Full Wave Simulation of RF Waves in Cold Plasma with the Stabilized Open-Source Finite Element Tool ERMES

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Abstract. When RF waves are applied in tokamaks with metal walls, sheath rectification effects associated with the fields induced in the scrape-off layer (SOL) may lead to enhanced plasma-wall interactions (i.e. heat-loads in the limiters, RF-induced impurity sources) which can endanger the integrity of the machine and limit the RF power. Currently, some numerical tools are being used to simulate the RF antenna near fields in the presence of magnetized plasmas, but they have their limitations. Some neglect completely the interaction of the RF waves with the low-density plasma close to the antenna. Other take into account these interactions but, due to numerical instabilities associated with its finite element formulation, they fail to find a stable solution close to the Lower Hybrid Resonance (LHR). Simplifications to reach convergence had also been tried (e.g. neglect gyrotropy, increase electron density to avoid LHR), but the fields obtained can be very different to the real ones and this difference can affect the accuracy of derived magnitudes which use these fields as an input, as the sheath rectification effects. In this work we try to overcome all the limitations mentioned above by customizing the open-source finite element code ERMES. This code implements a finite element formulation which allows to simulate the near fields of the RF antenna in a continuous gyrotropic non-homogeneous media without limits in the minimum value of the plasma density and provides stable solutions even in the presence of the LHR. Benchmarking of this approach is underway and comparison against measurements, semi-analytical approaches and other codes will be presented.

INTRODUCTION

Radio-Frequency (RF) electromagnetic waves are commonly employed as an auxiliary heating system in tokamak nuclear fusion devices. The RF waves are sent to the plasma core from antennas situated on the tokamak walls and, on its way to the core, they pass through the Scrape-Off Layer (SOL), a low-density plasma region situated between the antenna and the last closed magnetic surface. Unfortunately, in the SOL of a tokamak with metal walls the RF waves can induce sheath rectification effects which intensify the plasma-wall interactions. These enhanced interactions can heat the limiters to the point of melting and increase the contamination (and consequent cooling) of the plasma core with impurities coming from the increased heavy ion wall sputtering. Therefore, the RF waves in the SOL can endanger the integrity of the machine and limit the maximum RF heating power that can be safely sent to the plasma. In particular, it is especially important for the forthcoming DT experiments at the Joint European Torus (JET) where the avoidance of impurities in the plasma will be of key importance.

Although many codes are available to describe the wave-particle physics in the plasma core, the modelling of the RF wave interactions with the low-density plasma in the SOL is much less explored since the RF physics describing the involved mechanisms is not yet fully understood and the solution of the problem is numerically demanding due to the excitation of waves with wavelengths of just a few millimetres and the close interaction of these waves with the complex antenna and wall geometries. Moreover, the presence of resonances challenges the numerical formulations to its limits.

Currently, some numerical tools are being used to simulate the RF antenna near fields in the presence of magnetized plasmas, but they have their limitations. For instance, the well-known in-house code TOPICA [1], which is typically used to couple realistic antenna geometries with the hot plasma inside the reactor, needs a vacuum buffer area of separation between the antenna and the plasma. Therefore, it neglects all the physical phenomena related to the interaction of the RF waves with the low-density plasma close to the antenna surface. Other software such as the commercial packages CST, HFSS and COMSOL [2] or in-house codes such as MFEM [3], have been customized to take into account the close interaction of the near-fields with the low-density plasma. Unfortunately, they fail to find a stable solution around the Lower Hybrid Resonance (LHR) due to numerical instabilities associated with the finite element method (FEM) formulation implemented inside them. Simplifications can be used to reach convergence (neglect gyrotropy, increase electron density to avoid the LHR), but the fields obtained can be very different to the real ones (even if the input impedance of the antennas are similar to the ones measured) and this difference can affect the accuracy of derived magnitudes, as the sheath rectification effects, which use these fields as an input.

In this work we try to overcome all the limitations mentioned above by customizing the open-source finite element code ERMES [4]. We adapted ERMES for reading measured plasma density profiles and magnetic fields from files, incorporate these measurements in a realistic three-dimensional CAD representation of the RF antennas and tokamak walls and calculate the near-fields and other relevant magnitudes in the presence of a cold magnetized plasma. ERMES implements a stabilized finite element formulation [5] which allows to simulate the near fields of the antenna in a continuous gyrotropic non-homogeneous media without limiting the minimum value of the plasma density. As it is shown in [6] and at the end of this paper, ERMES obtains stable solutions near the LHR (although the work to demonstrate that this stable solution is the physical one is still on going). In the following, we show why the standard FEM approach fails close to the LHR and how ERMES stabilizes the numerical solution.

