Observation of Odd Toroidal Alfvén Eigenmodes

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Experimental evidence is presented for the existence of the theoretically predicted odd toroidicity induced Alfvén eigenmode (TAE) from the simultaneous appearance of odd and even TAEs in a normal shear discharge of the joint European torus. The modes are observed in low central magnetic shear plasmas created by injecting lower hybrid current drive. A fast ion population was created by applying ion cyclotron heating at the high-field side to excite the TAEs. The odd TAEs were identified from their frequency, mode number, and timing relative to the even TAEs.

At the time of the theoretical discovery of toroidicity induced Alfvén eigenmodes (TAE) in the low magnetic shear region of fusion plasmas, predictions were made in ideal MHD for two modes with the same toroidal mode number n in the TAE gap [1–3]. The two modes are formed by the coupling of two poloidal harmonics, m and $m + 1$. The even mode, which resides at the bottom end of the TAE gap, is formed by the coupling of the poloidal harmonics with the same sign, whereas the odd mode, which resides at the top end of the TAE gap, has opposite signs between its two poloidal components as can be seen in Fig. 1. Because of this difference the even TAE has a ballooning mode structure and the odd TAE has an antiballooning structure. These two core localized TAEs can exist when the following condition is fulfilled: $s^2 < \epsilon < s$ (with $s = r/q dq/dr$ the magnetic shear, $\epsilon = r/R$ the inverse aspect ratio, and R the tokamak major radius) [2]. When $\epsilon > s$ a spectrum of multiple TAEs differing in the number of radial nodes was found to exist in one gap [4]. The low shear TAEs can exist only when the normalized pressure gradient, $\alpha = -2(Rq^2/B^2)p'$ (with q the magnetic safety factor, B the magnetic field strength, and $p' = dp/dr$ the radial derivative of the pressure) is lower than a critical value. For the even mode this critical value is given by $\alpha_c^E \approx 3\epsilon + 2s^2$ and for the odd mode $\alpha_c^O \approx 3\epsilon - 2s^2$ [2]. The odd mode exists only when the shear is sufficiently low and its critical α is lower than the one for the even mode, which indicates that the odd mode ceases to exist before the even mode disappears if the central pressure gradient increases. This suggests that the even mode is more robust than the odd mode.

Experimentally so far, only the even mode has been observed unambiguously in large tokamaks [5–7], usually during sawtooth stabilization experiments with ion cyclotron range of frequency heating (ICRH). In those ICRH experiments an extended region of low shear is formed up to half of the plasma minor radius where the even TAEs reside. Until now there has been no unambiguous identification of the odd TAE in tokamak experiments

despite the early prediction of its existence. Experimental observation of those predicted odd TAEs would be a fundamental confirmation of TAE theory.

In this Letter we present compelling experimental evidence for the existence of the odd TAEs from the coexistence of even and odd TAEs in a joint European torus (JET) discharge. We further address the question why the odd TAEs are observed so rarely in tokamaks.

In Fig. 2 a spectrogram of the magnetic fluctuations as measured with a Mirnov coil at the plasma outer midplane is shown where a number of modes with n decreasing in time from 12 to 4 show up between 190 and 210 kHz. These modes chirp down in frequency. Another set of modes with n decreasing in time from 8 to 4 and chirping up in frequency when time progresses is
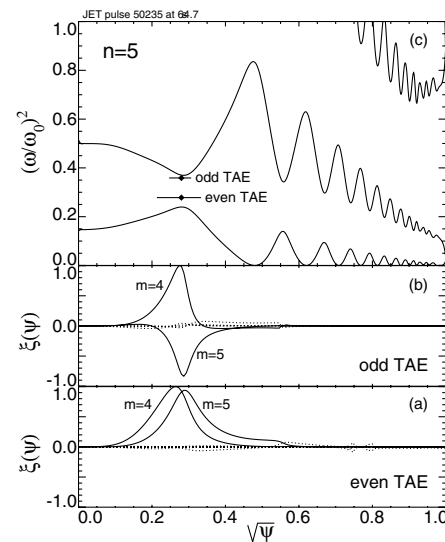

