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Physics driven scaling laws for similarity experiments: the role of the aspect ratio

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Abstract

Similarity experiments are conceived to study on existing tokamak facilities, characteristics of scenarios found on other devices or planned for new machines. The possibility of doing similarity experiments is linked to the physics processes studied and it gives in any case partial views which can be found in integrated way only on the planned devices. The paper presents scaling laws obtained to study pedestal physics , MHD limits and ELM behaviour, as well as bulk plasma confinement . *The focus is on the dependence upon the aspect ratio and ion mass*. The scaling laws are given in terms of plasma density(n), temperature(T) , current(Ip), magnetic field(B) and input power (see definition in sec 1, P_{input}) *versus major radius*(R), aspect ratio(A=R/a, R=major radius, a=minor radius) and ion mass(M)[12]. The introduction of ion mass is naturally included into the definition of the known set of dimensionless plasma physics parameters (q,ρ^*_T,ν^*,β_T) [6]. *In a first instance , the scalings are obtained indipendent from energy confinement scaling laws*. Further, if the scaling of heating power (P_{heat}) is obtained using the IPB(y,2) confinement scaling[6] or the DIIID/JET scaling [13-15] a sensible dependence of P_{heat} upon the geometry and aspect ratio is found. The scaling laws are obtained using the Kadomtsev[1] similarity scheme, where the alpha particle heating and atomic physics effects are neglected and the confinement is depending anly from the dimensionless set of parameters . The case of fusion reactor case is considered in detail in this paper: in this case the Kadomtsev scheme is NOT valid since the alpha particles heating is relevant, leading to a new scaling parameter useful to characterize the fusion reactors at fixed Q.

1.Introduction

Similarity in dimensionless parameters [1-4,12] can be used to extrapolate scenarios from existing to planned tokamak devices. To be completely rigorous, this would require identity in not only the known set of dimensionless plasma physics parameters (ρ^*_T , ν^*, β_p) but also similarity of plasma cross-section shape (including identity of aspect ratio A) , heating power deposition, and poloidal to toroidal field ratio (measured by q_{95}). In this rigorous form only few combinations of devices are in principle capable of truly equivalent operation, with the closest approach being Alcator C mod in conjunction with existing "mid-size" devices like DIII-D or ASDEX Upgrade.

A broader range of comparisons and extrapolations are however possible, if assumptions concerning the dominating physics effects are made to reduce the imposed number of constraints. In a different application of this principle, parallel experiments on pairs of devices can be conducted which are particularly discriminating with respect to model assumptions or theories. This allows also to include, as one option, the parameter P/R, characterizing divertor physics behaviour [5].

While the Kadomtsev[1] similarity can be considered as a 'global similarity' constraint in the sense that the similarity is devoted to reproduce identical (confinement) properties of scaled plasmas , the similarity scaling laws derived in the present paper reflect a 'restricted similarity' , since they are related to specific physics aspects of plasmas.

Table I shows the set of dimensionless parameters that can be used to define a plasma state. The present paper takes the view that a limited set of dimensionless parameters can describe particular physics aspects of tokamak plasma. On this basis, scaling laws can be derived of dimensional physics parameters (n, T, I_P, B_T) upon the major radius R, the aspect ratio A and mass M.

For the first time the complete dependences of the scaling laws are derived in this paper including both the aspect ratio (A) and the isotopic mass (M). The aspect ratio is one of the independent dimensionless plasma parameters characterizing a plasma state (see ref.1 and 12) , but the complete analysis of the scaling laws including A is still missing (to the author knowledge) in the literature.Three hypothesis (named hyp1, 2 and 3) are analyzed corresponding to a selection of dimensionless parameters and related to :1.Confinement of bulk plasma(hyp1); 2.Pedestal confinement and ELM physics(hyp2); 3.MHD stability and beta limits(hyp3). To derive the scaling law of the heating power the ELMy H-mode IPB(y,2)[6] or the JET/DIIID scaling [13-14] are used together with the relation.

