UKAEA-CCFE-CP(19)51

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This paper has been submitted to
46th European Physical Society Conference on Plasma Physics (EPS), Milan, 8-12 July 2019

Symmetry Breaking in Tokamak Plasmas

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Introduction

H-mode tokamak plasmas are characterised by the occurrence of quasi-periodic edge localised modes (ELMs) that lead to large particle and heat fluxes at the divertor region of the tokamak [1]. If extrapolated to large scale tokamaks, like ITER, the corresponding loads will exceed the material melting point [2], so active control of ELMs is needed. It has been experimentally demonstrated that the use of non-axisymmetric magnetic perturbations (MPs) allow active control of ELMs, where either ELM mitigation or complete ELM suppression is observed [3],[4]. However, reliable access to a suppressed state only occurs in a narrow regime of the parameter space and complete physics understanding of the cause or access to a such plasma state is still an area of active research. Although, several physics aspects could contribute to establish ELM suppression, this work focuses on the impact of the established non-axisymmetric magnetic equilibrium to the peeling-ballooning (PB) instability of the plasma edge. The PB instability is an ideal MHD instability linked with the onset of large type-I ELMs [5],[6].

Variational 3D MHD Stability

The stability of ideal PB MHD modes can be efficiently studied through the energy principle. In particular, if a displacement $\boldsymbol{\xi}$ exists such that the potential energy change δW around an equilibrium state is negative, then this equilibrium configuration is unstable. For a tokamak plasma the potential energy can be written as,

$$\delta W = \frac{1}{2\mu_0}\oint[\frac{1}{|\nabla\psi|^2}|\boldsymbol{B}\cdot\boldsymbol{\nabla}X|^2 + \frac{|\nabla\psi|^2}{B^2}|\boldsymbol{B}\cdot\boldsymbol{\nabla}U|^2$$
$$+(S-\mu_0\frac{J_\parallel}{B^2})(2\mathcal{R}e\{\boldsymbol{B}\cdot\boldsymbol{\nabla}UX^\dagger\}+S|X|^2)$$
$$-2\mu_0(\boldsymbol{\nabla}p\cdot\boldsymbol{\nabla}\psi)(\boldsymbol{\kappa}\cdot\boldsymbol{\nabla}\psi)|X|^2\,]\,\mathcal{J}\,\mathrm{d}\psi\mathrm{d}\theta\mathrm{d}\varphi + \delta W_V$$

where $\mathcal{J}$ is the jacobian of the coordinate system, $S=(\boldsymbol{n}\times\boldsymbol{b})\cdot\boldsymbol{\nabla}\times(\boldsymbol{n}\times\boldsymbol{b})$ is the local shear, $\boldsymbol{n}$ and $\boldsymbol{b}$ are the normal and parallel vectors, and δW_V is the vacuum magnetic energy. The variables (X,U) are related to the normal and binormal components of the plasma displacement. The δW derived here assumes that the compressional pressure and Alfven waves that can occur are minimized due to their either wise highly stabilizing contribution. The Hermitian nature of δW, together with the fact that unstable ideal MHD modes form a discrete spectrum, allows the use of linear stability analysis to determine the most unstable eigen mode of the system.

The stability of the axisymmetric tokamak system is routinely studied using the energy principle. A significant simplification arises through local ballooning analysis, that analytically relates the normal and binormal components of the displacement. As such, the potential and kinetic energy change can be expressed as a function of X and further minimization leads to a generalized eigenvalue problem that is efficiently solved using Fourier harmonics as basis functions for the poloidal and toroidal direction,