FIG. 1. NOVA-K solution for an $n = 5$ even (a) and odd (b) TAE and their locations in the Alfvén continuum gap (c) normalized to $\omega_0 = B/R\sqrt{\rho(0)}$ with $\rho(0)$ the central mass density. The modes are located at $\epsilon = 0.1$, with $s = 0.272$ and $\alpha = 0.131$. The dotted curves in (a) and (b) are poloidal components other than $m = 4$ and 5.
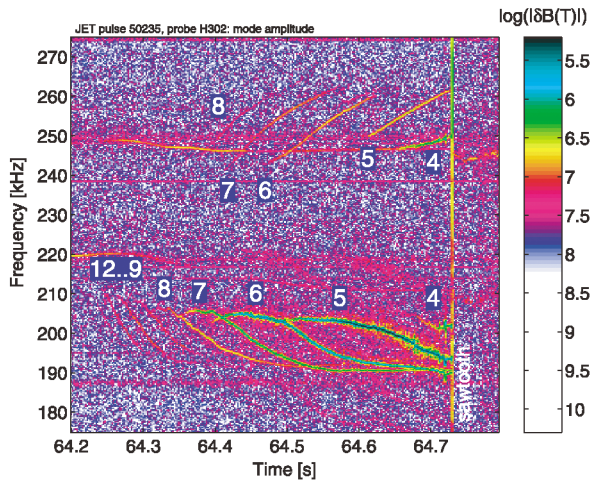
FIG. 2 (color). Spectrogram of the even ($f = 190$–210 kHz)
and odd ($f = 250$–265 kHz) TAEs labeled with respective
toroidal mode numbers.

visible between 240 and 265 kHz. At the giant sawtooth
crash of 64.75 s these modes disappear. The plasma cur-
rent in this discharge (JET pulse 50235) was 2.5 MA; the
toroidal magnetic field at the magnetic axis was 2.6 T. The
heating wave forms of ICRH and lower hybrid current
drive (LHCD, applied from 59 to 63 s) are shown in Fig. 3
together with the on-axis electron density and tempera-
ture. The first harmonic hydrogen ICRH resonance was at
2.81 m, which is 0.15 m to the high-field side from the
plasma center. No neutral beams were injected in this
discharge but from the appearance of the even TAEs the
evolution of the central q profile can be deduced [8]. The
Alfvén frequency at the $q = 1$ surface before the saw-
tooth crash at 64.75 s is 240 kHz, estimated from $\omega_{\text{TAE}} =
B/(\sqrt{\mu_0\rho}2R_0q)$ (ρ the mass density). A thorough com-
parison between measured and simulated frequencies is
difficult because the plasma rotation, which induces a
Doppler shift, was not measured. The rotation, however,
was estimated to be around 1 kHz (from a line integrated
nickel impurity measurement).

The pattern of the modes around 200 kHz is typical for
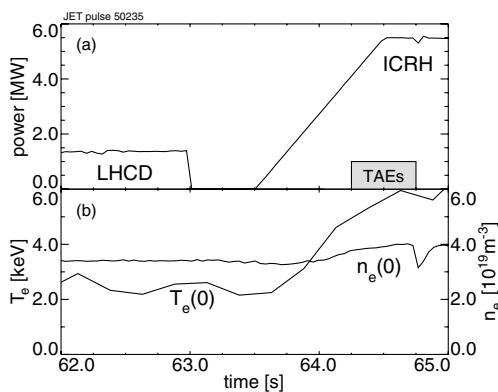even TAEs at the bottom of the TAE gap. The frequency

chirping is caused by a slowly decreasing central mag-
netic safety factor q_0 [5,9].

