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Introduction

The establishment of steep pressure gradient and large current density at the edge of magnetised thermonuclear plasmas are potentially destabilising for peeling-ballooning (PB) modes [1]. Those instabilities are manifested as edge localised modes (ELMs) and correspond to rapid bursts of particles and heat deposited at the wall of the reactor. Especially for large tokamaks like ITER, those transients will result in heat fluxes that current material technologies are unable to cope with [2]. Active ELM control methods are therefore required to minimise the potential damage of the reactor. One method that is widely applied in various devices and will be installed in ITER, uses external non-axisymmetric resonant magnetic perturbations (RMPs) produced from magnetic coils placed inside the tokamak vessel. Experimental observations indicate two main operational states, where ELM mitigation or complete suppression occurs. However, the exact physics mechanism that allows this ELM free regime is still debatable. This work focuses on understanding the impact of the 3D equilibrium on peeling-ballooning stability following a perturbative approach, in order to simulate the coupling of axisymmetric toroidal modes.

3D Perturbative Linear Equilibrium & Stability

In general, external 3D fields affect transport and MHD properties of the plasma. The change in equilibrium geometry can affect MHD instabilities leading to potential modification of stability boundaries that can directly affect the onset of ELMs. Ideal infinite-n ballooning analysis revealed that the modification of torsion, locally affects the instability [3]. However, for intermediate-n modes responsible for the occurrence of ELMs, a global 3D analysis for the edge is needed and has not yet been applied to an ELM control scenario. To some extend, such an investigation has been performed by non-linear fluid codes and mode coupling was observed to be the key mechanism to achieve a suppressed state [4][5].

The external field forces a new 3D equilibrium state characterised by large Pfirsch-Schluter current density around rational surfaces and a non-axisymmetric component that intrinsically results in geometrical mode coupling, with respect to the axisymmetric case. Mode coupling will allow energy transfer between neighbouring modes that can directly affect the evolution of instabilities. In order to examine this effect, perturbation theory is employed on ideal axisymmetric MHD modes as developed by C.C. Hegna [6]. In short, corrections to the linear axisymmetric growth rate ω_{n0} can be obtained considering a second order expansion for the force operator as given in Eqn.1.

$$\omega_n^2 = \omega_{n0}^2 + \sum_k \frac{|V_{nk}|^2}{\omega_{n0}^2 - \omega_{k0}^2} \tag{1}$$

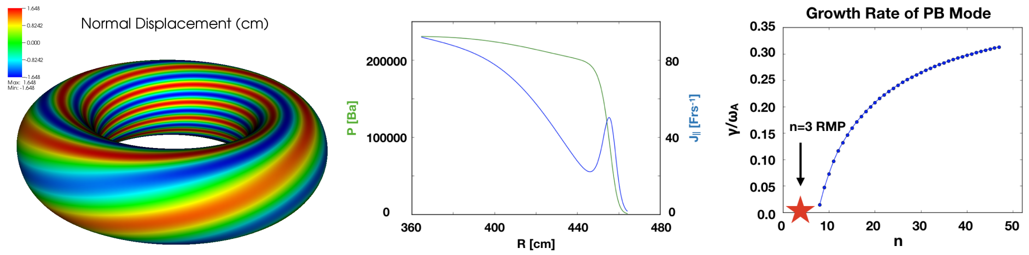
The coupling coefficients V_{nk} depend solely on the 3D equilibrium and axisymmetric modes that can be provided by tokamak stability codes. In such a way, existing axisymmetric stability codes could be used within a certain framework to simulate 3D plasma behaviour and parameter scans are possible allowing for deep understanding of the system.

ELITE[7]* is an axisymmetric stability code that can very efficiently simulate the linear ideal plasma response from low to high n toroidal modes. Therefore, PB instabilities are captured and the ideal nature of the plasma retains nested flux surfaces that are required for a perturbative stability analysis. The code solves the eigenvalue problem of the equation of motion normal to the plasma flux surfaces, for a displacement functional that minimises the axisymmetric energy principle. The 3D equilibrium part can be obtained for stable modes at marginal stability, when the variation of the kinetic energy is zero $\delta K \to 0$. The applied RMP field is introduced assuming a fixed boundary condition at the plasma-vacuum interface and using the induction equation the external field is related to the plasma displacement. As an example of such fixed boundary condition, Fig.1a illustrated the normal displacement as resulted from a resonant 3D field $n = 3$ even coil (current in up/down coils has no phase difference) configuration. The calculation of the 3D part of the equilibrium requires the knowledge of the linearised magnetic field $\vec{\delta B} = \nabla \times (\vec{\xi}_\perp \times \vec{B})$, current density $\mu_0 \vec{\delta J} = \nabla \times \vec{\delta B}$ and pressure $\delta P = -\vec{\xi}_\perp \cdot \nabla P$.

