Turbulent Particle Transport in Magnetized Plasmas

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Particle transport in magnetized plasmas is investigated with a fluid model of drift wave turbulence. An analytical calculation shows that magnetic field curvature and thermodiffusion drive an anomalous pinch. The curvature driven pinch velocity is consistent with the prediction of turbulence equipartition theory. The thermodiffusion flux is found to be directed inward for a small ratio of electron to ion pressure gradient, and it reverses its sign when increasing this ratio. Numerical simulations confirm that a turbulent particle pinch exists. It is mainly driven by curvature for equal ion and electron heat sources. The sign and relative weights of the curvature and thermodiffusion pinches are consistent with the analytical calculation.

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Particle transport is a key issue in magnetically confined fusion plasmas since the fusion power increases with density. The ionization source will be mainly peripheral in a future reactor. Therefore the density gradient is expected to be small on the basis of a diffusive transport. However, the density profile is often peaked in tokamaks, even when the fueling is peripheral. This observation is traditionally translated into a particle flux of the form $\Gamma_s = -D_s \nabla n_s + V_s n_s$, where V_s is the pinch velocity, D_s is the diffusion coefficient, and n_s is the density of the species "s." The theory of collisional transport shows that the inductive field in a tokamak induces a pinch of electrons, the Ware pinch [1]. Also transport analysis of tokamak plasmas indicates that the diffusion coefficient D_s is usually anomalous, i.e., larger than the collisional value. This anomaly is attributed to turbulent diffusion. A pending question is whether the pinch velocity is collisional or anomalous. Experimental results are quite contradictory regarding this issue. Density profiles were found to be consistent with a Ware pinch only in Asdex-U [2] and JET [3] for plasmas at high density in the H mode. On the other hand, an anomalous pinch has been observed in various devices in the L mode [4–6], including JET [7]. From the theory standpoint, two mechanisms leading to an anomalous pinch have been proposed. One is based on turbulent thermodiffusion [8,9] and predicts a velocity pinch proportional to the gradient of the temperature logarithm $\nabla T_s/T_s$. Thermodiffusion was found in simulations of electron drift wave turbulence [10]. The second type is often called "turbulence equipartition" (TEP) [11–13] and predicts a velocity proportional to the curvature of the magnetic field. This mechanism received some support from 2D simulations of interchange turbulence [14]. An anomalous particle pinch was also found in 2D simulations of ion temperature gradient (ITG) modes and trapped electron modes (TEM) [15]. The aim of this Letter is to investigate the existence of an anomalous pinch and its nature by using a 3D fluid model of ITG/TEM turbulence. Both an analytical quasilinear theory and numerical simulations are used to clarify this issue.

A set of five fluid equations is used here to describe a collisionless ITG/TEM turbulence:

$$d_t n_e = i\omega_{dte}(n_{e,\mathrm{eq}}\phi - p_e) + S_n, \tag{1a}$$

$$d_t p_e = i\omega_{dte}\Gamma(n_{e,\mathrm{eq}}\phi + T_{e,\mathrm{eq}}^2 n_e - 2T_{e,\mathrm{eq}} p_e) + S_{pe}, \tag{1b}$$

$$\begin{aligned} d_t\Omega = &- n_{e,\mathrm{eq}}\nabla_\parallel v_{\parallel i} - i\omega_{di}(n_{e,\mathrm{eq}}\phi + p_i) \\ &- i\omega_{dte} f_t (n_{e,\mathrm{eq}}\phi - p_e) + [p_{i,\mathrm{eq}}, \nabla_\perp^2 \phi] \\ &+ f_c [n_{e,\mathrm{eq}}, \nabla_\perp^2 \phi], \end{aligned} \tag{1c}$$

$$d_t v_{\parallel i} = -\nabla_\parallel(\phi + p_i/n_{e,\mathrm{eq}}) + S_v, \tag{1d}$$

$$\begin{aligned} d_t p_i = &- i\omega_{di}\Gamma[p_{i,\mathrm{eq}}(1 - f_c T_{i,\mathrm{eq}}/T_{e,\mathrm{eq}})\phi \\ &- T_{i,\mathrm{eq}}^2 n_e + 2T_{i,\mathrm{eq}} p_i] \\ &- \Gamma p_{i,\mathrm{eq}} \nabla_\parallel \mathbf{v}_{\parallel i} + S_{pi}, \end{aligned} \tag{1e}$$