$$P_{heat} \approx \frac{R a^2 k\ nT}{\tau_E}$$

Where R and a are the majior and minor radius of the tokamak, n and T the plasma density and temperature , k the plasma elongation, τ_E the confinement time. The input power can be expressed also by using the expression $P_{input} \approx nT^{3/2}R^2A^{-1}$. This is the power flux across the last closed magnetic surface.

$$aspect\ ratio\ A = \frac{R}{a} = \frac{major\ radius}{\min radius};$$

$$beta\ toroidal\ \beta_T \approx \frac{n\,T}{B_T^{\ 2}};$$

$$beta\ poloidal\ \beta_p \approx \frac{nT}{B_{p^2}} = \frac{n\,T\,a^2}{I_P^{\ 2}};$$

$$rho\ star\ toroidal\ \rho_i *_T \approx \frac{V_{T_i}}{a * \omega_c} = \frac{(MT)^{1/2}}{a\ B_T};$$

$$rho\ star\ poloidal\ \rho_{i_P} \approx \frac{V_{T_i}}{a * \omega_{c_{Pi}}} = \frac{(MT)^{1/2}}{I_P}$$

$$q = safety\ factor \approx \frac{R\ B_T}{A^2\ I_P}$$

$$v * collisiona\ lity \approx n\ R\ q\ T^{-2}\ A^{3/2}$$

TableI – Dimensionless parameters used to derive scaling laws. ,

n = plasma density,T = plasma temperature,

$B_{T,P}$ *= toroidal (poloidal) magnetic field,*

I_P *= plasma current,*

$\omega_{cT\,i}/2\pi$ *=ion toroidal cyclotron fraquency*

$\omega_{c_{Pi}}/2\pi$ *= ion poloidal cyclotron frequency)*

In the following the scaling laws will be obtained in a first instance NOT using the energy confinement scalings , after the comparison between the dependences of P_{input} and P_{heat} (which depends upon the scaling of the energy confinement time) is commented separately .

It is worth noticing that (*rigorously*) in the confinement scaling law the net power Pnet must be included instead of the heating power, the net power being defined by :

$P_{net}=P_{source} - P_{RAD}$

Where $P_{source} = P_{alpha} + P_{ohmic} + P_{heating}$. ; $P_{RAD} = P_{Bremsstrahlung} + P_{synchrotron} + P_{line-core}$.

The Pnet~Pheat is valid only in the conditions where the alpha power (P_{alpha}), ohmic power (P_{ohmic}) and radiation losses(P_{RAD}) are negligible, with respect to the heating power. These conditions are quite restrictive if it is considered that the fusion reactor will work in the opposite regime where the radiation is a substantial part of the alpha power and both (alpha and radiation losses) are definitely higher than the heating power. So all (and only) the consequences of the use of the ITER IPB(y,2) and other scaling laws in this paper must be considered having in mind these (strong) limitations.

The focus of the paper is the (strong) dependence of the physics quantities from the aspect ratio. This feature is independent from the scaling law of the confinement used.

All the mass scalings and aspect ratio scalings discussed in this paper arise through their appearances in the four basic dimensionless quantities. However aspect ratio could occur independently of this, through magnetic geometry effects (e.g. on ballooning stability) and trapped particle fraction for example. In fact, we have assumed that for the core plasma the aspect ratio dependency arises only from the four dimensionless parameters, we have therefore neglected the effect of e.g. trapped particles and ballooning modes. This is a reasonable assumption for low/medium beta/beta’ plasmas. We have extended Kadomtsev principle (see ref.1 eq. 5) to include A as an explicit dimensionless parameter while Kadomtsev invariance ensures that the plasma physics of two plasmas having same (q, ρ^*_T , ν^*, β_T) is invariant.

To show how the mass dependence appears from a different dimensionless parameter , the Mach number can be introduced as parameter describing the state of a plasma and the related scaling laws can be derived .

