tearing mode in [14]) is used when viscosity and resistivity dominate inertia in the layer. This layer response gives

$$\Delta'_L(p) = p\tau_L = p\frac{\tau_A^{1/3}\tau_\eta^{5/6}}{\tau_\mu^{1/6}}, \tag{12}$$

so that $\tau_L = \tau_A^{1/3}\tau_\eta^{5/6}/\tau_\mu^{1/6}$. This layer response applies whenever

$$0 < \Delta'_s < \min\left\{\frac{\tau_\eta^{7/6}}{\tau_\mu^{5/6}\tau_A^{1/3}}, \frac{\tau_\eta^{1/6}\tau_\mu^{1/6}}{\tau_A^{1/3}}\right\}. \tag{13}$$

3.3. Visco-ideal

The visco-ideal response is used when viscosity and inertia dominate resistivity in the layer. This layer response gives

$$\Delta'_L(p) = -\frac{1}{(p\tau_L)^{1/4}} = -\left(\frac{\tau_\mu}{p\tau_A^2}\right)^{1/4}. \tag{14}$$

so that $\tau_L = \tau_A^2/\tau_\mu$. It applies whenever

$$-\frac{\tau_\mu^{1/3}}{\tau_A^{1/3}} > \Delta'_s > -\frac{\tau_\eta^{1/6}\tau_\mu^{1/6}}{\tau_A^{1/3}}. \tag{15}$$

3.4. Ideal

Finally, the ideal response is used when inertia dominates viscosity and resistivity in the layer. This layer response gives

$$\Delta'_L(p) = -\frac{1}{p\tau_L} = -\frac{1}{p\tau_A}. \tag{16}$$

This layer response applies whenever

$$0 > \Delta'_s > \max\left\{-\frac{\tau_\eta^{1/3}}{\tau_A^{1/3}}, -\frac{\tau_\mu^{1/3}}{\tau_A^{1/3}}\right\}. \tag{17}$$

3.5. Visco-ideal to Ideal

The previous sections give discrete limits depending upon the dominant physics, however, it is possible to solve the layer equation exactly if resistivity is neglected. The equation that fully models the response from visco-ideal to ideal is

$$\frac{\mathrm{d}}{\mathrm{d}k}\left(k^2\frac{\mathrm{d}\xi(k)}{\mathrm{d}k}\right) - \delta_{in}^2 k^2\xi(k) - \delta_\mu^4 k^4\xi(k) = 0. \tag{18}$$

The solution of this yields the following

$$\Delta'_L(p) = -\frac{1}{2}\frac{\tau_\mu^{1/3}}{\tau_A^{1/3}}\frac{1}{Q_\mu^{1/6}}\left(\frac{\Gamma\left[(Q_\mu+1)/4\right]}{\Gamma\left[(Q_\mu+3)/4\right]}\right), \tag{19}$$

where $\Gamma[x]$ is the Gamma function and

$$Q_\mu = \left(\frac{\delta_{in}}{\delta_\mu}\right)^2 = \left(p^3\tau_A^2\tau_\mu\right)^{1/2}. \tag{20}$$

This layer response applies whenever

$$0 > \Delta_s' > \max\left\{-\frac{\tau_\eta^{1/3}}{\tau_A^{1/3}}, -\frac{\tau_\eta^{1/6}\tau_\mu^{1/6}}{\tau_A^{1/3}}\right\}. \tag{21}$$

3.6. Ideal to Tearing

An exact solution of the layer equation is also possible if viscosity is neglected. The equation that fully models the response from tearing to ideal is

$$\frac{\mathrm{d}}{\mathrm{d}k}\left(\frac{k^2}{1+\delta_\eta^2 k^2}\frac{\mathrm{d}\xi(k)}{\mathrm{d}k}\right) - \delta_{in}^2 k^2 \xi(k) = 0. \tag{22}$$

The solution of this yields the following dispersion relation

$$\Delta_L'(p) = -\frac{\tau_\eta^{1/3}}{\tau_A^{1/3}}\frac{Q_\eta^{5/6}}{8}\left(\frac{\Gamma\left[(Q_\eta - 1)/4\right]}{\Gamma\left[(Q_\eta + 5)/4\right]}\right), \tag{23}$$

where

$$Q_\eta = \frac{\delta_{in}}{\delta_\eta} = \left(p^3\tau_A^2\tau_\eta\right)^{1/2}. \tag{24}$$

