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1 **Observation of quantum de-trapping and transport of heavy defects in tungsten**

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25

26 The diffusion of defects in crystalline materials¹ governs macroscopic behaviour in a
27 wide range of processes, including alloying, precipitation, phase transformation, and
28 creep². In real materials, intrinsic defects are unavoidably bound to static trapping
29 centres such as impurity atoms, meaning that their diffusion is controlled by the de-
30 trapping process. It is generally held that de-trapping occurs only by thermal
31 activation. In this Letter we report the first direct observation of the quantum de-
32 trapping of defects below around 1/3 of the Debye temperature. We successfully
33 monitored the de-trapping and migration of self-interstitial atom clusters, strongly
34 trapped by impurity atoms in tungsten, by triggering de-trapping out of equilibrium at
35 cryogenic temperatures, using high-energy electron irradiation and in-situ
36 transmission electron microscopy. The quantum-assisted de-trapping leads to low-
37 temperature diffusion rates orders of magnitude higher than a naive classical estimate
38 suggests. Our analysis shows that this phenomenon is generic to any crystalline
39 material.

40

41

42 Under high-energy irradiation (or extreme mechanical deformation), atoms in a
43 crystal can be displaced significantly from their lattice positions, forming vacancy and
44 self-interstitial atom (SIA) defects. These are ultimately responsible for severe
45 degradation in the mechanical properties of materials, such as hardening, swelling,
46 and embrittlement³. Understanding the basic mechanisms controlling their formation
47 and diffusion⁴⁻⁶ is critical for the development of future next-generation energy
48 systems.

49

In the field of material science, to the best of our knowledge, all observed migration processes of species heavier than H or He^{7,8} have been interpreted as thermal activation characterized by the Arrhenius rate⁹, or phonon dragging^{10,11}; no apparent quantum effects have been detected¹², although they have been theoretically considered for SIAs¹³⁻¹⁵ and screw dislocations¹⁶. Quantum effects have also been observed on metal surfaces¹⁷. We focus here on the low temperature diffusion of SIA clusters in tungsten as a model for crystal defects in heavy-atom systems.

The lowest-energy SIA configuration in tungsten (and the other non-magnetic body-centred-cubic (bcc) transition metals) is the $\langle 111 \rangle$ crowdion, in which atomic displacements are confined almost entirely to the $\langle 111 \rangle$ string containing the extra atom. The defect is delocalized: it involves many more than one atom, as the displacement field is spread down the string, resulting in very low barriers to translation (known as *Peierls* barriers, see [Supplementary Discussion 1a](#) and [Fig. ED1](#)). Hence crowdions perform one-dimensional (1D) diffusion along their axis with a low (meV scale) activation energy^{10,18,19}. Similarly to single crowdions, SIA clusters in the form of $\mathbf{b} = \frac{1}{2}\langle 111 \rangle$ dislocation loops undergo 1D glide diffusion in the direction of the Burgers' vector \mathbf{b} . This phenomenon has been studied using classical molecular dynamics simulations (MD)²⁰⁻²⁴ and transmission electron microscopy (TEM)^{5,25} for α -iron and other metals and alloys.

According to MD studies, the activation energy (Peierls barrier) for cluster diffusion is less than 0.1 eV^{20,22}, meaning they are thermally mobile even at very low temperatures. In any real material however, impurity atoms (mainly carbon and

nitrogen) act as traps by binding to the clusters. Vacancies (expected at high density under irradiation) will mutually annihilate with SIAs at the cluster boundary.

Previous studies, using resistivity recovery and internal friction experiments⁹, have shown that low-temperature cluster migration in tungsten (and other bcc metals) is strongly influenced by the concentration of impurity atoms²⁶⁻²⁸.

These traps are deep enough (~ 1 eV, see [Supplementary Discussion 1b](#) and [Fig. ED2](#)) to prevent TEM observation of the clusters' thermal escape and subsequent motion on experimental timescales, even at 300 K, and they remain immobile. To overcome this, we used the electron beam in transmission electron microscopes such as a high-voltage electron microscope (HVEM) to enhance the vacancy mobility and reduce the effective trap depth. In the absence of the electron beam, vacancies are immobile up to 620-900K⁹, but in our experiment, the momentum imparted by the incident electrons moves the vacancies up to 100 times per second. The experimental system is shown schematically in [Fig. 1](#), and operates as follows.

First, a high energy (2000 keV) electron beam is used to create displacement damage, vacancies and SIAs at 105 K, before aging at 300 K. This allows the SIA clusters to nucleate and grow to the nanoscale, bound to impurities at their perimeters (where the binding energy is greatest). At these temperatures, the vacancies are thermally immobile and remain dispersed throughout the sample. A lower energy (100-1000 keV) beam is then turned on the sample. These energies are too low to create additional vacancies and SIAs, but high enough to athermally move the existing vacancies (see [Methods](#)), and the previously trapped clusters begin to move ([Fig. 1](#);

99 [Supplementary Video 1](#)). The principal quantity we monitor is the cluster motion
100 frequency. The precise definition of this quantity, together with its dependence on the
101 experimental irradiation conditions, is given in [Methods](#) and illustrated in [Fig. 2](#).
102 Perhaps the most striking feature of our study is its ability to resolve the SIA clusters'
103 thermal and quantum-mechanical motion, even under a ballistic flux of vacancies. In
104 [Methods](#) we describe in detail how this is possible.

105 The key features of the motion are:

- 106 i) hops are rare events, i.e. the clusters spend far more time trapped than
107 travelling between traps;
- 108 ii) clusters sometimes move back and forth between fixed points in the
109 sample;
- 110 iii) clusters are observed to shrink under the beam;
- 111 iv) motion frequency depends strongly on temperature.

112
113 i) and ii) tell us that the clusters are escaping from the impurity traps, moving quickly
114 through the lattice before being subsequently trapped again; iii) tells us how: the
115 radiation-mobilized vacancies move through the crystal, attracted to the high
116 compressive strain at the cluster boundaries. Here they annihilate the SIAs at the
117 cluster boundaries, shrinking the cluster, and increasing the separation between the
118 impurity atom and the cluster boundary. The impurity-cluster interaction is strong but
119 short-ranged (see [Supplementary Discussion 1b](#) and [Fig. ED2](#)), and falls off towards
120 zero within a few lattice spacings, so the traps are now much shallower, and escape is
121 easier ([Fig. 2abc](#)). We now turn to the temperature dependence, iv), which
122 demonstrates that the low temperature escapes are quantum mechanical in nature.

123

124 [Figure 3](#) is an Arrhenius plot showing the logarithm of the motion frequency vs. the
125 inverse temperature. Hops due to thermal escape from potential wells of depth
126 $\Delta V \gg k_B T$ have a characteristic rate $\propto \exp(-\Delta V/k_B T)$, i.e. a straight line on an

127 Arrhenius plot. This appears to be the case for the higher temperatures $T \geq 50$ K and
128 the slope suggests ΔV is higher than 10 meV. As the temperature is reduced, $17 \text{ K} \leq T$
129 ≤ 50 K, the slope flattens as the mechanism transitions from classical thermal escape
130 towards temperature-independent quantum mechanical diffusion.