As said before, the bulk plasma similarity depends on aspect ratio from the dependency of the four dimensionless parameters however, we have included the balooning modes in the pedestal stability, see for example hypothesis 2 (sec.2.4). We have highlighted the physics context where the aspect ratio is important. In the pedestal similarity the ballooning stability depends on the aspect ratio while the core plasma is Kadomtsev invariant.

The concept of the scaling developed so far , gives equal weight to the set of dimensionless parameters chosen to describe the plasma status . The concept of **partial similarity scaling** could be introduced in addition,where to each dimensionless parameter a weight can be given. This weight can be used to parametrize the strength of dependence of the plasma status from the dimensionless parameters. For example instead of taking fixed the set (q, ρ^*_T , ν^*, β_T) we could take fixed the set (q, ρ^*_T , $C_\nu\ \nu^*$, $C_\rho\ \beta_T$). The parameters (C_ν , C_ρ , both ≤ 1) can be used to see the level of sensitivity of the scaling to the related dimensionless parameter. A discussion of this new concept will be inserted in sec.2.

The extension of this methodology to burning plasma of a fusion reactor is addressed in this paper as well. The Kadomtsev scheme is derived under the hypothesis that alpha particle and atomic physics effects are negligible: of course this is not the case of fusion reactor plasmas where alpha particle effects are dominant. In this context two sets of physical conditions are taken as starting points; i) a reactor with fixed energy gain factor $Q=Q_0$, operating in H-mode (the alpha power P_α higher than the L-H mode power threshold P_{LH}) , and where the slowing down time of the alpha particles (τ_{SD}) is shorter than the energy confinement time (τ_E); ii) a reactor with fixed energy gain , operating in H-mode, with(τ_{SD})<~(τ_E) , and where the plasma radiation (P_{BR}) is of the order of the alpha power ($P_\alpha \approx P_{Br}$) and $P_\alpha - P_{Br} > P_{LH}$.

The paper is organized as follows: in sec.2 the scaling laws useful to test physics hypothesis limited to bulk plasma confinement(hyp1), pedestal confinement(hyp2) and MHD stability and beta limits(hyp3) are derived ; the concept of weak or partial scaling is introduced and discussed ; in sec.3 notes on scaling for edge similarity are outlined; in sec.3.1. the scaling laws obtained including the Mach number are derived ; in sec.4 general trends detected in the scaling laws obtained in sections 2-4 are summarized ; in sec.5 physics based scaling laws for tokamak fusion reactors are derived; in sec.6 the main results are discussed. In particular: i) the similarity parameters obtained are used in determining a ITER-similar device at low aspect ratio (A=2.5) and magnetic field B=6.5T; ii) the JET plasma parameters of a similarity experiment between JET and JT-60SA are outlined as well as the similarity parameters between TCV and MAST-U; iii) the parameters of a Q~1 neutron source are derived in two possible variants (tokamaks with A=3 and a low aspect ratio device); in sec.7 the conclusions are presented.

2.Derivation of the scaling laws

2.1.Scaling laws for bulk plasma (hyp1)

We assume that bulk plasma confinement can be described by the toroidal beta, collisionality, normalized ion Larmor radius and safety factor (q, ρ^*_T , ν^*, β_T). This scheme is denoted in sec.7 as Kadomtsev-Lackner scaling[1-5,12,16]. This choice has been used in studies of core transport similarity between JET and JT-60U[7] in particular for the optimized scenario (monotonic magnetic shear) and also for some pedestal identity study[8]. We suppose of making similarity experiments where the **dimensionless parameters** (q, ρ^*_T , ν^*, β_T) **given in Table I are fixed** , the derived scaling of the dimensional plasma physics quantities (n, T , Ip, B, P_{input}) upon aspect ratio A , ion mass M and major radius R obtained is reported in Table II (Hyp1)..