This layer response requires a more intricate set of conditions for applicability. If $\Delta_s' > 0$ then this layer response applies for

$$\frac{\tau_\eta^{7/6}}{\tau_\mu^{5/6}\tau_A^{1/3}} < \Delta_s' < \frac{\tau_\eta^{1/3}}{\tau_A^{1/3}}, \tag{25}$$

or if

$$\Delta_s' > \frac{\tau_\eta^{1/3}}{\tau_A^{1/3}} \quad \text{as long as} \quad \frac{\tau_\eta}{\tau_\mu} < 1. \tag{26}$$

If $\Delta_s' < 0$ then this layer response applies for

$$\Delta_s' > \max\left\{-\frac{\tau_\eta^{1/3}}{\tau_A^{1/3}}, -\frac{\tau_\mu^{1/3}}{\tau_A^{1/3}}\right\}, \tag{27}$$

or if

$$\Delta_s' < -\frac{\tau_\eta^{1/3}}{\tau_A^{1/3}} \quad \text{as long as} \quad \frac{\tau_\eta}{\tau_\mu} < 1. \tag{28}$$

If no viscosity is assumed, which amounts to $\tau_\mu \to \infty$, then this layer response is valid for all Δ_s'.

3.7. Dispersion relation

The dispersion relation is formed by matching the solution inside and outside the layer. The RWM is defined to have an unstable ideal mode with no wall and a stable resistive mode with a perfect wall at the plasma boundary assuming a non-rotating plasma. The layer theory above can be used to frame the outer Δ'_s more succinctly than (7).

For an RWM two conditions must apply. First, the mode must be ideally unstable with no wall, which is equivalent to taking $\tau_w \to 0$. Matching the no wall outer Δ'_s to the ideal response inside the layer, see (16), and defining ϵ

$$-\frac{1}{p\tau_A} = \frac{\delta_0}{\psi_0(r_s)} = -\frac{1}{\epsilon}, \tag{29}$$

where $\psi_0(r_s) < 0$ and $\delta_0 > 0$ for ideal instability and $\epsilon > 0$. This guarantees that $p = \epsilon/\tau_A > 0$ and so the mode is unstable.

Next, the mode must be stable with a perfect wall at the plasma boundary, which is equivalent to taking $\tau_w \to \infty$. The perfect wall outer solution is matched to the visco-resistive response, see (12), inside the layer. The visco-resistive response is used because the mode must be resistively stable with a perfect wall. The matching produces

$$p\tau_L = \frac{\delta_\infty}{\psi_\infty(r_s)} = -\delta, \tag{30}$$

where $\psi_\infty(r_s) > 0$ for ideal stability and $\delta > 0$. This guarantees that $p = -\delta/\tau_L < 0$ so the mode is stable.

It can also be shown, by considering the edge region of the plasma as a near vacuum, that $\psi_\infty(r_s)/\delta_0 \simeq 1$, hence, dividing (7) through by δ_0 the outer solution can be expressed as

$$\Delta'_s = \frac{1 - \delta p\tau_w}{-\epsilon + p\tau_w}, \tag{31}$$

where $\epsilon > 0$ and $\delta > 0$. It will be assumed that $\delta = O(1)$ and calculations based on approximating the region outside the resonance as vacuum indicate that this is reasonable.

As remarked earlier, the dispersion relation is formed by setting the outer and inner solutions equal

$$\Delta'_L(p) = \Delta'_s = \frac{1 - \delta p\tau_w}{-\epsilon + p\tau_w}. \tag{32}$$

The growth rate of the mode p can be found be solving this equation with the appropriate layer response. There are several possible roots of the dispersion relation, but the mode of interest is the mode which is ideally unstable when there is no wall. This mode can be followed as the wall is made more perfect, therefore as $\tau_w \to \infty$. It moves from unstable when $\tau_w = 0$ to $p \to 0$ as $\tau_w \to \infty$. The effect of rotation will be considered next.