131

132 The measured rates result from three independent processes: the athermal radiation-
133 driven vacancy migration under the beam (rate Γ_{vac}), the fluctuation-driven escape of
134 the cluster from the trap (depth ΔV_{trap} , rate Γ_{trap}), and finally the traversal of the Peierls
135 barrier intrinsic to the crystal (depth ΔV_{p} , rate Γ_{p}) (see [Methods](#)).

136

137 [Figure 3a](#) shows attempted classical fits for all barriers
138 $10 \text{ meV} \leq \Delta V = \Delta V_{\text{p}} + \Delta V_{\text{trap}} \leq 90 \text{ meV}$. Note that the Peierls traversal rate is non-
139 Arrhenius (since ΔV_{p} is not more than $k_{\text{B}}T$, see [Methods](#)), but no possible classical
140 form for the rate can explain the observed values. (We are confident that the sample
141 temperatures continue to decrease below 50 K, and are not significantly increased by
142 beam heating – see [Supplementary Discussion 2 and Fig. ED3](#)).

143

144 In [Fig. 3b](#), we use a quantum mechanical form for the escape rate Γ^{QM} , derived from
145 the quantized nature of the crystal phonons (see [Methods](#)). These obey Bose-Einstein
146 rather than Boltzmann statistics, and their zero-point fluctuations increase the average
147 energy available for the cluster to overcome the barrier, thus increasing the low
148 temperature rates in excellent agreement with the experimental values. **Moreover, the**
149 **same quantum rates simultaneously fit two datasets acquired at different voltages.**
150 **This proves that the same quantum mechanism underlies both datasets.**

151

152 However, we still obtain acceptable fits for all barriers between 10 and 90 meV. To
153 narrow this down, we considered the critical temperature τ_c below which classical
154 physics breaks down (see [Methods](#)), which depends on the barrier height: [Figure 2](#)
155 shows the 90 meV fit clearly failing below 140 K, whereas the 10 meV one appears
156 reasonable down to around 50 K. τ_c depends on the phonon density of states, and is
157 estimated²⁹ to be 101 K for pure tungsten (about 1/3 of the Debye temperature). Fitted
158 values for τ_c are also shown in [Fig. 3](#), and the value 101 K is consistent with a barrier
159 height of 30 – 44 meV. We note that the resistivity recovery and internal friction
160 experiments obtain a barrier height of 15 – 60 meV^{9,26-28}.

161

162 Other manifestations of quantum behaviour are in principle possible, in particular the
163 deep tunneling of the entire cluster. However, fitting the data to this functional form
164 requires unrealistic values for the cluster's effective mass (see [Methods](#)), and we
165 conclude that, over the range of temperatures probed by our experiment, quantized
166 phonons facilitating the clusters' escape from traps 30 – 44 meV deep provide the
167 optimal explanation of the data.

168

169

170 In this study we have performed the first direct investigation of cryogenic defect
171 diffusion using in-situ TEM. Our unique experimental system allowed us to
172 manipulate the effective potential wells encountered by SIA clusters, reducing their
173 depth until we could probe the quantum mechanical nature of their de-trapping. The
174 quantum transport becomes dominant below around 1/3 of the Debye temperature.
175 Moreover, the behaviour derives from quantized phonons, which drive the stochastic
176 fluctuations of objects that are themselves too heavy to tunnel significantly. This

177 likely affects low temperature defect transport in many crystalline materials. Our
178 results demonstrate the importance of quantum effects on low temperature defect
179 evolution even in heavy atom systems.

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247

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266

267 Author contributions

268 K.A., M.C.M. and L.P. designed the study. K.A., T.Y., T.A., S.A., Y.Y., K.H., N.T., H.Y., T.Y. and
269 H.M. performed the experiments. M.C.M., S.P.F., L.P., D.N.M., A.M.G., S.L.D., P.W.M. and T.D.S.
270 performed the theoretical works. K.A., M.C.M., S.P.F., and S.L.D. wrote the main draft. All authors
271 discussed the results and commented on the manuscript.

272

273 **Additional information**

274 Supplementary information is available in the online version of the paper. Reprints and permissions
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276 should be addressed to K.A.

277

278 **Competing financial interests**

279 The authors declare no competing financial interest.

280

281 **METHODS**

282 **Specimen preparation.** We cut (011) discs from one grain of an ingot of high-purity
283 coarse-grained polycrystalline tungsten (99.9999 mass % JX Nippon Mining &
284 Metals Co., Tokyo, Japan; impurity amounts of the ingot are given in Ref. [30]). The
285 discs were thinned to 0.1mm, using spark erosion and mechanical polishing, then
286 perforated at the centre by electropolishing so the periphery of the hole became cross-
287 sectionally wedge-shaped for TEM observations.

288

289 **Production of SIA clusters.** We used high-energy electron irradiation in a HVEM
290 (Hitachi H-3000) to create SIAs and vacancies in the thin foil specimens. The
291 acceleration voltage was 2000 kV, and a temperature of 105 K was maintained using
292 a liquid-nitrogen-cooled specimen holder (Oxford Instruments). We note that the
293 thermal migration of vacancies is frozen at temperatures below 620-900 K⁹. The beam
294 flux was $1 \times 10^{24} \text{ m}^{-2}\text{s}^{-1}$, and the dose was $4 \times 10^{25} \text{ m}^{-2}$.

295 During 2000-keV electron irradiation, pairs of SIAs and vacancies are produced³¹ via
296 knock-on displacement. Based on our recent work^{19,30}, the point defect reactions
297 proceed as follows: most of the highly mobile 1D-moving SIAs react with vacancies,
298 or escape to the foil surface, where they are annihilated. Surviving SIAs bind to
299 impurity atoms and form embryonic SIA clusters, that grow by absorbing other SIAs,
300 and take the form of $\mathbf{b} = \frac{1}{2}\langle 111 \rangle$ dislocation loops. These clusters are intrinsically
301 highly mobile, yet they are trapped by impurities and remain stationary. Vacancies
302 that do not react with SIAs accumulate throughout the irradiated area of the specimen.

303 Using TEM, the average size and density of the SIA formed clusters under the above
304 condition were found to be approximately 3-4 nm and $4 \times 10^{22} \text{ m}^{-3}$, respectively.
305 Accumulated vacancies are not visible in the TEM. After the irradiation, the specimen
306 was aged at approximately 300 K. This allows the clusters trapped by weak impurity
307 atoms with shallow potential wells to thermally escape and move, leading to
308 coalescence with other clusters³², escape to the specimen surfaces, or to trapping by
309 stronger impurities with deeper wells. However, even after aging for several months,
310 we did not see any significant change in the cluster density, demonstrating that
311 thermal escape of SIA clusters from the deeper wells hardly occurs even at 300 K.