MODELLING RF FIELDS IN THE SOL WITH FEM

The most widespread finite element formulation for solving the Maxwell's equations in frequency domain is the so-called double-curl formulation [7]. This approach discretizes with edge elements the weak form of

$$\nabla \times \left(\frac{1}{\mu} \nabla \times \mathbf{E} \right) - \omega^2 \varepsilon \, \mathbf{E} = i \omega \mathbf{J}, \tag{1}$$

where $\mathbf{E}$ is the electric field, μ is the magnetic permeability, ε is the electric permittivity, ω is the angular frequency, i is the imaginary unit and $\mathbf{J}$ is a divergence-free current source. The double-curl formulation is the one used by commercial codes as CST, HFSS and COMSOL and by the in-house code MFEM mentioned in the introduction. In the SOL, the permittivity ε takes the form of the cold plasma permittivity tensor given in [8]:

$$\varepsilon = \epsilon_0 \begin{bmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{bmatrix}, \tag{2}$$

where ϵ_0 is the electric permittivity of vacuum and the parameters S, D and P are functions of the position, electron density, magnetic field, the species composing the plasma and the frequency ω. Close to the LHR, we have that

$$\nabla \sim i\mathbf{k} \rightarrow \infty \ , \ S \rightarrow 0 \ , \tag{3}$$

where $\mathbf{k}$ is the wave vector of the incident field. As a consequence of Equation 3, the operators in Equation 1 satisfy

$$\nabla \times \nabla \times \mathbf{E} \gg \omega^2 \varepsilon \mu \, \mathbf{E}, \tag{4}$$

which means that, close to the LHR resonance, we are effectively solving:

$$\nabla \times \nabla \times \mathbf{E} \approx 0. \tag{5}$$

This equation is ill-posed and produces ill-conditioned matrices which are difficult to solve. Equation 5 has an infinity set of possible solutions, indeed, any field coming from a gradient $\mathbf{E} = -\nabla\phi$ is a solution. In practical terms, this translates in the presence of spurious solutions that pollute the calculated fields at the vicinity of the LHR, as can be observed in the COMSOL simulations shown in [6] and in the LAPD [9] simulations of the code [3] shown in [10, 11]. We can try to overcome this difficulty by increasing the plasma density above the resonance, add some artificial damping mechanism (e.g. add an imaginary frequency) or even try to emulate the plasma behaviour with metamaterials (e.g. a multilayer set of anisotropic materials). But, even though, we can obtain reflection coefficients similar to the measurements, fields calculated in such a ways can be completely different to the physical ones, and this will affect derived magnitudes that use these fields as an input (e.g. induced rectification voltages on plasma sheaths).

ERMES

Instead of the double-curl formulation shown above, ERMES implements the so-called regularized formulation [5, 4]. This approach discretizes with nodal (Lagrangian) elements the weak form of

$$\nabla \times \left(\frac{1}{\mu}\nabla \times \mathbf{E}\right) - \bar{\varepsilon}\,\nabla\left(\frac{1}{\|\bar{\varepsilon}\varepsilon\|\,\mu}\nabla \cdot (\varepsilon\mathbf{E})\right) - \omega^2\varepsilon\mathbf{E} = i\omega\mathbf{J}, \tag{6}$$

where, for the cold plasma case, $\bar{\varepsilon}$ is the complex conjugate of the permittivity tensor given in Equation 2 and $\|\bar{\varepsilon}\varepsilon\|$ is the Frabenious norm of the matrix product $\bar{\varepsilon}\varepsilon$. Close to the LHR, now we have that

$$\nabla \times \nabla \times \mathbf{E} - \bar{\varepsilon}\,\nabla\left(\frac{1}{\|\bar{\varepsilon}\varepsilon\|\,\mu}\nabla \cdot (\varepsilon\mathbf{E})\right) \gg \omega^2\varepsilon\mu\,\mathbf{E}, \tag{7}$$

which means that, close to the LHR resonance, we are effectively solving:

$$\nabla \times \left(\frac{1}{\mu}\nabla \times \mathbf{E}\right) - \bar{\varepsilon}\,\nabla\left(\frac{1}{\|\bar{\varepsilon}\varepsilon\|\,\mu}\nabla \cdot (\varepsilon\mathbf{E})\right) \approx 0. \tag{8}$$

This operator has a null-kernel and it only admits one solution, which eliminates the spurious solutions around the LHR. We will show this with two examples in the next section.

