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Proposal for measuring magnetic fluctuations in tokamaks by Thomson scattering^{a)}

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Thomson scattering is proposed for the direct measurement of magnetic field fluctuations in a tokamak. The analysis is based on the electron fluid equations. The ordering $\lambda \ll \lambda_{\text{turb}} \ll a$, where λ and λ_{turb} are the incident and turbulence wavelengths, respectively, and a is the torus minor radius, is suggested by spectral information from observations on the TEXT tokamak, as well as general theoretical arguments. With this ordering, temperature effects are unimportant and an expression is derived for probe radiation-induced plasma polarization that depends upon density and magnetic fluctuations only. By choosing the incident probe beam to have its plane of polarization parallel to the local mean magnetic field \mathbf{B}_0 , and observing the scattered power in the plane of polarization perpendicular to \mathbf{B}_0 , it is shown that contributions from density fluctuations are automatically excluded. Faraday rotation is shown to have negligible influence on the scattered signal, which is accordingly determined by magnetic field fluctuations alone.

I. INTRODUCTION

Fluctuations are thought to play a role in the anomalous transport of particles and energy in magnetically confined fusion research plasmas. Possible mechanisms have been extensively reviewed in the literature.¹

Density fluctuations are routinely measured by laser light scattering, but techniques for measuring fluctuations of the magnetic field are less well developed, consisting in the main of probing the plasma by means of small coils, whose presence may be perilous, both for plasma and probe. So field fluctuation measurements have generally been confined to the edge region. But here, the magnetic fluctuations seem to be too small, according to current theories, to contribute significantly to anomalous transport, at least in tokamaks. Even from measurements limited to the edge region, however, there is intriguing evidence that field fluctuations increase towards the plasma axis, in contrast to fluctuations in other quantities like density, plasma potential, and temperature.² So perhaps the most important questions about magnetic activity are its amplitude and spectrum in the confinement zone and near the plasma axis, and how its behavior there correlates with the transport properties. Indeed, on the basis of an assumed magnetic spectrum, a possible interpretation of particle and energy losses in TEXT has recently been advanced by one of us.³ There is evidently need for a new, nonintrusive, diagnostic technique to measure magnetic turbulence in the hot core of these plasmas. We suggest that Thomson scattering affords such a technique.

Other approaches have been put forward. Thomas *et al.*⁴ propose crossed sightline correlation of electron cyclotron emission, while Lehner, Rax, and Zou⁵ discuss the use of linear-mode conversion using a pump wave. How-

ever, many years ago, Thompson⁶ showed that scattered radiation could be used to observe magnetic activity. Assuming a uniform plasma and magnetic field, he calculated the change in polarization due to magnetic fluctuations at an incident wave frequency large compared to the electron gyrotron frequency ω_{ce} . Following Thompson, we also start from the electron fluid equations, but make approximations consistent with an application specifically to a tokamak plasma.

Because they are typical of many small tokamaks, the parameters of TEXT ($a/R = 0.27 \text{ m}/1.0 \text{ m}$) are chosen as the basis for our discussion.⁷ TEXT densities are in the neighborhood of 10^{19} – 10^{20} m^{-3} , and the electron temperature can be as high as 1 keV, corresponding to a collision frequency of $3 \times 10^4 \text{ s}^{-1}$. The toroidal field on axis $B_z(0)$ is between 1.5 and 2.8 T, and plasma current lies in the range 150–400 kA. TEXT density fluctuations have wave numbers between 500 and 1500 m^{-1} , and amplitudes as great as a few percent of the mean density. These wave numbers are consistent with $k_{\perp} \rho_i < 0.3$, ρ_i being the ion Larmor radius. Thus a largest poloidal mode number about $m = 70$ corresponds to a turbulence wavelength $\lambda_{\text{turb}} \gtrsim 0.02 \text{ m}$. Guided by theoretical arguments³ and in the absence of any direct experimental evidence, we will assume that broadband magnetic turbulence has the same characteristic wavelengths as these density fluctuations.