312

313 **TEM observation of the 1D motion of SIA clusters in response to high-energy**

314 **electron irradiation.** We then used the electron beam to induce the vacancy mobility,
315 with acceleration voltages of 100, 150, 300, 500 (Hitachi H-9000UHV), 1000, and
316 2000kV (H-3000) – all except 2000kV are below the threshold for point defect
317 generation in tungsten³¹. Additional very intense irradiations were carried out at
318 1000kV using a JEOL JEM 1000K RS. Beam fluxes ranged from 5×10^{22} to 2×10^{25}
319 $\text{m}^{-2}\text{s}^{-1}$, and temperatures ranged from 17-300 K (where no thermal migration of
320 vacancies takes place⁹). We achieved these temperatures using liquid-helium-cooled
321 specimen holders (Oxford Instruments), in which the temperature is measured with a
322 thermocouple attached to the specimen mount, so the measured temperature is the
323 average over the whole specimen.

324 The specimen thickness ranged from 50 to 70 nm (measured using equal-thickness
325 fringes³³). The observations were carried out using the weak-beam dark-field
326 technique³⁴ with a reflection of $\mathbf{g} = 200$. Under this condition, all SIA clusters in the
327 form of prismatic dislocation loops with a $\mathbf{b} = \frac{1}{2}\langle 111 \rangle$ type Burgers vector and a
328 diameter greater than approximately 2 nm were imaged. The dynamic response of the
329 clusters was monitored and recorded with CCDs having frame rates of 30 fps for H-
330 9000UHV and H-3000, and 15 fps for JEM 1000K RS.

331 We define the motion frequency of the clusters as the ratio of the number of cluster
332 hops observed per unit time divided by the number of observable clusters, i.e. the
333 average motion frequency of individual SIA clusters.

334

335 **Motion frequency and the ballistic and kinetic rates of SIA clusters.** Our
336 experiments measure the average motion frequency of SIA clusters as observed in the
337 transmission electron microscopes. The average motion frequency is defined
338 as $\nu_{\text{MF}}(t) = n_{\text{m}}/(n t)$: the ratio of the number of clusters that move (n_{m}) divided by
339 the total number of observed clusters (n) in the observation time (t).

340 The measured rates ν_{MF} are the combined results of motion induced by directly by the
341 irradiation, and stochastic motion induced by the underlying phonon bath.
342 Consequently, the motion frequency is impacted by irradiation conditions, in
343 particular the electron beam flux Φ and energy E . The temperature T also influences
344 the experimental observations through the phonon bath, meaning that the motion
345 frequency is a function defined on a 4 dimensional space $\nu_{\text{MF}}(t, T, \Phi, E)$. Figure 3
346 illustrates the temperature dependence, and Fig. 2d-g shows the behaviour of the
347 motion frequency with respect to the other variables. Here we derive an expression

348 for the motion frequency in the context of the experiments.

349 Detailed experimental analysis suggests that the shrinkage of the clusters (Fig. 2a)
 350 originates from irradiation-induced vacancy motion (Fig. 2b). Since the impurities are
 351 immobile, the erosion of the clusters increases the distance between them and the
 352 impurities. We call the mechanism of the cluster de-trapping due to this process the
 353 *indirect de-trapping mechanism*. Since it depends on the radiation-mobilized
 354 vacancies eroding the SIA clusters, the cluster motion frequency is proportional to the
 355 vacancy concentration, c_V . These vacancies are absorbed by the clusters, and other
 356 sinks such as the specimen surface, at a rate proportional to the concentration itself:
 357 $\dot{c}_V \propto -c_V$. So long as no new Frenkel pairs are created, this leads to an exponential
 358 decay in time of the vacancy concentration, and hence the cluster motion frequency.
 359 This is precisely what we observe in Fig. 2cd, in the short time limit.

360 In the indirect mechanism, cluster de-trapping is also impacted by the thermal rate at
 361 which the clusters escape from the impurities. At a given cluster-impurity separation,
 362 d_k , sufficiently large that the trapping energy is low, the thermal escape rate Γ_{th}^k is
 363 governed by the cluster-impurity trapping energy ΔV_{trap}^k for that distance (see next
 364 section). If we have n_k cluster-impurity sets at given cluster-impurity separation d_k ,
 365 then the number of clusters that jump within the observation time is $a c_V n_k \Gamma_{th}^k t$. The
 366 factor a incorporates the impact of the beam flux and energy on the observations.

367 Since the incident electron energy is high, what we call the *direct de-trapping*
 368 *mechanism* – direct collision of the electron with the impurity that traps the cluster –
 369 can also release the cluster. The rate Γ_d of this direct mechanism is athermal and
 370 uniform in time, depending only on the concentration of cluster-trapping impurities
 371 and the flux and energy of the electrons. The probability to release a cluster from an
 372 impurity via the direct mechanism is $n\Gamma_d t$.

373 Consequently, the measured motion frequency can be written as

$$\nu_{MF} = \frac{n_{indirect} + n_{direct}}{n t} = \frac{\sum_k a c_V n_k \Gamma_{th}^k(T) t + n \Gamma_d t}{n t}$$

374 Or, in a simpler form, if we assume that in the system the initial vacancy density
 375 $c_v(0)$ decreases in time with a decay factor α_v :

$$\nu_{MF} \sim \sum_k a c_V(0) e^{-\alpha_v t} \frac{\Gamma_{th}^k(T) n_k}{n} + \Gamma_d = e^{-\alpha_v t} \left[\sum_k a c_V(0) \frac{\Gamma_{th}^k(T) n_k}{n} \right] + \Gamma_d$$

376 This theoretical expression for the motion frequency is fully compatible with all the

377 experimental evidence described in the body of the paper and illustrated in Fig. 2.

378 Firstly, the experimental observations shown in Fig. 2de indicate that the motion
 379 frequency decreases exponentially in time, and after several hundred seconds, the
 380 frequency's exponential decay transitions to a constant plateau. This reflects the local
 381 exhaustion of vacancies near the clusters, and the transition to the direct mechanism.
 382 The $t \rightarrow \infty$ limit provides the frequency associated with the direct mechanism
 383 $\nu_{\text{MF}} \rightarrow \Gamma_{\text{d}}$. In the limit of $t \rightarrow 0$:

$$\nu_{\text{MF}}(t \rightarrow 0) \rightarrow \left[\sum_k a_k c_v(0) \frac{\Gamma_{\text{th}}^k(T) n_k}{n} \right] + \Gamma_{\text{d}} \sim \text{const} \times \Gamma_{\text{th}}^0(T) + \Gamma_{\text{d}},$$

384 we have access, up to multiplicative (const) and additive (Γ_{d}) constants, to the
 385 dominant thermal/quantum rate $\Gamma_{\text{th}}^0(T)$ on whose nature, classical or quantum, our
 386 study is focused. Moreover, the higher the beam energy, the greater the mobility
 387 enhancement and the sooner this happens. The plateaus are also higher for higher
 388 beam energies, reflecting the direct mechanism's expected dependence on beam
 389 energy.