4. Rotation

It is possible to investigate the effect of rotation on certain instabilities using the dispersion relation. The plasma rotation is introduced by including a simple Doppler shift of the growth rate, p, at the resonant layer. This rotation is equivalent to a stationary plasma with a rotating wall (or a bulk toroidally rotating plasma with a stationary wall). If a tearing response is assumed the dispersion relation is

$$(p\tau_L)^{5/4} = \frac{1 - \delta p\tau_w}{-\epsilon + p\tau_w}. \tag{33}$$

Gimblett and Hastie [7] found that it is possible to stabilize the mode with rotation, but only in a very small parameter window

$$0 < \epsilon\delta < 0.04. \tag{34}$$

Indeed, this condition is only a necessary one. Gimblett and Hastie [7] further identified that stabilization only occurs in part of this parameter space. If a visco-resistive or ideal response is used then stabilization is not possible within this model.

4.1. Visco-ideal response

The dispersion relation for the visco-ideal layer response, assuming that the wall is rotating with frequency Ω and the plasma is fixed is

$$-\frac{1}{(p\tau_L)^{1/4}} = \frac{1 - \delta(p - \mathrm{i}\Omega)\tau_w}{-\epsilon + (p - \mathrm{i}\Omega)\tau_w}. \tag{35}$$

The mode becomes marginally stable with rotation when $\mathrm{Re}(p) = 0$, where $\mathrm{Re}(x)$ denotes the real part of x, therefore when $p = \mathrm{i}\omega$ where without loss of generality $\omega > 0$ and real. An indicative dispersion relation

$$-\frac{1}{(\mathrm{i}\omega)^{1/4}} = \frac{1 - \mathrm{i}\delta(\omega - \Omega)}{-\epsilon + \mathrm{i}(\omega - \Omega)}, \tag{36}$$

is found by assuming that $\tau_L = \tau_w$. The equations arising from the real and imaginary parts of this equation give

$$\delta X^2 + (\sqrt{2} + 1)(1 - \epsilon\delta)X + \epsilon = 0, \tag{37}$$

where $X = (\omega - \Omega)$. The discriminant of this quadratic equation gives the condition for X to be real to be

$$(\sqrt{2} + 1)^2(1 - \epsilon\delta)^2 > 4\epsilon\delta. \tag{38}$$

This condition is satisfied for $\epsilon\delta \le 0.446$ or $\epsilon\delta \ge 2.240$. Recalling (29), $p\tau_A = \epsilon$ is the dispersion relation without a wall from which it can be seen that typical values of ϵ would be less than 0.1, say. Values of $\epsilon > 0.1$ would represent a strong ideal instability. The consequent value of $\delta > 22$ for the second inequality is then considered unrealistic. This can be seen by considering the edge of the plasma as a near vacuum. The second region is therefore not considered further. This indicates that it is possible to stabilize the RWM in this regime and that the parameter space for which stabilization is possible is apparently an order of magnitude larger than for the tearing response, (34).

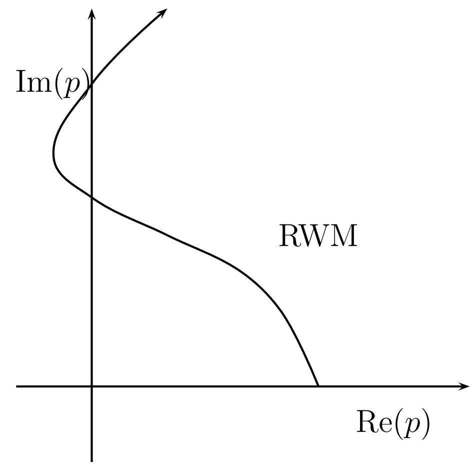

Figure 1. Root locus for the RWM with visco-ideal response with a stability window.

4.2. Root loci with respect to rotation

The analysis in [7] showed that the roots of the dispersion relation for the tearing response (33) had a non-trivial structure. Figure 4 in [7] shows four different classes of root loci with respect to rotation. Two of these classes have stable windows present and two have no stable window. However, the structure of the roots of the dispersion relation with the visco-ideal response only displays two varieties of behaviour; one with a stable window and one without. If a stable window exists the imaginary part of the growth rate increases with plasma rotation while the real part decreases until $\mathrm{Re}(p) = 0$, which is the marginal point. Increased plasma rotation then stabilizes the mode. However, if the plasma rotation is increased still further the mode is destabilized. The locus of the growth rate with increasing plasma rotation is shown schematically in figure 1. If no stable root exists the locus of the growth rate follows a similar path to the case with a stable window but does not cut the imaginary axis, shown schematically in figure 2.