VALIDATION EXAMPLES

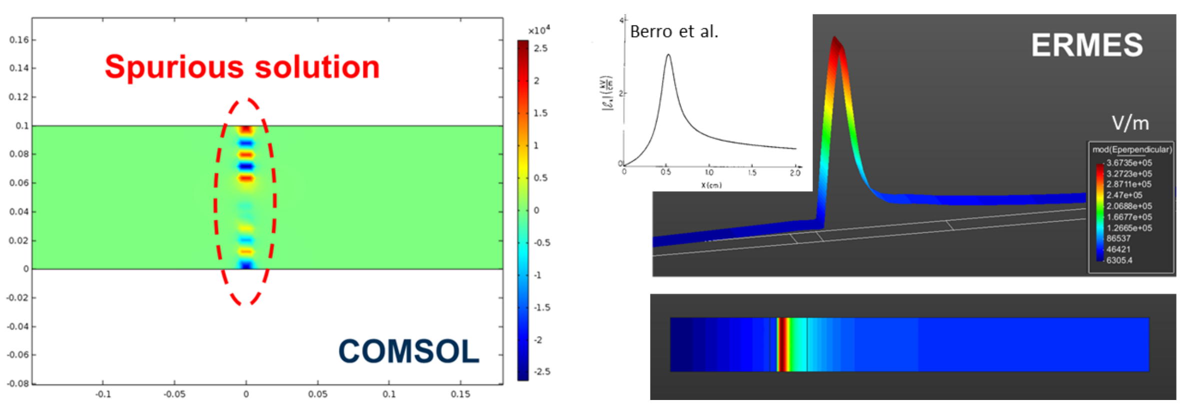

FIGURE 1. COMSOL and ERMES simulations of the problem described in [12].

In Figure 1 is shown the solution of the problem described in [12]. It consists on a simple RF antenna in a continuous plasma density variation that presents a LHR in front of the strap. Figure 1 shows the spurious solution provided by the double-curl formulation implemented in COMSOL and the spurious-free solution calculated with ERMES, which also match quite well with the field calculated in [12].

The Large Plasma Device (LAPD) at UCLA [9] is an experimental facility that allows the direct measure of the electric field created by a RF antenna in a magnetized cold plasma. Figure 2 shows ERMES simulations of the LAPD and measurements results. The fields compare well and the spurious solution around the LHR are avoided, contrary to the double-curl code simulations showed in [10, 11], with spurious solutions clearly visibly around the S=0 layer.

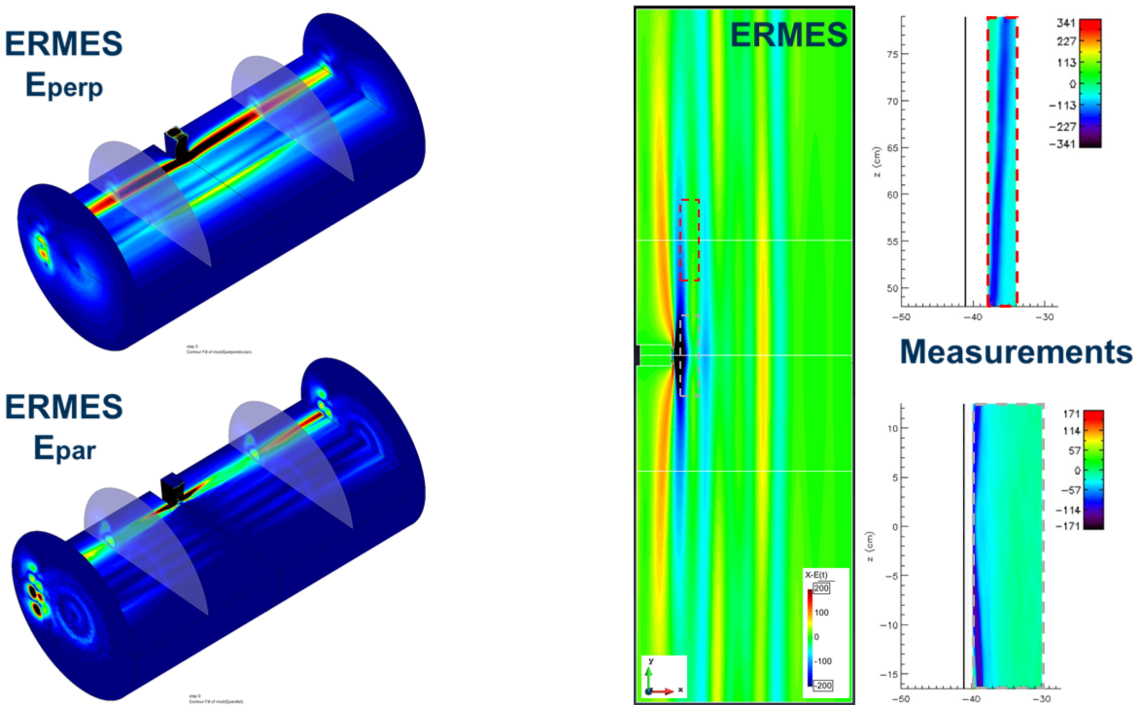

FIGURE 2. ERMES simulations and experimental measurements of the LAPD [9].

SUMMARY

We have presented a customization of the open-source code ERMES for RF-cold plasma interaction modelling which implements a finite element formulation that avoids the numerical instabilities on the lower hybrid resonance associated with the standard double-curl formulation. Work is on-going to further validate the approach and more stabilized formulations are being investigated.

ACKNOWLEDGMENTS

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training programme 2014-2018 under grant agreement No 633053. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

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