390 Secondly, Fig. 2f shows the cluster motion frequency's strong dependence on beam
 391 intensity at 300 kV, clearly illustrating the essential role the irradiation plays through
 392 the multiplicative constants. Note that no further Frenkel pairs are created with beam
 393 energies at or below 1000 kV.

394 Finally, Fig. 2g shows the electron energy dependence of $\nu_{\text{MF}}(t \rightarrow 0)$, together with
 395 the athermal radiation-driven vacancy migration rate under the beam Γ_{vac} . The Γ_{vac}
 396 value is proportional to the product of beam flux and the cross section for radiation
 397 induced vacancy migration³⁵,

$$\sigma_{\text{mig}} \approx \int_{E_{\text{mig}}^V}^{E_{\text{K, max}}} \frac{E_{\text{K}}}{E_{\text{mig}}^V} \frac{d\sigma}{dE_{\text{K}}} dE_{\text{K}},$$

398 where E_{K} is the kinetic energy transferred from an incident electron to a tungsten
 399 atom neighbouring a vacancy, E_{mig}^V is the vacancy migration energy (1.78 eV³⁶), and
 400 $d\sigma$ is the differential cross section for the electron-tungsten atom collision calculated
 401 using the McKinley-Feshbach formula³⁷. The high correlation is clear, further
 402 emphasizing the vacancy migration origin of the indirect mechanism.

403 A natural question is whether this approach has sufficient accuracy to reveal the
 404 classical or quantum nature of this rate. The quantity of interest is the logarithm of the
 405 motion frequency, which can be written as:

$$\ln \nu_{\text{MF}}(t \rightarrow 0) = \ln[\Gamma_{\text{th}}^0(T) + \Gamma_{\text{d}}] \sim \ln \Gamma_{\text{th}}^0(T) + \frac{\Gamma_{\text{d}}}{\Gamma_{\text{th}}^0(T)}$$

406 The second term of the right side is easily estimated from the ratio of asymptotic
 407 limits $\nu_{\text{MF}}(t \rightarrow 0)/\nu_{\text{MF}}(t \rightarrow \infty)$. This quantity is in the order of 10^{-1} and 10^{-2} at
 408 1000 keV and 500 keV, respectively, for 289-298 K (Fig. 2d). Also, it is shown to be
 409 at most 0.2 at 300 keV even for 31 K (Fig. ED4). This analysis shows that in the
 410 measured logarithm of $\nu_{\text{MF}}(t \rightarrow 0)$ the effect of the direct de-trapping mechanism is
 411 relevant only from the first up to the second decimal place. Hence, the direct and
 412 indirect contributions to the motion frequency can be reliably separated.

413 We provide the statistical procedure in the measurement of $\nu_{\text{MF}}(t)$. One specimen
 414 involved 1×10^2 areas for 2000-keV electron irradiation for the SIA cluster
 415 production, at maximum. The n value within one area of interest (AOI) centred at a
 416 2000-keV electron irradiated area was $(1 - 2) \times 10^2$ for $t = 0$ s. This n value was the
 417 practical upper limit under the lowest TEM magnification enabling the observation of
 418 the cluster motion. In the $\nu_{\text{MF}}(t)$ data shown in Figs. 2d-g, Fig. 3, and ED4, each data
 419 symbol corresponds to an AOI. The error in the $\nu_{\text{MF}}(t)$ value was evaluated under the
 420 assumption that both the distributions of n and n_{m} for a given area independently obey
 421 the Poisson distribution. Then, error in a measured $\nu_{\text{MF}}(t)$ value becomes
 422 $\nu_{\text{MF}}(t) \sqrt{\frac{1}{n} + \frac{1}{n_{\text{m}}}}$. Series data for temperature dependence of $\nu_{\text{MF}}(t)$ under fixed other
 423 conditions (Fig. 3) were taken from the areas belonging to an identical TEM specimen
 424 so that the impurity amount over the measured areas was in a very similar level.

425

426 **Diffusion rates in quantum and classical phonon baths.** The archetypal problem of
 427 a particle traversing a potential barrier has been treated exhaustively; see Ref. [38] for
 428 a thorough review. For a barrier height $\Delta V \gg k_{\text{B}}T$, the classical escape rate is given
 429 by the Arrhenius function $\Gamma_{\text{th}}^{\text{cl}} = f_{\text{cl}} \exp(-\Delta V/k_{\text{B}}T)$, where the classical prefactor f_{cl}
 430 can be loosely interpreted as an attempt frequency. As $k_{\text{B}}T$ rises towards ΔV the
 431 Arrhenius function breaks down, and the rate transitions to a form linear in the
 432 temperature^{11,23} (manifested as a sharp steepening on an Arrhenius plot). For barriers
 433 $\Delta V \sim k_{\text{B}}T$ or less, the particle migrates stochastically, being slowed only by the
 434 dissipative coupling between the particle and the underlying phonon bath. This is
 435 quantified by the friction parameter γ , and the rate is proportional to $k_{\text{B}}T/\gamma$ ^{11,23,39}. If
 436 $\Delta V \ll k_{\text{B}}T$, the friction can be absorbed into f_{cl} ^{38,40}. Both standard rate formulae
 437 originate from the classical Boltzmann distribution for the phonons. For clusters
 438 escaping from traps, the barrier to be overcome is $\Delta V = \Delta V_{\text{p}} + \Delta V_{\text{trap}}$, the sum of the

439 Peierls barrier and the critical binding energy of the impurity or vacancy respectively.
 440 Therefore the diffusion rate is the product of two independent probabilities: the
 441 probability related to the free migration of the SIA cluster through the Peierls
 442 potential in the absence of a trap, and the escape probability from the trap itself:
 443 $\Gamma_{\text{th}}^{\text{cl}}(T) = \Gamma_{\text{P}}(T) \times \Gamma_{\text{trap}}(T)$. $\Delta V_{\text{trap}} \gg k_{\text{B}}T$, so Γ_{trap} is Arrhenius in the classical limit.
 444 Since the Peierls barrier ΔV_{P} for SIA clusters (a.k.a. $\frac{1}{2}\langle 111 \rangle$ loops) is small, i.e. of
 445 order $k_{\text{B}}T$, the total classical rate becomes:

$$\Gamma_{\text{th}}^{\text{cl}}(T) = \text{const.} \times k_{\text{B}}T \times \exp\left(-\frac{\Delta V_{\text{trap}}}{k_{\text{B}}T}\right) \quad (1)$$

446 We note that the constant prefactor above can take on a weak temperature dependence
 447 in other formulations of the rate; we obtain similar fits in either case and our
 448 conclusions are unaffected.