Gimblett and Hastie [7] were able to analyze where the stabilization window closes with rotational frequency. The mode is stabilized with rotational frequency Ω_{st} and subsequently destabilized with rotational frequency Ω_{unst}. The stabilization window closes when $\Omega_{st} = \Omega_{unst}$. The analysis of the visco-ideal case starts with the visco-ideal dispersion relation with bulk toroidal plasma rotation and static wall

$$-\frac{1}{((p - \mathrm{i}\Omega)\tau_L)^{1/4}} = \frac{1 - \delta p \tau_w}{-\epsilon + p\tau_w}. \tag{39}$$

Again, the point of marginal stability, $p = \mathrm{i}\omega$, where ω is real, is taken to give

$$-\frac{1}{(\mathrm{i}(\omega - \mathrm{i}\Omega)\tau_L)^{1/4}} = \frac{1 - \mathrm{i}\delta\omega\tau_w}{-\epsilon + \mathrm{i}\omega\tau_w}. \tag{40}$$

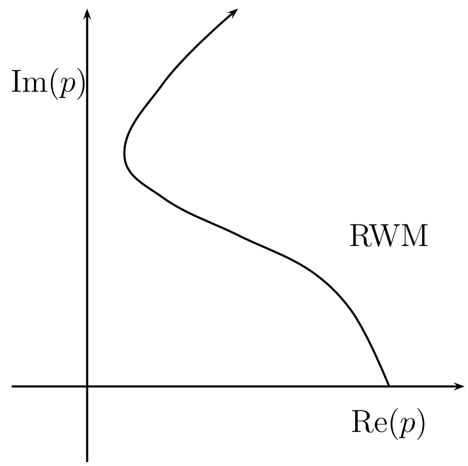

Figure 2. Root locus for the RWM with visco-ideal response with no stability window.

The real and imaginary parts of this equation give two equations which are rearranged to yield

$$\omega^2\tau_w^2\delta + \omega\tau_w(1-\epsilon\delta)(\sqrt{2}+1) + \epsilon = 0, \tag{41}$$

and

$$\Omega = \omega - \frac{1}{\tau_L}\left(\frac{\epsilon^2+\omega^2\tau_w^2}{1+\delta^2\omega^2\tau_w^2}\right)^2. \tag{42}$$

The two solutions of (41) are then used to determine the values of Ω_{unst} and Ω_{st} by using (42). It is found that in the visco-ideal case stability windows exist in two regions; $\epsilon\delta \leq 0.446$ or $\epsilon\delta \geq 2.240$. These regions for the existence of a stability window are exactly the same as the necessary conditions for stabilization from (38). This indicates that for the visco-ideal regime the roots always have the same locus shape with rotation so these conditions are necessary and sufficient for stabilization, unlike the tearing layer case.

4.3. Regime change

The layer responses given in (19) and (23) can be used to model the dispersion relation when the increase in rotation takes the layer response from one regime to another, say from tearing to ideal. The resulting dispersion relations are difficult to investigate analytically, so they will be solved numerically with experimentally relevant timescales in the next section.

5. Comparison with experiments

Reimerdes *et al* [2] give some of the typical plasma parameters for the JET and DIII-D machines where RWM stabilization is seen. This information can be used to check that stabilization is possible in these experimental regimes.

Timescales similar to those seen on DIII-D will be used. The important parameters are: $\tau_w \simeq 7ms$ wall time; $\tau_A \simeq 0.5\mu s$ Alfvén time; $\tau_\mu \simeq 0.1s$ 'phenomenological' viscous time; and $\tau_\eta \simeq 50s$ resistive diffusion time. The 'phenomenological' viscous timescale based on the momentum confinement time typically seen in the machine has been used here rather than the 'Braginskii' classical viscous timescales, which would be much longer. The resistive diffusion timescale has been estimated using the typical Lundquist number $S = \tau_\eta/\tau_A \simeq 10^8$ for a tokamak plasma at the $q = 2$ layer.