where n_s, T_s, p_s, $v_{\parallel s}$, and ϕ are, respectively, the normalized density, temperature, pressure, parallel velocity, and electric potential (the labels "e" and "i" are for electrons and ions; no impurity is included). The generalized vorticity Ω is defined as $\Omega = n_{e,\mathrm{eq}}[f_c(\phi - \phi_{\mathrm{eq}})/T_{e,\mathrm{eq}} - \nabla_\perp^2 \phi]$. The normalization is of the gyroBohm type, $n_e \rightarrow a/\rho_{s0} n_e/n_0$, $p_{e,i} \rightarrow a/\rho_{s0} p_{e,i}/p_0$, $\phi \rightarrow a/\rho_{s0} e\phi/T_0$, $v_{\parallel i} \rightarrow a/\rho_{s0} v_{\parallel i}/c_{s0}$, where ρ_{s0} is the ion gyroradius [$\rho_{s0} = m_i c_{s0}/e_i B_0$, c_{s0} is the sound speed $(T_0/m_i)^{1/2}$], a

and R are the minor and major radius, and n_0, T_0, and $p_0 = n_0 T_0$ are arbitrary reference values. Time and spatial coordinates are normalized to a/c_{s0} and ρ_{s0}. The geometry of flux surfaces is circular concentric, (r, θ, φ) being the labels of the minor radius, poloidal and toroidal angles ($\rho = r/a$ is the normalized minor radius). The fraction of trapped (respectively, passing) electrons is $f_t = 2/\pi(2r/R)^{1/2}$ (respectively, $f_c = 1 - f_t$). The electron precession drift and the ion curvature drift operators are $\omega_{dte} = -i2\varepsilon_a \lambda_t \rho_{s0} q r^{-1} \partial_\varphi$ and $\omega_{di} = -i2\varepsilon_a \rho_{s0}(\cos(\theta) r^{-1}\partial_\theta + \sin(\theta)\partial_r)$, respectively. The function $\lambda_t = \frac{1}{4} + 2s/3$ characterizes the dependence of the precession frequency on the magnetic shear $s = \rho dq/q d\rho$ and $\varepsilon_a = a/R$ parametrizes the curvature ($\varepsilon_a < 1$). The Lagrangian time derivative is defined as $d_t = \partial_t + [\phi,] - D$, where D is a "collisional" diffusion operator and $[f, g] = r^{-1}(\partial_r f \partial_\theta g - \partial_\theta f \partial_r g)$. The functions S_n, S_v, S_{pe}, S_{pi} are particle, momentum, ion, and electron heat sources, respectively. A label "eq" indicates a flux surface average with normalization to the corresponding reference value. Note that the perturbed part of $f_t n_e$ is the fluctuating density of trapped electrons, whereas $n_{e,\mathrm{eq}}$ is the total equilibrium electron density normalized to n_0. The adiabatic compression index is $\Gamma = 5/3$.

Using fluid equations instead of kinetics is obviously an important simplification, in particular, in view of the instability threshold and the dynamics of zonal flows. However, a fluid approach allows separating the various contributions in the following way. The vorticity equation (1c) expresses an ambipolarity condition. The vorticity is coupled via the curvature drifts to electron and ion pressure, which are governed by Eqs. (1b) and (1e). This coupling is responsible for TEM and toroidal ITG instabilities, while the coupling with the parallel momentum equation (1d) is responsible for the slab ITG instability. Anticipating a density gradient of the order of $\varepsilon_a = a/R$, the contributions of the perturbed electron density n_e in the electron and ion heat equations are found to be a factor ε_a smaller than the other terms. The subset, Eqs. (1b)–(1e), is therefore quasiautonomous. When pressure fluctuations are small ($p_e \approx 0$), Eq. (1a) can be recast as $d_t(Hn_e) = 0$, where $H = \exp[\varepsilon_a \int^\rho d\rho(\frac{1}{2} + 4s/3)]$. Thus Hn_e behaves as a passive scalar in this case. If the transport due to velocity fluctuations is diffusive, the "natural" density profile is proportional to $1/H$, in agreement with the TEP prediction [11–13]. Also electron pressure fluctuations are small when the electron pressure gradient is weak. Hence trapped electrons behave as "test particles" if the turbulence is mainly driven by ITG modes.