449 The full quantum-mechanical development is more complicated. Here, the Boltzmann
 450 distribution is replaced by either the Bose-Einstein (BE) or Fermi-Dirac distribution,
 451 for bosons or fermions respectively. For tungsten or impurity atoms the ground state
 452 has integer spin and hence obeys Bose-Einstein statistic. A simple way to recover the
 453 BE phonon distribution whilst retaining the form of the classical rate formulae is to
 454 renormalize the temperature to mimic the true quantum statistics^{15,40,41}. Consider a
 455 crystal with periodic boundary conditions represented by N atoms in a box. Imposing
 456 equality of the classical and quantum energies, the (renormalized, effective) classical
 457 temperature and the (true) quantum temperature should be related by the relation:

$$(3N - 3)k_{\text{B}}T_{\text{c}} = \int d\omega \hbar\omega \left(\rho_{\text{BE}}(\omega, T_{\text{q}}) + \frac{1}{2} \right) n(\omega)$$

458 where T_{c} and T_{q} are the (renormalized, effective) classical and (true) quantum
 459 temperatures respectively. $n(\omega)$ is the density of states of the phonon gas, normalized
 460 to the number of modes, and $\rho_{\text{BE}}(\omega, T)$ is the BE distribution function. Therefore, the
 461 effective classical temperature is a function of the true quantum temperature $T_{\text{c}} =$
 462 $f(T_{\text{q}})$.

463 For temperatures higher than the Debye temperature T_{D} , $\hbar\omega \ll k_{\text{B}}T$, the energy of
 464 one oscillator becomes:

$$\hbar\omega \left(\rho_{\text{BE}}(\omega, T_{\text{q}}) + \frac{1}{2} \right) \approx \frac{\hbar\omega}{2} + k_{\text{B}}T_{\text{q}} \left(1 - \frac{\hbar\omega}{2k_{\text{B}}T_{\text{q}}} + K \right) = k_{\text{B}}T_{\text{q}},$$

465 and the classical and quantum temperatures are very close. When the (true) quantum
 466 temperature T_q tends to zero K, the effective classical temperature T_c tends to a finite
 467 limit, capturing the zero point energy:

$$(3N - 3)k_B T_c = \int d\omega \frac{1}{2} \hbar \omega n(\omega)$$

468 The simple form $T_c = \sqrt{\tau_c^2 + T_q^2}$ satisfies these limits (see Fig. ED5). Therefore, the
 469 quantum rates can be estimated by simply renormalizing the temperature in equation
 470 (1) yielding:

$$\Gamma_{th}^{QM}(T) = \text{const.} \times k_B \sqrt{\tau_c^2 + T^2} \times \exp\left(-\frac{\Delta V_{\text{trap}}}{k_B \sqrt{\tau_c^2 + T^2}}\right) \quad (2)$$

471 We also attempted to fit the data with up to three distinct classical barrier escape
 472 mechanisms operating simultaneously. Only the quantum rates explain the observed
 473 temperature dependence.

474 **Quantum TST rates.** For deep tunneling, we computed the rate by numerically
 475 integrating the quantum transition state theory rate expression⁴⁰

$$\Gamma_{th}^{QTST} = (hZ_0)^{-1} \int W(E) e^{-E/k_B T} dE,$$

476 where h is the Planck constant and $W(E)$ is the transfer integral at energy E for the
 477 sech-squared impurity interaction potential predicted by the Frenkel Kontorova model,
 478 (see Supplementary Discussion 1). The data can be fitted with a barrier height of 55
 479 meV, but requires an unrealistically low effective cluster mass of $m_W/200$ (m_W is the
 480 mass of one tungsten atom). The remaining parameters (potential width and
 481 curvature) are fixed by the Arrhenius limit, which applies to the highest temperature
 482 points in the dataset.

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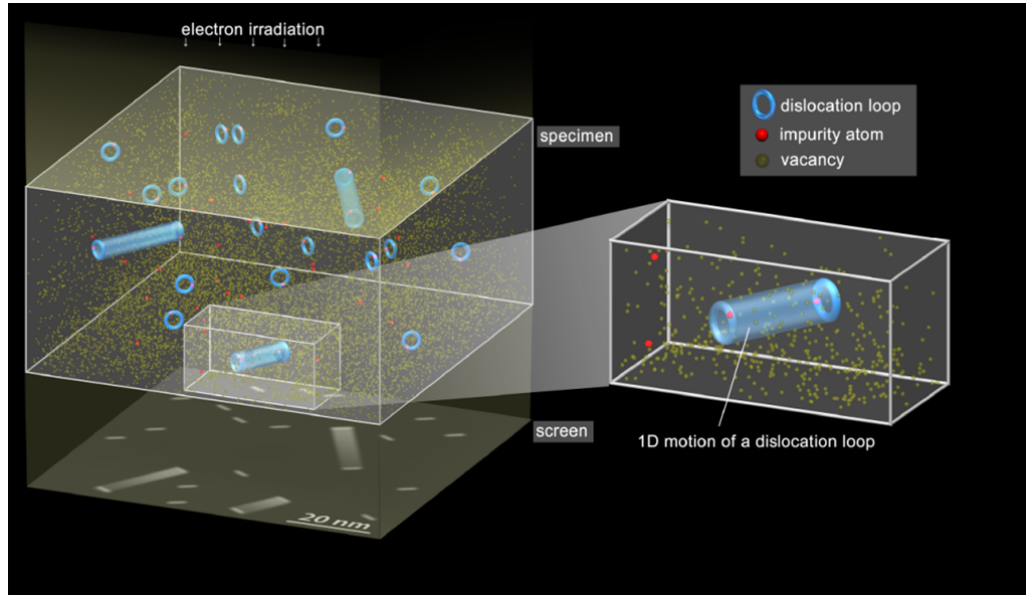
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518 Figures