The visco-ideal result derived earlier can be re-examined in the light of these experimental timescales. If the dispersion relation (36) is solved for ω and this is substituted back to find the values of Ω, using $\tau_A/\tau_w << 1$, then

$$\epsilon \approx \left(\frac{\Omega}{\Omega_A}\frac{\tau_A}{\tau_\mu}\right)^{1/4}. \tag{43}$$

In experiments $\Omega/\Omega_A \leq 1\%$ and $\tau_A/\tau_\mu \simeq 5 \times 10^{-6}$ so that $\epsilon \simeq 0.015$ which is of similar order to the tearing case. This indicates that viscosity at the rational surface does not explain the stabilization of the RWM.

The other mechanism proposed here for the stabilization of the RWM is stabilization in the transition from one regime to another. The two dispersion relations which handle transitions of regimes can now be solved numerically with the above typical values. First, we consider the tearing to ideal response

$$-\frac{\tau_\eta^{1/3}}{\tau_A^{1/3}}\frac{Q_\eta^{5/6}}{8}\left(\frac{\Gamma\left[(Q_\eta-1)/4\right]}{\Gamma\left[(Q_\eta+5)/4\right]}\right) = \frac{1-\delta(p-\mathrm{i}\Omega)\tau_w}{-\epsilon+(p-\mathrm{i}\Omega)\tau_w}. \tag{44}$$

This dispersion relation was solved for increasing values of ϵ. The values of rotation where the mode was stabilized and destabilized can be seen in figure 3. This figure shows that stabilization is only possible for $\epsilon < 0.04$ when $\delta = 1$. This agrees with the analytical result from the pure tearing response.

Next, we consider the visco-ideal to ideal response

$$-\frac{1}{2}\frac{\tau_\mu^{1/3}}{\tau_A^{1/3}}\frac{1}{Q_\mu^{1/6}}\left(\frac{\Gamma\left[(Q_\mu+1)/4\right]}{\Gamma\left[(Q_\mu+3)/4\right]}\right) = \frac{1-\delta(p-\mathrm{i}\Omega)\tau_w}{-\epsilon+(p-\mathrm{i}\Omega)\tau_w}, \tag{45}$$

This dispersion relation was also solved for increasing values of ϵ. The values of rotation where the mode was stabilized and destabilized can be seen in figure 4. This figure shows that stabilization is only possible for $\epsilon < 0.0068$ when $\delta = 1$. This agrees with the estimate produced for the visco-ideal regime.

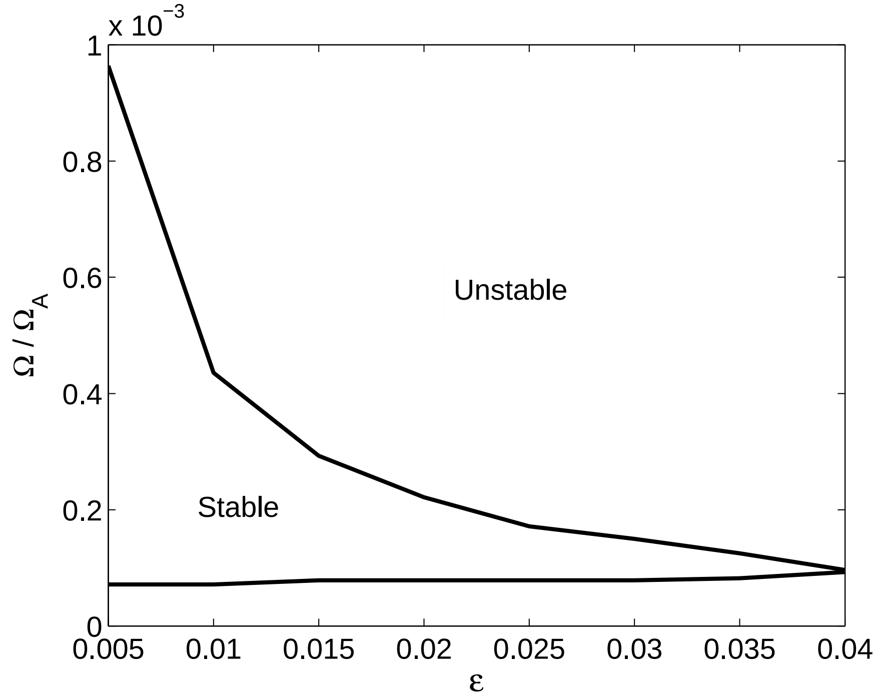

Figure 3. Stable and unstable regions with increasing rotation for the tearing to ideal resonant layer response.