A quasilinear particle flux can be calculated using Eqs. (1a) and (1b):

$$\Gamma_e = -f_t D_{\mathrm{ql}}\{\partial_\rho n_{e,\mathrm{eq}} + 2\varepsilon_a \lambda_t n_{e,\mathrm{eq}} - 4\varepsilon_a \lambda_t V_{\mathrm{phe}} \partial_\rho p_{e,\mathrm{eq}}\}, \tag{2}$$

where

$$D_{\mathrm{ql}} = \sum_{\mathbf{k}\omega} \frac{k_\theta^2}{\Delta\omega_{\mathbf{k}}} |\phi_{\mathbf{k}\omega}|^2, \tag{3}$$

$$V_{\mathrm{phe}} = \langle \omega/k_\theta \rangle - 2\varepsilon_a \Gamma \lambda_t T_{e,\mathrm{eq}} \langle 1 \rangle,$$

$$\langle F \rangle = \frac{1}{D_{\mathrm{ql}}} \sum_{\mathbf{k}\omega} \frac{k_\theta^4}{\Delta\omega_{\mathbf{k}}^3} |\phi_{\mathbf{k}\omega}|^2 F. \tag{4}$$

Here $\Delta\omega_{\mathbf{k}}$ is a turbulent frequency broadening and $\mathbf{k} = (k_\theta, k_\varphi)$ labels the poloidal and toroidal wave numbers. The calculation is done at order one in$\omega/\Delta\omega_{\mathbf{k}} \sim o(\varepsilon_a^{1/2})$. The "phase velocity" V_{phe} is a shifted poloidal phase velocity ω/k_θ averaged over the turbulence spectrum. The expression (2) indicates that both curvature and thermodiffusion pinches appear in this turbulence model. As expected the TEP result is recovered for zero electron pressure gradient since $dH/Hd\rho = 2\varepsilon_a \lambda_t$. We stress here that the particle pinch velocity depends on the magnetic shear via the precession drift frequency of trapped electrons. Also the TEP result is expected to hold for a well developed turbulence. This excludes cases with negative magnetic shear since TEM modes are then stable. For an arbitrary geometry, the generalized peaking factor is $dn_{e,\mathrm{eq}}/n_{e,\mathrm{eq}} d\psi = -e\omega_{de}/T_{e,\mathrm{eq}}$, where ψ is the poloidal flux and ω_{de} is the precession frequency averaged over the phase space.

In fact, the phase velocity $\langle \omega/k_\theta \rangle$ cannot be chosen freely in Eq. (2). It is constrained by the ambipolarity condition. This property is illustrated here by writing a simplified equation for the ion density using Eqs. (1a) and (1c) and the electroneutrality condition $n_i = n_{e,\mathrm{eq}} f_c(\phi - \phi_{e,\mathrm{eq}})/T_{e,\mathrm{eq}} + f_t n_e$. The ion flux is similar to Eq. (2) (with the transformation $f_t \to 1$, $\lambda_t \to 1$, $\partial_\rho p_{e,\mathrm{eq}} \to -\partial_\rho p_{i,\mathrm{eq}}$) when ignoring finite Larmor radius effects and assuming ballooned modes. The latter assumption implies that finite parallel wave vector $k_\parallel$ must be accounted for in the calculation. This leads to an additional contribution $\langle (k_\parallel/k_\theta)^2 \rangle \partial_\rho p_{i,\mathrm{eq}}$ in the thermodiffusion flux. The average phase velocity $\langle \omega/k_\theta \rangle$ is determined by equating the electron and ion fluxes. It is then used to recalculate the (now equal) ion and electron fluxes,

$$\begin{aligned}\Gamma_e &= \Gamma_i \\ &= -f_t D_{\mathrm{ql}} \frac{1 + \lambda_t \tau_e}{1 + f_t \lambda_t \tau_e} \Big\{ \partial_\rho n_{e,\mathrm{eq}} + 2\varepsilon_a \lambda_t \frac{1 + \tau_e}{1 + \lambda_t \tau_e} n_{e,\mathrm{eq}} \\ &\quad + [8\Gamma \varepsilon_a^2 \langle 1 \rangle (\lambda_t T_{e,\mathrm{eq}} + T_{i,\mathrm{eq}}) \\ &\quad - \langle k_\parallel^2/k_\theta^2 \rangle] \frac{\lambda_t \partial_\rho p_{e,\mathrm{eq}}}{1 + \lambda_t \tau_e} \Big\},\end{aligned} \tag{5}$$

where $\tau_e = \partial_\rho p_{e,\mathrm{eq}}/\partial_\rho p_{i,\mathrm{eq}}$. The structure of Eq. (5) is similar to Eq. (2). In the limit of strong ion heating $\tau_e \to 0$, the pinch velocity due to curvature is identical to Eq. (2), and the TEP result is recovered.