Application to External RMPs

The calculation of the non-axisymmetric part of the equilibrium requires an initial axisymmetric equilibrium that is stable for low-n toroidal modes, relevant for experimental RMPs fields. Such an equilibrium is the one used for this academic case and the plasma profiles and PB stability are illustrated in Fig.1b,1c. The external field is based on an even n=3 RMP perturbation. ELITE provides the normal displacement of the plasma and considering perturbations

* The low-n version is used as developed by Ref.[8]

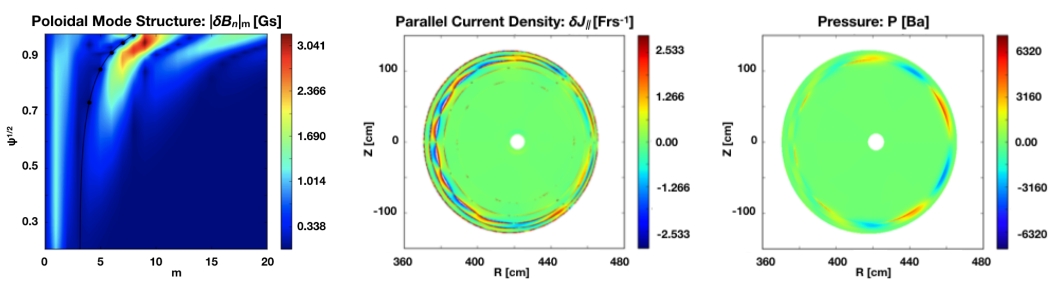


(a) (b) (c)

Figure 1: a) Fixed B.C. for the normal displacement $\vec{\xi} \cdot \hat{n}$ as inserted in ELITE for an n=3 even RMP configuration. Equilibrium radial plasma profiles for b) the plasma pressure and current density as well as c) the growth rate of the axisymmetric PB mode where the n=3 RMP is stable.

that minimise stabilising contributions the binormal component can be obtained accordingly [9]. As such the magnetic field can be fully reconstructed taking into account the plasma response and Fig.2a illustrates the field-aligned poloidal spectrum of the normal magnetic field. The ideal nature of the plasma leads to screening at rational surfaces preventing the formation of islands and leading to a saturated kink-like response. The normal displacement together with the full knowledge of the magnetic field allows the calculation of the current density and pressure gradient. As such all equilibrium quantities are known. The main feature of such helical equilibrium state is the formation of significant Pfirsch-Schluter current density in comparison to the axisymmetric equilibrium. Fig.2b and Fig.2c illustrates the 3D equilibrium current density and pressure.

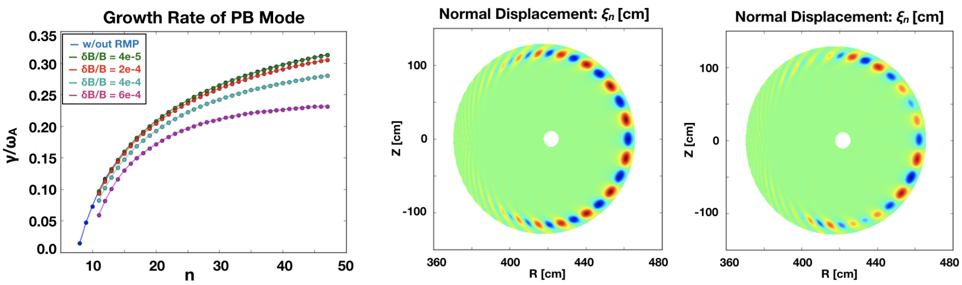
The coupling coefficients can be calculated using the above 3D equilibria quantities and axisymmetric toroidal PB eigenfunctions. It is observed that above a certain external field amplitude the impact of mode coupling is significant and that the dependence is nonlinear with respect to the applied field strength. Fig.3a shows the stabilising influence of the 3D nature



(a) (b) (c)

Figure 2: 3D equilibrium a) poloidal mode structure of normal magnetic field, b) parallel current density and c) pressure.

of the equilibrium geometry, due to stronger coupling with higher n toroidal modes, that have a stabilising influence to the new 3D mode. In addition, the reconstruction of the 3D normal displacement of the PB mode results in a localised mode structure with respect to the poloidal location, due to the interplay of different axisymmetric modes. Despite having an opposite effect on the growth of the mode, this characteristic is consistent with 3D infinite-n ballooning analysis and experimental observations [10]. A comparison between the mode structure the corresponding axisymmetric and non-axisymmetric mode is illustrated in Fig.3b and Fig.3c.



(a) (b) (c)

Figure 3: a) Perturbative 3D Peeling-Ballooning stability under the influence of external even n=3 RMP coil configuration as calculated using the modified ELITE. Normal plasma displacement $\vec{\xi} \cdot \hat{n}$ for a $n = 12$ b) axisymmetric and c) non-axisymmetric PB mode.

Discussion

The mode coupling is observed to have a significant impact on linear MHD growth rates. Effectively energy is transferred between modes and in such a way the growth rate is decreased leading to a saturated plasma state. This semi-analytical work shares qualitatively similar features with nonlinear HPC simulations which are far more computationally expensive and time consuming. Although, future work will focus on the validation of this simplified model with such codes. Last but not least, future work will also involve a free boundary 3D equilibrium calculation to capture qualitative differences between realistic equilibrium configurations.

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