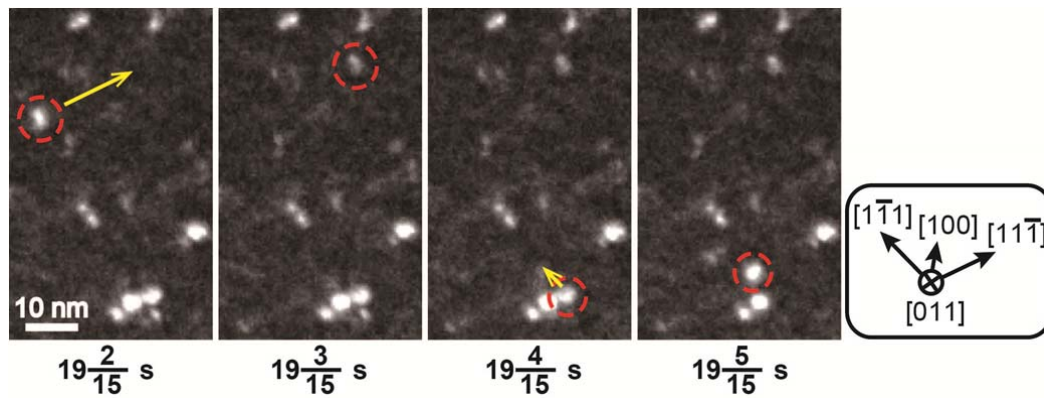
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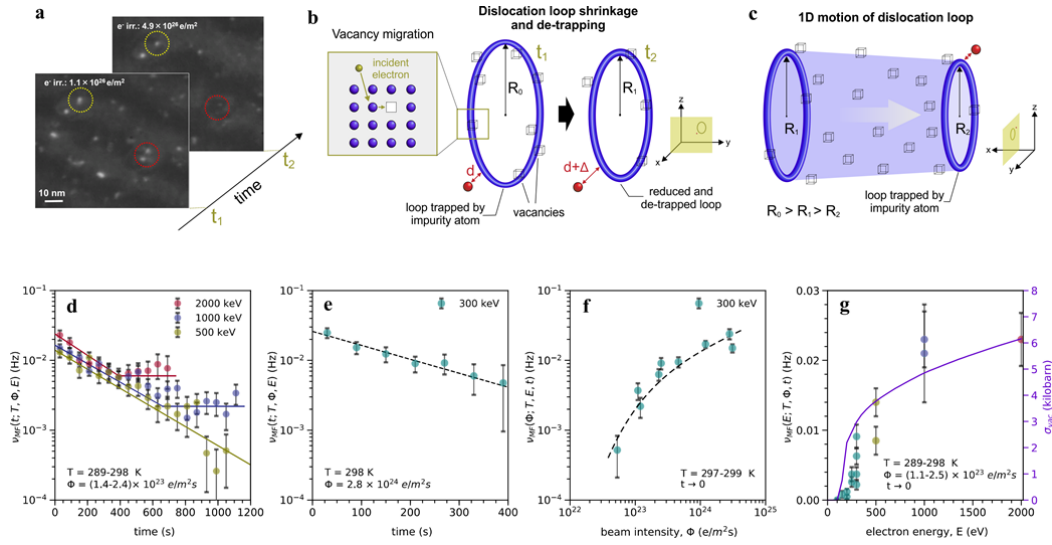


523

524 **Figure 1 | 1D SIA cluster motion.** a: Experimental setup. In a high-purity tungsten
 525 specimen, SIA clusters in the form of nanoscale $\frac{1}{2}[111]$ dislocation loops are trapped
 526 by impurity atoms at their boundary. b: High-energy electron irradiation enables
 527 clusters to escape, and subsequently undergo fast 1D glide diffusion before being
 528 trapped by other impurity atoms. This 1D motion was monitored simultaneously
 529 (acceleration voltage: 1000 kV; beam intensity: $2 \times 10^{25} \text{ m}^{-2}\text{s}^{-1}$; temperature: 260 K,
 530 see [Supplementary Video 1](#)). Circled clusters move in the directions indicated by

531 arrows, parallel to the $\frac{1}{2}11\frac{1}{2}$ -type cluster Burgers vectors. The clusters hop distances
532 of several nm to a few tens of nm within a single 1/15 s movie frame.

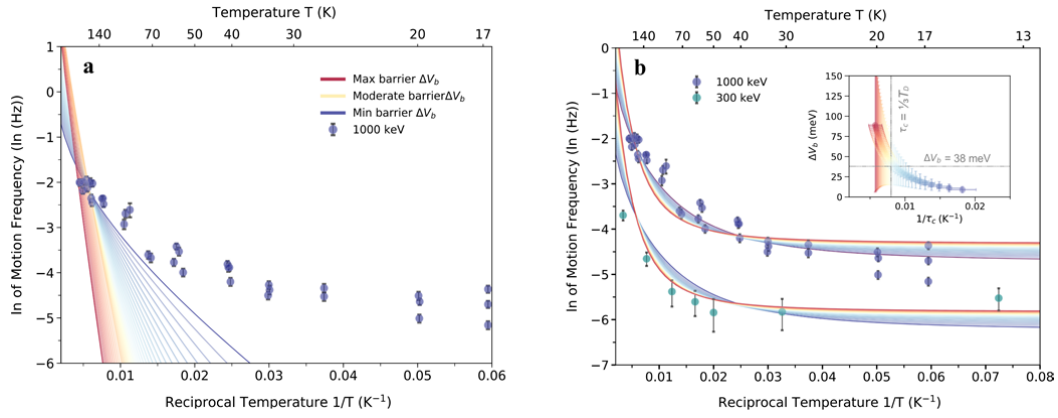
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535 **Figure 2 | Characterization of the motion frequency of SIA cluster de-trapping.**

536 **a:** SIA cluster (dislocation loop) shrinking under the beam (acceleration voltage: 300
537 kV; beam intensity: $3.1 \times 10^{24} \text{ m}^{-2}\text{s}^{-1}$; temperature: 299 K). Vacancies in tungsten are
538 thermally immobile at 299 K, and so the only way the SIA clusters can shrink is via
539 the absorption of radiation-mobilized vacancies. **b:** The clusters escape by increasing
540 the distance between their perimeter and the impurity, from d to $d + \Delta$, as they shrink
541 from radius R_0 at time $t_1 \rightarrow R_1 < R_0$ at time t_2 . This reduces the binding energy (see
542 [Supplementary Discussion 1](#)) **c:** Stop-and-go motion of the loop in the clouds of
543 vacancies and impurities. Once the loop has escaped from the impurity, it migrates
544 until is trapped by another impurity. During this macro-jump, over many Peierls
545 barriers, the loop sweeps through the surrounding vacancy clouds, decreasing its
546 effective radius to $R_2 < R_1$. **d, e:** Motion frequency decaying exponentially with time
547 under irradiation which corresponds to indirect mechanism (see [Methods](#)). Plateaus
548 are reached when the supply of vacancies local to the clusters is exhausted by
549 annihilation, and the direct mechanism takes over (see [Methods](#)). **f:** Motion frequency
550 increasing with beam intensity (time: 0 – 60 s). **g:** Motion frequency vs. beam energy
551 and cross section for radiation-induced vacancy migration (time: 0 – 60 s) (see
552 [Methods](#)).