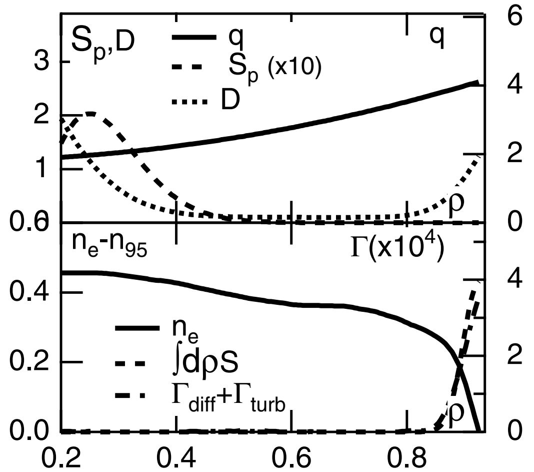

FIG. 1. Profiles of safety factor, heat source, and collisional diffusion coefficient (top panel). Lower panel: time average density profiles, sum of diffusive and turbulent fluxes, particle flux calculated from the source.

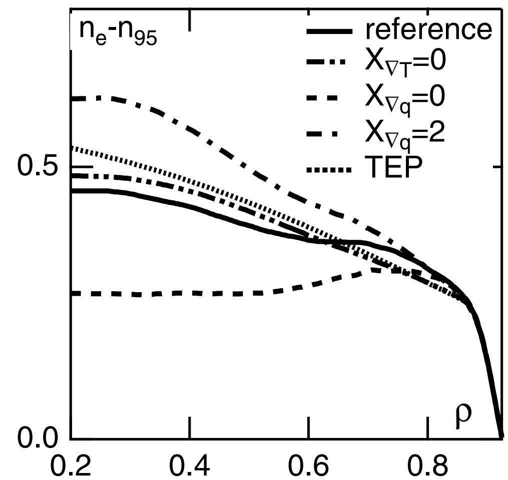

FIG. 2. Density profiles when suppressing ($X_{\nabla q} = 0$) or doubling ($X_{\nabla q} = 2$) the coupling with the electric potential or when suppressing the coupling with the electron pressure equation ($X_{\nabla T} = 0$). The reference case ($X_{\nabla q} = 1, X_{\nabla T} = 1$) is also shown. The dotted line is the TEP prediction with the same boundary edge profile.

Thermodiffusion induces an inward pinch if the average parallel wave number $k_{\parallel}$ is large enough. In the opposite limit $\tau_e \gg 1$, the curvature pinch velocity is controlled by the ion curvature drift $V = 2\varepsilon_a$. Thus the curvature driven pinch depends at least on the ratio τ_e of the electron to ion pressure gradients. A recent analysis indicates that it also depends on the collisionality [16]. The thermodiffusion flux is directed outward if the electron temperature is large enough. This change of sign is due to a change of direction of the average phase velocity. The latter result depends on the closure assumption in the electron and ion pressure equations (parameter Γ), and on the statistical properties of the turbulence (brackets). Also passing electrons are not included here and may also affect the thermodiffusion flux [10].

Equations (1a)–(1e) have been simulated with the spectral TRB code [17]. Numerical details are given in Ref. [18]. All simulations were done for a normalized gyroradius $\rho_{s0}/a = 7.510^{-3}$ with an aspect ratio $R/a = 3$, typical of a JET plasma. The perturbed fields are set to zero at $r = a$, whereas the equilibrium edge density and temperature were set to $n_{e,\mathrm{eq}}(a) = 0.3$ and $T_{e,\mathrm{eq}}(a) = 0.1$. The profiles of safety factor, heat, and particle sources are shown in Fig. 1. Electron and ion heating sources are equal, $S_{\mathrm{pe}} = S_{\mathrm{pi}} = 0.01$. The heat sources are high enough to maintain the temperature gradient well above the instability threshold, i.e., to establish a well developed turbulence. The simulations have been run over 12 000 time units, i.e., three energy confinement times. In these simulations, the fluxes are fixed rather than the gradients. The particle flux is thus maintained to zero so that any density peaking is an unambiguous signature of a turbulent pinch (the Ware pinch is not implemented in this code). Particle flux and density profiles are shown in Fig. 1. The density gradient is finite in the region of zero flux, thus giving evidence of a turbulent pinch. To clarify the nature of this pinch, Eq. (1a) has been replaced by the equation $d_t n_e = i\omega_{dte}(n_{e,\mathrm{eq}} X_{\nabla q}\phi - X_{\nabla T} p_e) + S_n$, where $X_{\nabla q}$ and $X_{\nabla T}$ are adjustable coefficients. Following the quasilinear calculation above, setting $X_{\nabla q} = 0$ should suppress the curvature driven pinch, whereas $X_{\nabla T} = 0$ should suppress thermodiffusion (in the test particle approximation). The results are shown in Fig. 2. It is found that $X_{\nabla q} = 0$ enforces a flat density profile, whereas $X_{\nabla q} = 2$ enhances the peaking by a factor close to 2. Conversely setting $X_{\nabla T} = 0$ affects weakly the density profile. Electron and ion pressure profiles remain almost the same. This analysis shows that curvature is the main drive for particle pinch when $S_{pe} = S_{pi}$. The