$$X \equiv \sum X(\psi)_{nm} e^{i(m\theta - n\varphi)}$$

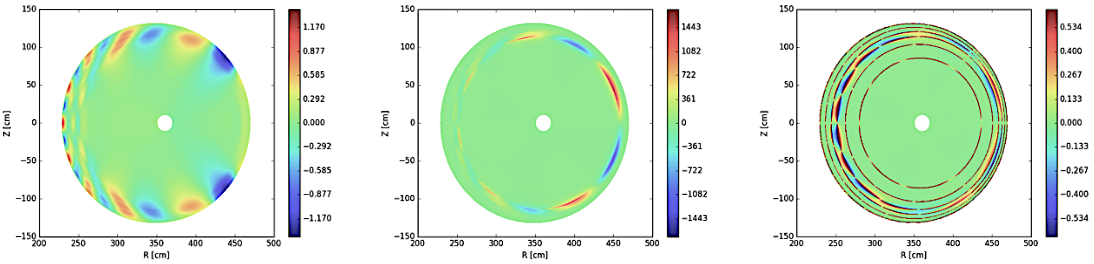
The application of basis functions allows the representation of the kinetic and potential energy change in a matrix form, where minimization leads to a generalized eigenvalue problem, such as

$$\sum_{m,n} \int < (\delta W_{nm} - \omega^2 \delta K_{nm}) | X_{nm} > \mathrm{d}\psi = 0$$

where ω is the eigenvalue (or mode frequency), *n* and *m* represent the toroidal and poloidal mode number respectively. In principle, the selection of "good" basis functions provides an asymptotic higher bound for the eigenvalue. Therefore, a framework is visualized and implemented where axisymmetric stability codes, in this case ELITE [6] which is routinely used to solve the axisymmetric version of δW and δK, can be used to provide trial functions for the full 3D system. The trial functions X_{nm} represent the axisymmetric eigenfunctions of the PB instability. This procedure requires the knowledge of the non-axisymmetric equilibrium, such that the 3D component of δW and δK can be computed. This information can be obtained once again by axisymmetric stability codes run at marginal stability and applying appropriate boundary conditions at the plasma-vacuum interface. In this work, a fixed boundary condition is used to represent the non-axisymmetric normal displacement of the last flux surface.

Application to MPs

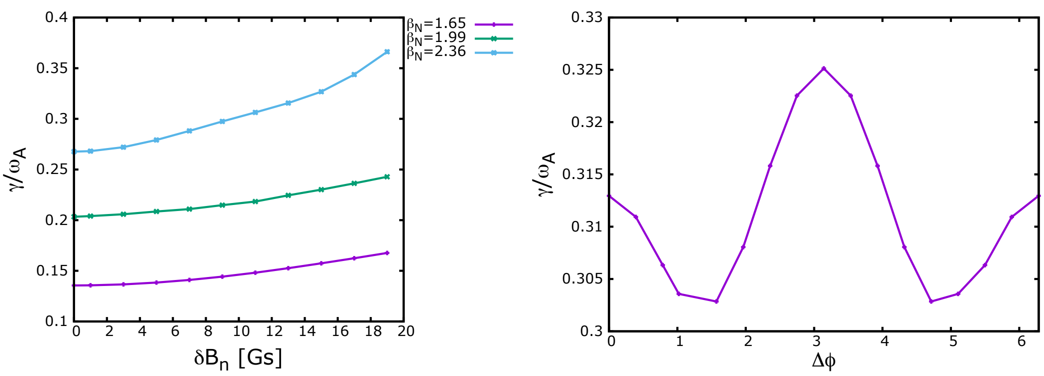
Due to the fact that ***δB***<<***B₀*** only second order coupling is retained leading to the triplet of modes {*n-N,n,n+N*}, where N is the toroidal mode number of the applied MP field. In principle an arbitrary number of toroidal harmonics can be included. The above method is applied to a circular large aspect ratio tokamak plasma that is unstable to ballooning modes. An example of the linear response in terms of the 3D normal magnetic field δB_n, pressure δp and parallel current density $\delta j_{\|}$ is given in Fig.[1].



A) B) C)

Figure.[1] The non-axisymmetric A) normal magnetic field δB_n [Gs], B) pressure p [Ba] and C) parallel current density $\delta j_{\|}$ [$Frs^{-1}cm^{-3}$] as obtained for an even $N = 3$ external resonant MP field of the $\beta_N = 2.36$ equilibrium.