The CASTOR code [10] was used with experimentally
determined plasma profiles reconstructed with the equi-
librium fitting code EFIT to evaluate the frequency behav-
ior of the TAEs as a function of q_0. It was found that the
low shear region satisfied the condition $s^2 < \epsilon < s$, so
that only two TAEs with the same n are expected to exist
[2]. Furthermore, α stays below α_c throughout the whole
plasma for the even TAEs and from $r/a = 0.0$ to 0.32 for
the odd TAEs. As soon as the threshold q_0 is passed for
exciting the TAEs with mode number n, the even and odd
modes are found from the code (Fig. 4). The even TAEs
show a small frequency up chirp followed by a steady
down chirp that eventually comes to a halt. The odd TAEs
show a small frequency down chirp followed by a steady
up chirp that does not show signs of frequency saturation
as the even modes do which is consistent with the up-
down asymmetry of the TAE gap [11] which is caused by
thermal ion Larmor radius effects and splits the upper
boundary of the TAE gap into kinetic TAEs (KTAEs). The
odd TAEs that exist in the ideal MHD limit are different
from KTAEs whose frequencies are in the upper Alfvén
continuum and are due to finite Larmor radius [11,12].

The different frequency behavior of the odd and even
TAEs is explained as follows. When q_0 decreases, the even
TAEs follow the bottom of the TAE gap to a region where
they can couple with global TAEs that reside outside the
$q = 1$ surface and consist of many popoidal harmonics,
thereby becoming far less sensitive to variations in q_0.
The change from a core localized to global mode struc-
ture is revealed from CASTOR calculations. The odd TAEs
follow the top of the TAE gap when q_0 decreases and they
eventually disappear in the upper Alfvén continuum
without coupling to global modes [3].

The simulated frequency behavior of the even and odd
TAEs compares very well with the modes observed
experimentally at around 200 and 250 kHz. The observed
modes are excited sequentially with the highest n first as
expected for a decreasing q_0. The frequency of the modes

FIG. 3. (a) ICRH and LHCD heating wave forms and
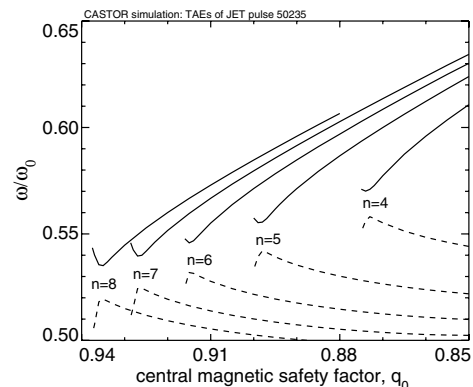(b) central electron density and temperature.

FIG. 4. CASTOR simulation of the frequency behavior of the
observed TAEs as a function of (decreasing) q_0. Solid curves,
odd TAEs; dashed curves, even TAEs; and ω_0 as in Fig. 1.

around 200 kHz show a slight up chirp when they appear followed by a down chirp that comes to a stop after a while similar to the even TAEs in the simulations. The modes at around 250 kHz in the experiment chirp up monotonically with a rate of 120 kHz/s and they show no sign of a decreasing frequency chirp before they disappear, which matches well with the frequency behavior of the odd TAEs as found in the CASTOR simulations.

From the observation of the evolution of the even TAE mode numbers, the rate of change of q_0 with time is obtained as 0.19 s^{-1} [8]. This gives a chirp rate for the calculated odd $n = 6$ TAE of 100 kHz/s, in good agreement with the observed rate. The estimated frequency chirp rate for the $n = 6$ KTAE is 480 kHz/s [12], much faster than the experimental one. In addition, due to the large value of the nonideal parameter ($\propto 1/r^{3/2}$) the core localized KTAE should appear at higher frequencies (310 kHz for $q = 0.92$ at $r/a = 0.28$), which is inconsistent with the experimental observations.