Assuming the wavelength λ of the probe radiation to be about $1 \times 10^{-3} \text{ m}$, the following ordering prevails:

$$\lambda \ll \lambda_{\text{turb}} \ll a \quad \text{and} \quad v \gg v_{\text{turb}} \gtrsim v_{\text{coll}}$$

where a is the minor radius. When comparing sizes of various terms we shall take $\lambda/\lambda_{\text{turb}} \sim 10^{-1}$ and $\lambda/a \sim 10^{-2}$.

^{a)}The abstract for this paper appears in the *Proceedings of the 8th Topical Conference on High Temperature Plasma Diagnostics* in Part II, Rev. Sci. Instrum. 61, 3072 (1990).

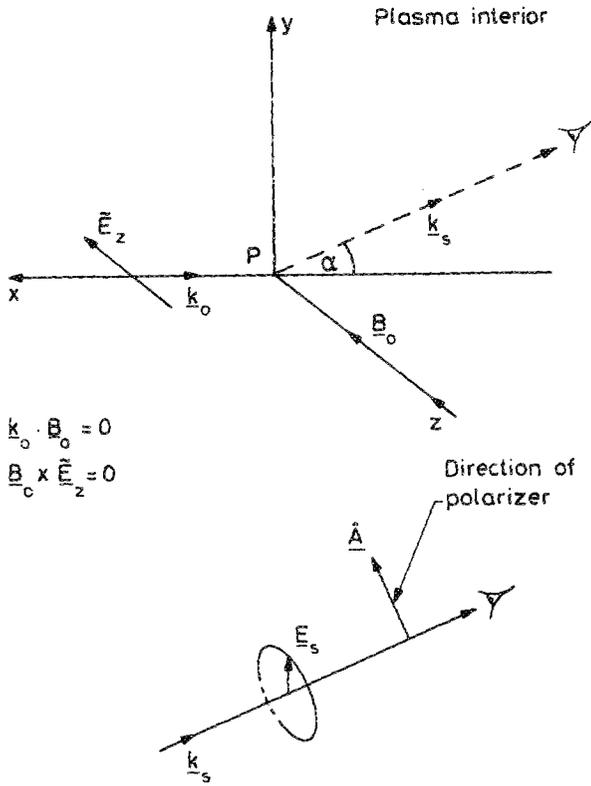


FIG. 1. Scattering geometry. Probe radiation, wave number k_0 , electric vector \vec{E}_z , is incident along the $-x$ direction. Scattering takes place at the origin, where the mean magnetic field is $B_0 \hat{z}$. Scattered radiation, wave number k_s , is observed in the xy plane, at scattering angle α , through polarizer A .

II. POLARIZATION OF THE PLASMA BY THE INCIDENT RADIATION

We begin by constructing the bulk polarization P of the plasma, in the presence of a fluctuating magnetic field, due to the probe radiation. Physical quantities of interest, such as density n_e , are composed of a mean, n_{e0} , a part due to the turbulence, Δn_e , and an oscillation, \tilde{n}_e , at the probe frequency. Thus

$$n_e = n_{e0} + \Delta n_e + \tilde{n}_e \quad (1)$$

where $n_{e0} \gg \Delta n_e \gg \tilde{n}_e$. Experiments indicate that $\Delta n_e / n_{e0} \sim 10^{-2}$ in the confinement zone. In the absence of comparable measurements of fluctuations in either electron temperature or electron fluid velocity, we will assume $\Delta u_e / u_{e0} \sim \Delta T_e / T_{e0} \sim 10^{-2}$ as well. Ions are ignored throughout.