A series of equilibria, where the pressure is increased such that $\beta_N = [1.65, 1.99, 2.35]$ and $q_a = [2.97, 3.01, 3.04]$, are used to examine the stability of PB modes under the application of an even $N = 3$ RMP field that is nearly resonant with the surface of the plasma. It is observed that symmetry braking leads to further destabilization of ballooning modes, as it can be observed from Fig.[2]. In addition, the increase in the axisymmetric equilibrium pressure, results in additional destabilization that is not attributed only to the larger plasma response. Moreover, as can be seen from Fig.[2] by varying the phase $\Delta\phi$ of the normal magnetic field δB_n, the change in growth rate is larger in the non-resonant ($\Delta\phi \sim \pi$) than the



A) B)

Figure.2 A) The normalized growth rate γ/ω_A of a $n = 15$ ballooning mode as a function of applied resonant $N = 3$ MP normal field δB_n for different values of normalized beta β_N. B) The normalized growth rate γ/ω_A as a function normal field phase $\Delta\phi$ for $\delta B_n = 10$ [Gs] of the $\beta_N = 2.36$ equilibrium case.

resonant ($\Delta\phi \sim 0$) case. Although, the plasma response is stronger in the resonant case it does not necessarily lead to further destabilization and potentially indicating the importance of the non-resonant harmonics of the 3D equilibrium in the variation of curvature and local shear. Finally, Fig.[3] illustrates a comparison between the axisymmetric and non-axisymmetric mode structure, where for large enough MP field the ballooning mode localizes at specific poloidal locations that coincide with locations of increased pressure.

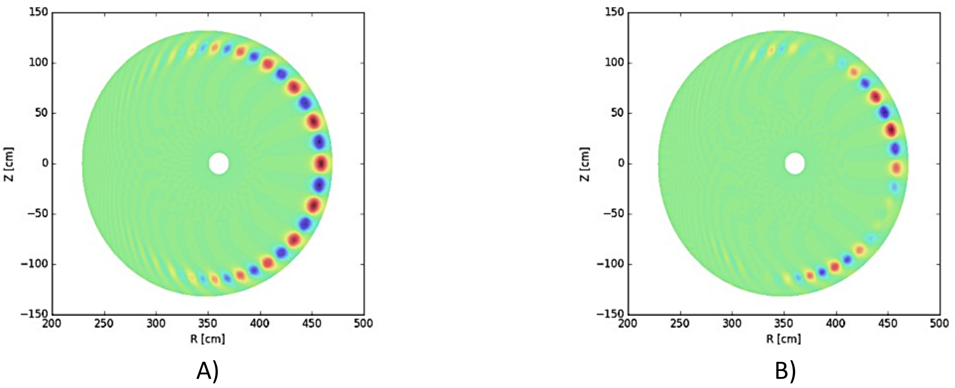


Figure.3 The mode structure of A) an axisymmetric $n = 15$ PB mode and the mode structure of B) a non-axisymmetric $n = 15$ PB mode under the influence of a $N = 3$ MP with $\Delta\phi = 0$, $\delta B_n = 10\ [Gs]$ of the $\beta_N = 2.36$ equilibrium case.

Discussion

A numerical framework around the ELITE stability code is presented for the calculation of 3D MHD stability of tokamak plasmas, based on the variational formulation of the energy principle. It is observed that the ballooning mode is in general destabilized by the application of external MP fields in accordance with local ballooning analysis. The non-axisymmetric equilibrium leads to toroidal as well as poloidal coupling such that retaining the individual poloidal harmonics of the toroidal basis functions becomes important. Finally, strong coupling is observed with increasing external field and additional toroidal harmonics will lead to more accurate results.

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Acknowledgements

This work has been carried out within the framework of the EUROfusion Consortium and has received funding from the Euratom research and training program 2014-2018 under grant agreement No 633053 and from the RCUK Energy Program [grant number EP/P012450/1]. To obtain further information on the data and models underlying this paper please contact PublicationsManager@ccfe.ac.uk*. The views and opinions expressed herein do not necessarily reflect those of the European Commission. In addition, this work is supported by the Fusion CDT program through EPSRC [grant number EP/L01663X/1].