For the modes to be excited in experiments, the fast particle drive has to overcome the mode damping. The NOVA-K code [13] was used to study the various damping and fast particle drive mechanisms to investigate if the drive is sufficient for the excitation of the even and odd modes. In a TAE study at JT-60u it was reported that the critical fast particle β for the odd TAEs is more than 10 times the one for the even TAEs [14] and because of that the odd TAEs were not observed there.

We have calculated the damping rates for the odd and even TAEs with n between 5 and 8. The profiles that were needed for NOVA-K were obtained from TRANSP [15] at 64.5 s. At that time q_0 computed by TRANSP was 0.87. From the results for the damping rates as shown in Table I it can be seen that the total damping rate for both the odd and even modes decreases with decreasing n. The main damping mechanism for most of the odd modes is the collisional damping [16]. Only for the $n = 5$ mode is the electron Landau damping the strongest. Except for the $n = 8$ mode where the collisional damping is the strongest, the radiative damping is the main damping mechanism for the even modes due to its frequency being close to the lower continuum. (The radiative damping was calculated along the lines of Refs. [3,17]). All the modes studied here were located well inside the TAE gap so that they did not interact with the Alfvén continuum; hence, the continuum damping was negligible.

The ICRH induced fast particle drive was also calculated with NOVA-K for the even and odd modes with n between 5 and 8 as a function of the hot ion tail temperature (Fig. 5). In the calculations for the fast ion drive, finite orbit width and finite Larmor radius effects [18–20] were included. The maximum drive for the even TAEs was calculated to be between 3.0% and 7.5%, which is sufficient to excite the modes with n between 5 and 7. The maximum drive for the odd TAEs, between 0.8% to 1.4% depending on n, is reached for tail temperatures less than 250 keV and is sufficient to overcome the damping

TABLE I. Damping rates γ/ω from NOVA-K for odd (top) and even (bottom) TAEs at $q_0 = 0.87$.

| | Odd TAEs | | | |
n	5	6	7	8
Ion Landau [%]	−0.008	−0.004	−0.002	−0.002
Electron Landau [%]	−0.031	−0.028	−0.037	−0.025
Collisional [%]	−0.016	−0.090	−0.368	−0.601
Radiative [%]	−0.001	−0.010	−0.017	−0.019
Total [%]	−0.056	−0.132	−0.424	−0.647
	Even TAEs			
n	5	6	7	8
Ion Landau [%]	−0.041	−0.016	−0.007	−0.005
Electron Landau [%]	−0.048	−0.004	−0.016	−0.014
Collisional [%]	−0.273	−0.196	−0.373	−4.387
Radiative [%]	−1.286	−2.059	−2.395	−2.704
Total [%]	−1.648	−2.275	−2.791	−7.110

(Table I). In this experiment the ICRH resonance layer was located 15 cm away from the plasma center toward the high-field side. This helped to increase the fast ion drive for the odd TAEs significantly compared to on-axis ICRH. For on-axis ICRH the threshold fast ion tail temperature for exciting the odd TAEs shifted to higher tail temperatures, 100 to 200 keV depending on n. The maximum odd TAE drive for on-axis heating was reached at much higher tail temperatures, between 200 and 400 keV, and the maximum drive was found to be more than 3 times less than for the high-field side heating, insufficient to excite the odd TAEs.