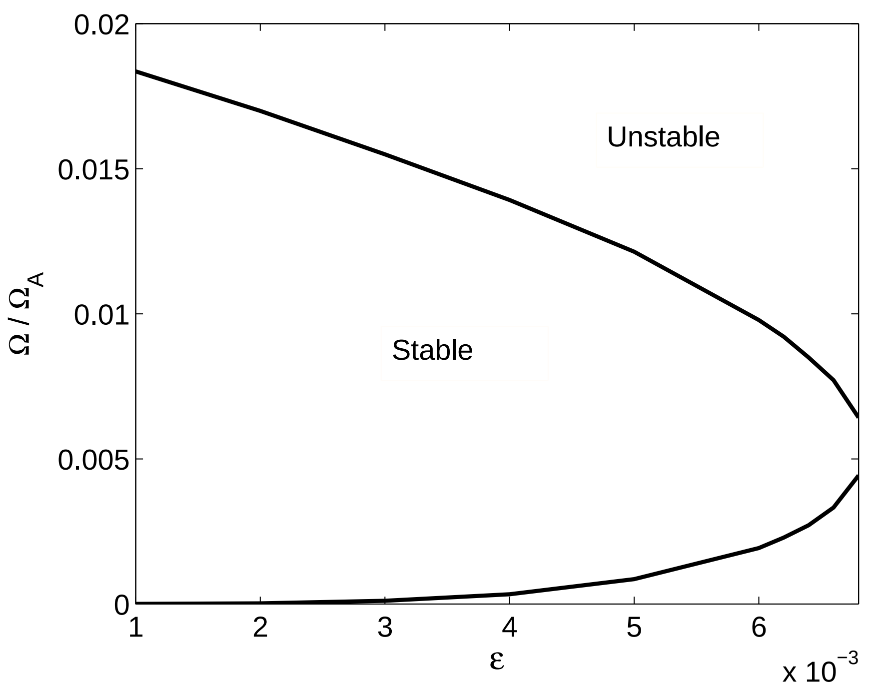

Figure 4. Stable and unstable regions with increasing rotation for the visco-ideal to ideal resonant layer response.

6. Conclusions and discussion

A dispersion relation has been derived for a cylindrical plasma with one resonant surface within the plasma and an electromagnetically thin resistive wall. This cylindrical model provides a qualitative model of a toroidal plasma. A specific cylindrical equilibrium has not been studied here but instead a generic form, which has the characteristics of the RWM, has been used. Ideal MHD has been assumed outside of the resonant surface and resistive wall. Inside the resonant layer the theory of Porcelli [14] has been used to provide solutions of the MHD equations including resistivity and viscosity. Several layer responses have been catalogued.

A succinct outer solution, which encapsulates the RWM equilibrium, has been matched with the inner solutions provided by the layer theory to produce dispersion relations. The solution to the dispersion relation with bulk plasma rotation has been calculated for the visco-ideal layer response and it was found that the equilibrium parameter space which allowed rotational stabilization was apparently an order of magnitude larger than for the tearing layer response.

When timescales for experimental devices were used it was found that the actual parameter space for stabilization of the visco-ideal regime was of the same order as the tearing response. Viscosity in the resonant layer is therefore unlikely to explain the experimentally observed RWM stabilization.

The other mechanism proposed, which looks for stabilization in the transition between responses was investigated. The dispersion relations which allow for transition from tearing to ideal response and from visco-ideal to ideal response were solved numerically also using the experimental timescales. The results here also showed very small windows for stabilization.

These results indicate that only very weak RWMs can be stabilized with the two mechanisms suggested in this paper. It seems unlikely that rotational stabilization of the RWM by coupling to a dissipative resonant surface is the mechanism responsible for stabilization of the RWM, especially since experiments have demonstrated plasmas operating in excess of 50% above the no wall limit for many resistive wall times, τ_w [2]. It should be noted that a cylindrical model has been used for the outer equilibrium here. It may be that toroidal mode coupling in the outer equilibrium is important for stabilization.

Acknowledgments

This work was jointly funded by the UK Engineering and Physical Sciences Research Council and by the European Commission under the contract of association between EURATOM and UKAEA. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

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