When the continuity equation for electrons is expanded in terms of the perturbations in n_e and u_e , and the relative magnitudes of the various terms calculated, we find that, to leading order,

$$\tilde{n}_e / n_{e0} \sim \tilde{u}_e / c. \quad (2)$$

Using this together with the same ordering, the bulk polarization becomes

$$\mathbf{P} = -\frac{en_e \mathbf{u}_e}{i\omega} = -\frac{e(n_{e0} + \Delta n_e) \tilde{\mathbf{u}}_e}{i\omega}. \quad (3)$$

To evaluate $\tilde{\mathbf{u}}_e$ we turn to the electron momentum equation

$$m_e n_e \frac{d}{dt} \mathbf{u}_e = -\nabla p_e - en_e (\mathbf{E} + \mathbf{u}_e \times \mathbf{B}) - m_e n_e \nu_{\text{coll}} \mathbf{u}_e, \quad (4)$$

in which the symbols have their usual meanings. In the unperturbed state (mean plus turbulence) collisions play an important role. At the probe beam frequency, linearizing about this state leads to an equation in which the collision term is insignificant. Only the time part of the convective derivative on the left-hand side survives the ordering analysis, while on the right-hand side the pressure term is dropped, being about three orders of magnitude smaller than the inertial term and other terms retained. Probe beam frequency quantities are taken to vary as $\exp(i\omega t)$, so that $\partial \tilde{\mathbf{u}}_e / \partial t = i\omega \tilde{\mathbf{u}}_e$. The resulting equation for electron momentum is

$$i\omega m_e \tilde{\mathbf{u}}_e = -e[\tilde{\mathbf{E}} + \tilde{\mathbf{u}}_e \times (\mathbf{B}_0 + \Delta \mathbf{B})]. \quad (5)$$

This is solved for $\tilde{\mathbf{u}}_e$ and the result substituted into Eq. (3) to obtain an expression for the bulk polarization:

$$\mathbf{P} = \frac{en_e \omega (m_e / e) \tilde{\mathbf{E}} - (e / m_e \omega) \mathbf{B} \cdot \tilde{\mathbf{E}} \mathbf{B} - i \mathbf{B} \times \tilde{\mathbf{E}}}{\omega (B^2 - \omega^2 m_e^2 / e^2)}. \quad (6)$$

Here we have written n_e for $n_{e0} + \Delta n_e$ and \mathbf{B} for $\mathbf{B}_0 + \Delta \mathbf{B}$.

III. MEASUREMENT OF MAGNETIC FLUCTUATIONS

Next we consider how Eq. (6) can be used to infer information about the plasma magnetic turbulence. A set of orthogonal coordinates x, y, z is defined with its origin at the point at which scattering occurs (see Fig. 1). The z axis lies along the mean magnetic field \mathbf{B}_0 at the point and the x axis along a vertical minor diameter of the toroidal plasma, the magnetic surfaces being assumed circular and essentially concentric. The plane polarized probe beam is incident along the negative x direction with its electric vector in the z direction parallel to \mathbf{B}_0 . A detector is arranged to observe radiation scattered in the xy plane at an angle α to the incident beam direction.

Let $\hat{\mathbf{k}}_s$ be a unit vector in the direction from the scattering point to the detector. Thus $\hat{\mathbf{k}}_s = -\hat{\mathbf{x}} \cos \alpha + \hat{\mathbf{y}} \sin \alpha$. Then the scattered electric field at the detector is proportional to $\hat{\mathbf{k}}_s \times (\hat{\mathbf{k}}_s \times \mathbf{P})$.