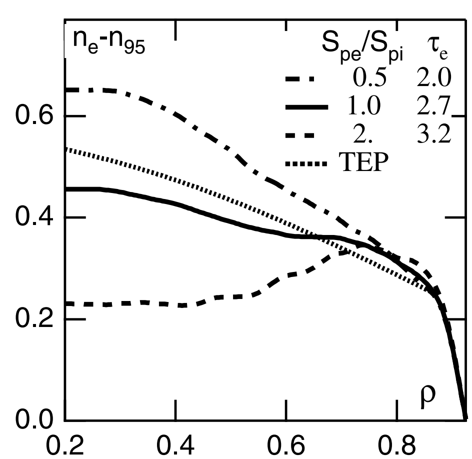

FIG. 3. Density profiles when varying the ratio of electron to ion heating $S_{pe}/S_{pi} = 0.5$, 1, and 2. The corresponding values of τ_e at $\rho = 0.5$ are indicated.

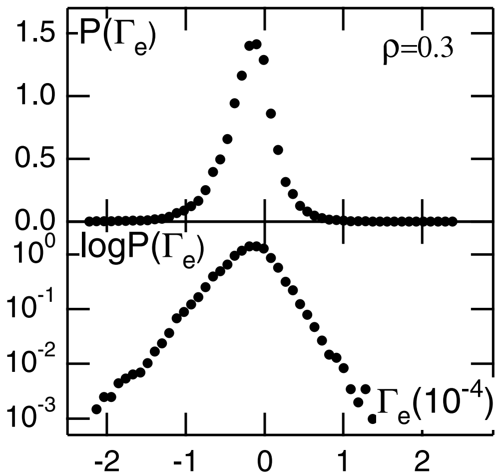

FIG. 4. Probability distribution function of the turbulent particle flux at $\rho = 0.3$ in linear (top) and log-linear (bottom) scales

case $X_{\nabla T} = 0$ agrees fairly well, as expected, with the TEP profile $1/H$ normalized at $\rho = 0.8$ to the profile for $X_{\nabla T} = 0$. We note, however, that the density profile is slightly hollow when $X_{\nabla q} = 0$, which is the indication of a (small) outward thermodiffusion flux. To assess the effect of thermodiffusion, the ratio of ion to electron heating has been changed at constant ion heating source. The density profiles are shown in Fig. 3 for three values of $S_{pe}/S_{pi} = 0.5$, 1, and 2. In this set of simulations, the electron pressure profile increases whereas the ion pressure remains mostly unchanged. In the case of dominant ion heating, the profile is more peaked than expected on the basis of a TEP theory alone. This indicates that an inward thermodiffusion pinch takes place as predicted by Eq. (5) for large enough values of $\langle (k_\parallel/k_\theta)^2 \rangle$. The density profile becomes flatter with increasing electron heating. In fact, an outward pinch is observed in the edge, consistently with the outward thermodiffusion driven by the electron pressure gradient found in expression (5). It is also consistent with the previous test $X_{\nabla q} = 0$. This outward pinch is visible only in the limit of a large ratio of electron to ion pressure gradient $\tau_e \approx 3$. The probability distribution function (PDF) of turbulent flux for $S_{pe}/S_{pi} = 2$ is shown in Fig. 4. It is calculated with a sample of 10^4 values taken at $\rho = 0.3$, i.e., far from the particle source. The average is -0.2 and the variance 0.12 (in units of10^{-4}). This figure suggests that the pinch comes from many events of all sizes (bulk of the PDF) and not from a few exceptional events that would appear in the tail of the distribution.

In conclusion, clear evidence of an anomalous particle pinch has been found when using a fluid model of ITG/TEM turbulence in a tokamak plasma. The pinch velocity due to field curvature agrees with the TEP prediction in the limit of small electron pressure gradient. Also, turbulence simulations indicate that the density profile is close to the TEP prediction for equal electron and ion heating sources. Thermodiffusion plays an increasing role when changing the ratio of electron to ion temperatures. The corresponding flux is inward for a dominant ion heating and becomes outward when the ratio of electron to ion temperatures is large enough. The latter result depends on the turbulence statistical properties and on the closure assumption. We note that electron density profiles remain peaked in devices where electron heating is dominant [4,5]. This suggests that the outward component may not be sufficient to overcome the curvature pinch, albeit it may explain a reduction of profile peaking.

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