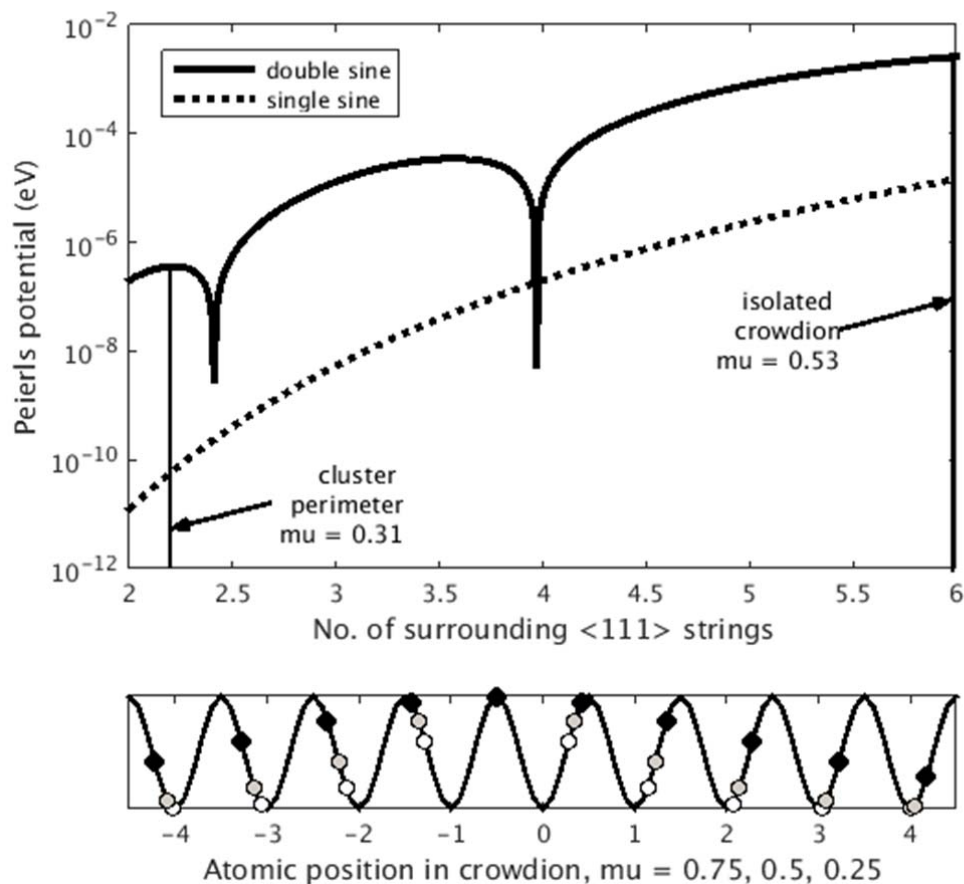
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554 **Figure 3 | Motion frequency of SIA cluster de-trapping vs. temperature.** Data
555 points show measured motion frequency vs. temperature (data taken in first 60 s of
556 irradiation. Blue points: beam energy 1000 keV, beam intensity $2 \times 10^{25} \text{ m}^{-2}\text{s}^{-1}$; green
557 points: beam energy 300 keV, beam intensity $(2 - 4) \times 10^{24} \text{ m}^{-2}\text{s}^{-1}$). Some error bars
558 are too small to be visible. **a**: All possible classical fits of one single dataset, at beam
559 energy of 1000 keV, for activation barriers between 10 meV (blue) and 90 meV (red).
560 Thin lines between are intermediate values. No classical fit can capture the
561 temperature dependence. **b**: As panel **a** but using quantum mechanical rate function.
562 Both 1000 and 300 keV datasets were fitted simultaneously, with a single parameter
563 to account for the ratio of the two (we obtained a value of 4.52 for the ratio, consistent
564 with Fig. 2g, see Methods). Inset: fitted correlation between activation barrier and
565 critical temperature τ_c (see text and Methods), with corresponding error bars. The
566 value of the effective activation barrier at $\tau_c = \frac{1}{3}T_D$ (T_D : Debye temperature) is 38
567 meV.

568

569 Extended Display Items

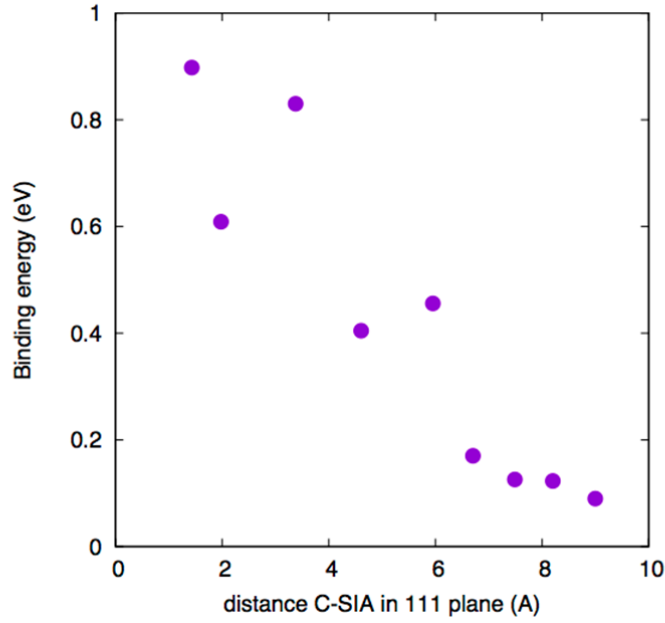
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572 **Figure ED1** | Top: suppression of Peierls potential as delocalization increases (and μ
 573 decreases). Both the standard single-sine and more accurate double-sine Frenkel-
 574 Kontorova models predict a negligibly small barrier for cluster diffusion after escape
 575 from the traps. Bottom: atomic positions showing increased delocalization as μ
 576 decreases from 0.75 (open circles) through 0.5 (grey circles) to 0.25 (solid circles).

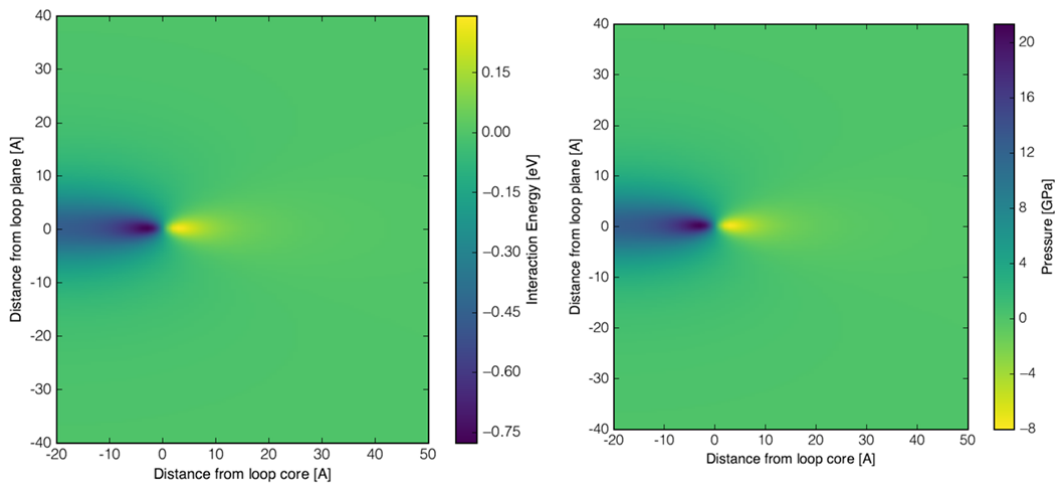
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579 **Figure ED2a** | DFT calculation of the SIA-carbon binding energy vs. separation in
 580 plane transverse to the crowdion axis.