Bulk polarization \mathbf{P} is divided into mean and turbulent fluctuating parts: $\mathbf{P} = \mathbf{P}_0 + \Delta \mathbf{P}$. By differentiating Eq. (6), we find the former is

$$\begin{aligned} \mathbf{P}_0 &= \frac{\epsilon_0 \omega_{pe}^2}{(\omega_{ce}^2 - \omega^2)} \left[\tilde{\mathbf{E}} - \left(\frac{e}{m_e \omega} \right)^2 B_0^2 \tilde{\mathbf{E}} \right] \\ &= -\epsilon_0 (\omega_{pe}^2 / \omega^2) \tilde{\mathbf{E}}, \end{aligned} \quad (7)$$

and the latter has components

$$\Delta P_x = P_0 \gamma [ib_y + (\omega_{ce} / \omega) b_x], \quad (8)$$

$$\Delta P_y = -P_0 \gamma [ib_x - (\omega_{ce} / \omega) b_y], \quad (9)$$

$$\Delta P_z = P_0 \{ (\Delta n_e/n_{e0}) - 2\gamma(w_{ce}/w)(b-b_z) \}, \quad (10)$$

where

$$P_0 = -\epsilon_0(w_{pe}^2/w^2)\bar{E}, \quad \gamma = (w_{ce}/w)(w_{ce}^2/w^2 - 1)^{-1},$$

$$b_x = \Delta B_x/B_0, \quad b_y = \Delta B_y/B_0, \quad b_z = \Delta B_z/B_0,$$

and

$$b^2 = b_x^2 + b_y^2 + b_z^2.$$

Note that density fluctuations contribute only to ΔP_z .

The mean polarization P_0 gives rise to the well-known incoherent Thomson scattering from unperturbed plasma. This will be trivially small compared to the coherent scattering from turbulence originating in the fluctuation parts of the polarization. It will accordingly be omitted from further consideration.

So from the foregoing,

$$\begin{aligned} \hat{\mathbf{k}}_s \times (\hat{\mathbf{k}}_s \times \mathbf{P}) &= \hat{\mathbf{x}} (-\Delta P_x \sin^2 \alpha - \Delta P_y \sin \alpha \cos \alpha) \\ &\quad + \hat{\mathbf{y}} (-\Delta P_x \sin \alpha \cos \alpha - \Delta P_y \cos^2 \alpha) \\ &\quad + \hat{\mathbf{z}} (-\Delta P_z). \end{aligned} \quad (11)$$

It will be seen that the z component of the fluctuating $\Delta \mathbf{P}$ contributes exclusively to the z component of the scattered electric vector.

Therefore, to discriminate against density fluctuations, the scattered light is passed through a linear polarizer rotated so as to exclude electric field components in the z direction. Under these circumstances, contributions from density fluctuations are suppressed and magnetic fluctuations alone determine the observed scattered radiation.

Let the polarizer be represented by a unit vector \mathbf{A} perpendicular to $\hat{\mathbf{k}}_s$ and capable of being rotated about $\hat{\mathbf{k}}_s$ as an axis, e.g.,

$$\mathbf{A} = \hat{\mathbf{x}} \sin \alpha \cos \theta + \hat{\mathbf{y}} \cos \alpha \cos \theta + \hat{\mathbf{z}} \sin \theta.$$

Applying this to the scattered electric field vector Eq. (11), one finds the scattered electric field seen by the detector through the polarizer is proportional to

$$\mathbf{A} \cdot [\hat{\mathbf{k}}_s \times (\hat{\mathbf{k}}_s \times \mathbf{P})] = \Delta P_x \sin \alpha \cos \theta + \Delta P_y \cos \alpha \cos \theta + \Delta P_z \sin \theta. \quad (12)$$

Substituting for ΔP_x , ΔP_y , and ΔP_z from Eqs. (8)–(10), and assuming the angle α is small, then

$$\begin{aligned} \mathbf{A} \cdot [\hat{\mathbf{k}}_s \times (\hat{\mathbf{k}}_s \times \mathbf{P})] &= P_0 \gamma (w_{ce}/w) [b_y \cos \theta - 2(b-b_z) \sin \theta] \\ &\quad + P_0 (\Delta n_e/n_{e0}) \sin \theta - iP_0 \gamma (b_x \cos \theta). \end{aligned}$$