The large difference in the drive between the odd and even TAEs can be understood from the fact that the even TAE has a ballooning structure (localized at the low-field side), whereas the odd TAE has an antiballooning structure (localized at the high-field side). The TAEs are excited by the trapped fast particles so it is much more difficult to excite the odd TAE than the even TAE. Shifting the resonance layer to the high-field side
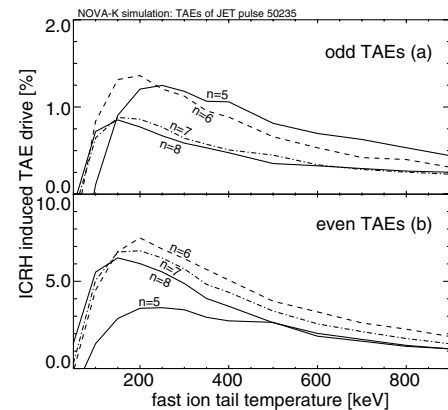

FIG. 5. NOVA-K simulation of the ICRH induced fast particle drive for (a) the odd TAEs and (b) the even TAEs as a function of the fast ion tail temperature.

increased the interaction between the fast particles and the odd TAE to a level where the odd modes were excited.

The ICRH induced fast ion tail temperature was estimated to be in the range of 200 to 400 keV from the TRANSP calculations, so the fast particle drive as calculated with NOVA-K was sufficient to destabilize both the even and the odd TAEs in this discharge.

The mode amplitude saturation is related to the ratio of the drive and damping rates. The TAE amplitude saturation is directly related to the quotient of the linear TAE growth rate, γ_L, in our case from the accelerated hydrogen minority ions, and the damping rate, γ_d. Close to the threshold the nonlinear mode saturation is proportional to $\sqrt{\gamma_L/\gamma_d - 1}$, and well above the threshold it is proportional to $(\gamma_L/\gamma_d)^{2/3}$ [21,22].

Experimentally, we observe that the modes with lower n have a larger amplitude than the high n modes. For the same n we also observe that the even TAE (around 200 kHz) is excited more strongly than the odd TAE (around 250 kHz). These observations are in qualitative agreement with the NOVA-K simulations where it was found that high-n TAEs have larger damping rates than low-n TAEs. Moreover, the NOVA-K modeling of the ICRH drive showed that the odd TAEs have a much weaker drive than the even TAEs, which is consistent with the observed mode saturation level.

From all of the above evidence we conclude that the modes around 200 kHz are the even TAEs and the modes around 250 kHz are the odd TAEs. The final question that remains is why the odd and the even TAEs appear together in this discharge whereas in all other discharges only the even TAEs are observed.

As pointed out in the introduction, two conditions have to be fulfilled for the existence of odd TAEs in tokamaks: (i) a low magnetic shear region in the core and (ii) a flat central pressure profile. Moreover, the ICRH accelerated fast ion population should supply enough drive to excite the odd TAEs. These conditions were all met in this discharge.

When the TAEs were excited, the plasma inductance (ℓ_i) was 1.1, which is significantly lower than $\ell_i = 1.2$ to 1.3 for discharges where only the even TAEs are observed. This low ℓ_i indicates that the current density profile was broader and hence the q profile was flatter than usual in this discharge. The LHCD that was injected before the ICRH was switched on had reduced the central magnetic shear in such a way that both the odd and the even TAEs can exist. The odd and the even TAEs appeared during the ICRH ramp-up phase (Fig. 3) when the fast ion pressure was building up. The ICRH resonance layer was located at the high-field side of the plasma center. This increased the drive for the odd TAEs to a level where the modes were excited.

In normal shear discharges with ICRH driven TAEs, the LHCD pulse before the application of ICRH is absent so the ICRH has to create the low central shear region first. During that time, the central pressure is able to become unfavorably peaked for odd TAEs. Moreover, the on-axis ICRH that is often used in experiments is highly unfavorable for the excitation of the odd TAEs. We can thus explain the successful observation of the odd TAEs in this discharge with the combination of LHCD before the ICRH and the high-field off-axis ICRH deposition.

For the first time odd TAEs, which were predicted to exist in the core of large scale fusion plasmas, have been identified by comparing JET experimental results with CASTOR and NOVA-K simulations. The odd TAEs were excited because of the creation of a central low magnetic shear region with LHCD. Moreover, the ICRH was deposited away from the plasma center at the high-field side, which enhanced the fast particle drive significantly for the odd TAEs.

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