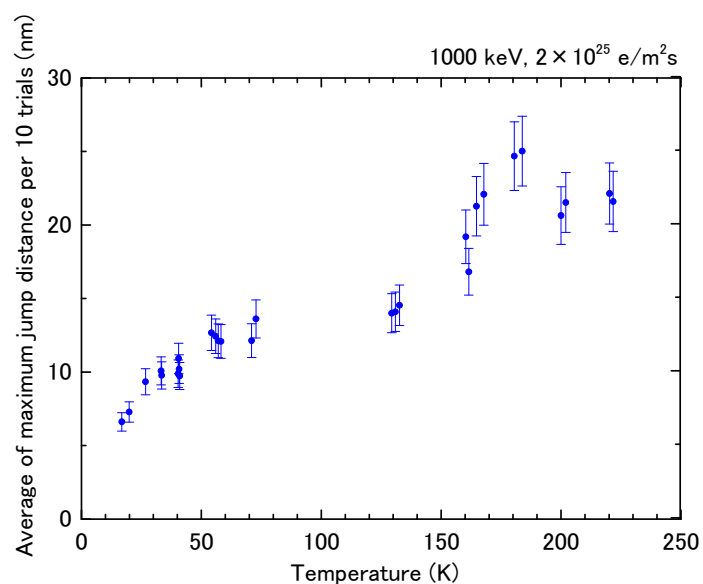
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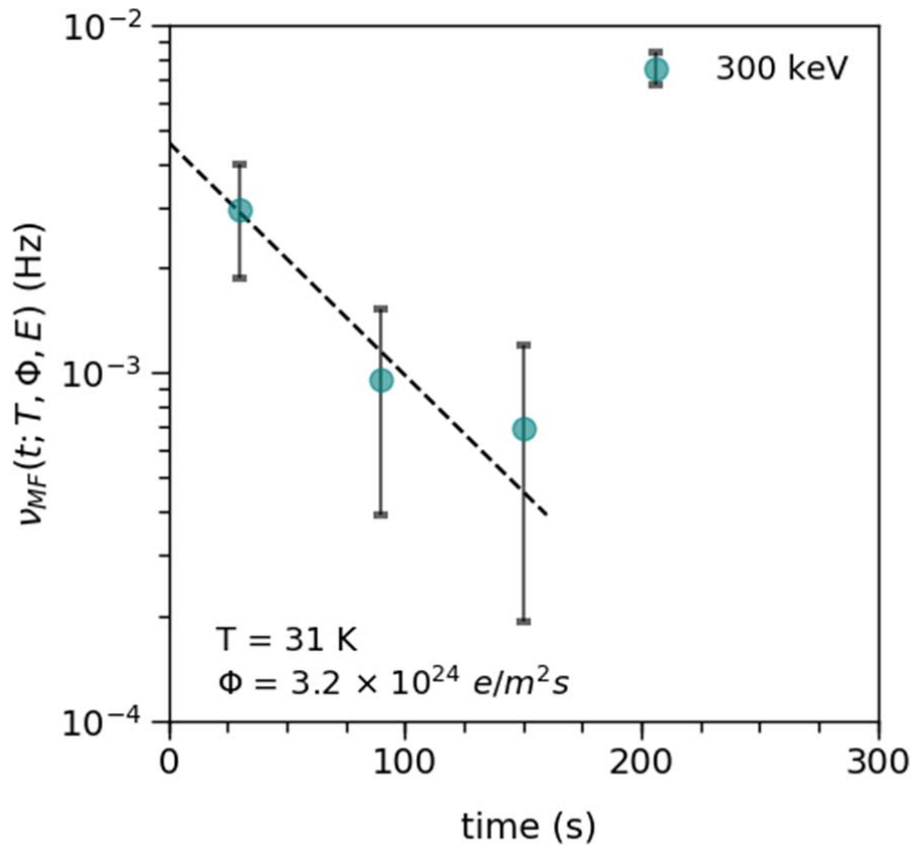
583 **Figure ED2b** | Elastic calculation of the SIA cluster-dilatation centre binding energy
 584 (left) and cluster pressure field (right).

585



586

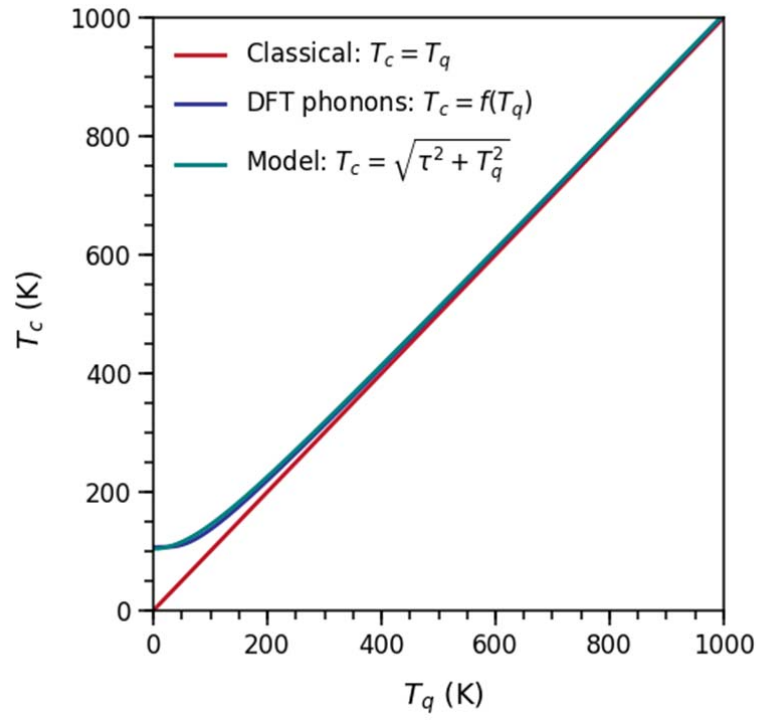
587 **Figure ED3** | Average maximum hop distance per 10 hops vs temperature. A range of
 588 binding energies exist, corresponding to different cluster-impurity separations. This
 589 means more impurities are effective traps at lower temperatures, leading to a reduced
 590 hop distance.



591

592 **Figure ED4** | Motion frequency vs irradiation time at 31 K, with beam energy 300
 593 keV. The decrease in motion frequency, attributed to the depletion of vacancies near
 594 the clusters, is still clear, and demonstrates that the direct mechanism (which would
 595 induce a motion frequency constant in time) is not wholly responsible for the cluster
 596 motion. Indeed, at short times the motion is dominated by the indirect mechanism, by
 597 at least a factor of 5.

598



599

600 **Figure ED5** | The correspondence between the effective classical temperature T_c (our
601 model) and the quantum (true) temperature T_q of perfect bulk bcc W. The classical,
602 DFT phonons and our model are shown in red, dark blue and light blue respectively.