The scattered intensity at the detector is proportional to the Laplace transform of the time autocorrelation of this electric field as described in, for example, Panofsky and Phillips.⁸ Using Poynting's theorem and multiplying by the area of the detector to produce a solid angle gives the following approximate expression for the scattered intensity at small scattering angle:

$$\begin{aligned} I &= r_e^2 I_0 L \Omega n_e^2 \{ [\gamma (w_{ce}/w) b_y \cos \theta \\ &\quad + (\Delta n_e/n_{e0}) \sin \theta]^2 + \gamma^2 b_x^2 \cos^2 \theta \}, \end{aligned} \quad (13)$$

where I_0 is in units of watts, I in W Hz^{-1} , r_e is the classical electron radius, and L is the length over which scattered light is collected into solid angle Ω . Note that the quantity in curly braces has the dimensions $\text{m}^3 \text{Hz}^{-1}$, since it is the Fourier transform of the enclosed expression, and not the square of the Fourier transform that is involved.

When the polarizer is aligned with the z axis, $\theta = 90^\circ$ and only the density fluctuations are observed; when the polarizer is rotated until $\theta = 0^\circ$, no z component is admitted to the detector and the scattered intensity is due entirely to field fluctuations.

If the probe frequency w equals the gyrofrequency w_{ce} , this expression diverges by virtue of the denominator in γ . Retaining temperature fluctuations or collisions formally prevents divergence, but still allows the scattered intensity to reach an unrealistic level. However, by choosing a sufficiently high probe beam frequency, the resonant denominator can be approximated to unity. But the choice of w is limited by the turbulence wavelength and the need to keep the scattering angle large enough to retain spatial resolution.

Suppose, following practice on TEXT, the probe wavelength is 1.22 mm, corresponding to a scattering angle of about 10° , and suppose the toroidal magnetic field $B_0 = 2$ T. Then $w = 1.55 \times 10^{12} \text{ rad s}^{-1}$ while $w_{ce} = 3.5 \times 10^{11} \text{ rad s}^{-1}$. Assuming also that $\Omega \sim 10^{-3} \text{ Sr}$, $n_e \sim 10^{19} \text{ m}^{-3}$, $L = 0.01 \text{ m}$, and $\Delta B/B_0 \sim 10^{-4}$,² then $I/I_0 \sim 5 \times 10^{-6} \text{ Hz}^{-1}$. That is to say, if an incident power of 1 W were focused onto 1 cm^2 , the scattered intensity would be $5 \mu\text{W}$ per Hz bandwidth.

More meaningfully, we compare the strengths of the field and the density scattering terms. Their ratio is about

$$(w_{ce}/w)^2 (\Delta B_x/B_0)^2 / (\Delta n_e/n_{e0})^2.$$

Near the edge of TEXT, $\Delta B/B_0 \sim 10^{-4}$,² rising towards the middle, whereas $\Delta n_e/n_{e0} \sim 10^{-2}$ near 0.16 m radius,⁹ falling towards the middle. Thus, towards the plasma edge, the ratio is about 10^{-5} , and may be expected to rise towards the center. For example, if $\Delta B/B_0$ increased and $\Delta n_e/n_{e0}$ decreased each by an order of magnitude, then the ratio would increase to about 0.1.

Plasma radiation imposes no limitation to the density fluctuation scattering measurements already performed in TEXT.⁷ It may jeopardize the very much smaller field fluctuation scattering anticipated near the plasma edge, but according to the foregoing argument, we may anticipate its threat will diminish towards the center.

IV. INFLUENCE OF FARADAY ROTATION

The Faraday effect can undermine the field measurement by rotating the polarization of either the probe or the scattered radiation. It is to guard against the first that the incident beam is introduced along a minor diameter, which is assumed to be everywhere perpendicular to the mean

field, at least for circular concentric flux surfaces. A vertical diameter has been chosen here so that the toroidal field component along it will be constant, but an alternative and perhaps better choice, engineering constraints permitting, might be a horizontal diameter. The scattering volume is located on this diameter beyond the minor axis of the torus, thereby minimizing the path length of the scattered radiation through plasma. Because of the nonzero scattering angle the path of the scattered radiation cannot be perpendicular to the magnetic field, so some Faraday rotation is inevitable. This leads to a small angular offset θ_F in the orientation to which the polarizer has to be set to get zero transmission of the density fluctuation component. It is easy to see that

$$\sin \theta_F \ll (\omega_{ce}/\omega) (\Delta B_x/B_0) / (\Delta n_e/n_{e0}),$$

that is, about 0.33×10^{-2} or $\theta_F \ll 0.2^\circ$ for the plasma far from the center, just considered. Near the center, if both field and density fluctuations become about 10^{-3} , $\sin \theta_F \ll 0.33$ or $\theta_F \ll 20^\circ$.

We have computed the Faraday rotation that might be experienced by the scattered radiation as it travels from the scattering point, a distance x_0 along the vertical diameter above the magnetic axis, through the plasma in the direction \mathbf{k}_s to the plasma edge. The incident frequency is assumed to be sufficiently large with respect to both ω_{pe} and ω_{ce} to justify the use of¹⁰

$$\theta^{\circ} = 1.5 \times 10^{-11} \lambda^2 \int n_e(s) \hat{\mathbf{k}}_s \cdot \mathbf{B} ds \quad (14)$$

to calculate the rotation induced in the plane of polarization. We have taken into account that the plane of scattering, defined by the vectors \mathbf{k}_0 and \mathbf{k}_s , is rotated with respect to the poloidal plane, since the z axis of the former must be parallel to the mean field direction at the scattering point, and particle and current density profiles have been assumed parabolic.

The x_0/a dependence of the rotation integral in Eq. (14) is found to have the form displayed in Fig. 2. Thus for $n_{e0} \sim 10^{19} \text{ m}^{-3}$, $\alpha = 0.1 \text{ rad}$, $a = 0.27 \text{ m}$, and $I_0 = 200 \text{ kA}$, we find $\theta^{\circ} < 2.5 \times 10^{-2}$, approximately one order of magnitude smaller than the value required to corrupt the scattered signal. Further reduction in θ could be effected by reducing the probe beam wavelength, but this would have the undesirable consequence of diminishing the already small spatial resolution.

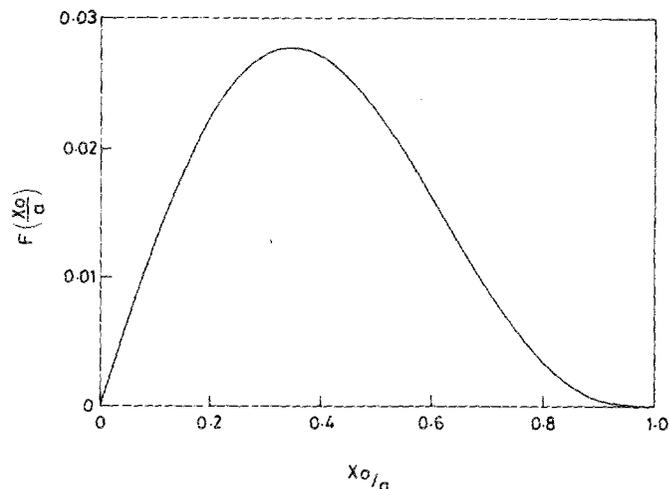


FIG. 2. Dependence of the Faraday rotation experienced by the scattered radiation as a function of the location, x_0/a , of the scattering point on the minor radius. The ordinate is defined in the text.

V. CONCLUSIONS

Magnetic field fluctuations in a direction normal to the mean field can be measured in a tokamak plasma by a Thomson scattering technique. By a judicious selection of the scattering geometry and the polarization of the incident and scattered radiation, the contribution to the scattered intensity from density fluctuations may be suppressed. As a result, the measured scattered intensity will be due to magnetic fluctuations alone.

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