Hyp1 bulk plasma similarity (q, ρ^*_T , ν^*, β_T) fixed	Hyp2 pedestal similarity (β_p, ν^*, ρ^*_P, A q) fixed	Hyp3 MHD stability (q, β_T,ρ^*_P,ν^*) fixed.	edge similarity (ρ^*_T,T,ν^*,q) fixed
n =M R^{-2} A^2 T =$M^{½}$ $R^{-1/2}$ $A^{7/4}$. Ip =$M^{¾}$ $R^{-1/4}$ $A^{-1/8}$. B =$M^{¾}$ $R^{-5/4}$ $A^{15/8}$. P_{input}=$M^{7/4}$ $R^{-3/4}$ $A^{29/8}$.	n=M R^{-2} A^2 T=$M^{1/2}$ $R^{-1/2}$ $A^{5/4}$ Ip= $M^{3/4}R^{-1/4}$ $A^{5/8}$. B= $M^{3/4}R^{-5/4}A^{13/8}$. P_{input}=$M^{7/4}$ $R^{-3/4}$ $A^{23/8}$.	n = M R^{-2} A^2 T = M $R^{-1/2}$ $A^{7/4}$ Ip = M $R^{1/4}$ $A^{-1/8}$ B = M $R^{-3/4}$ $A^{15/8}$. P_{input} = $M^{7/4}$ $R^{-3/4}$ $A^{29/8}$.	n = R^{-1} $A^{-3/2}$. T constant Ip=A^{-1} $M^{1/2}$. B= R^{-1} A $M^{1/2}$. P_{input} = R $A^{-5/2}$.

Table II – Scaling laws for the bulk plasma(Hyp1) , pedestal similarity(Hyp2) , MHD stability (Hyp3) and edge similarity .

It's useful to remark that: i) the scalings in Table II are the 'engineering' expression of the realization of similar bulk plasmas with the same dimensionless (q, ρ^*_T , ν^*, β_T) values; ii) the scaling laws are NOT depending from the energy confinement : they are a general expressions which take into account only the **Kadomtsev constraint[1,4] that the plasma state can be described by dimensionless parameters** , and the plasma confinement depends upon these dimensionless parameters. **The dependences upon the aspect ratio are a consequence of the Kadomtsev constraint** and definitions of Table I.

The equivalent tokamak family in terms of transport properties has the following new scaling parameter (see Tab. II) :

$$S_K \approx R B^{4/5} A^{-3/2} M^{-3/5} = a B^{4/5} A^{-1/2} M^{-3/5} \approx I A^{-1/2} M^{-3/5}$$

$$I \approx a B^{4/5} \approx aB$$

The new scaling parameter S_K includes the mass M and the aspect ratio A : this dependence derives from the hypothesis that the transport properties are preserved taking fixed the dimensionless parameter set (q, ρ^*_T , ν^*, β_T).
This means that discharges with the same confinement properties can be obtained decreasing both the aspect ratio and the plasma current at fixed ion mass and consequently:

$$\frac{\Delta I}{I} = \frac{1}{2}\frac{\Delta A}{A}$$

It must be stressed that the invariance of the transport properties does not means the invariance of the MHD properties , for example , or the invariance of the pedestal properties, which can be seen also in the context of MHD stability .

To get some insight on the consequences of these dependences we could consider similarity experiments between devices in two different conditions with constant set (q, ρ^*_T , ν^*, β_T) :

I) e*qual major radius, same ion mass but different aspect ratio.*

Moving from low to high aspect ratio all the plasma parameters must be increased (*only the plasma current Ip remain nearly constant, see Table II*) : the input power and magnetic field must be increased by $\Delta P_{input}/P_{input}$=36% and $\Delta B/B$=18.7% for an increase of aspect ratio of $\Delta A/A$=10%. A device with relatively low aspect ratio (A<3) has the characteristic of operating at the same (q, ρ^*_T , ν^*, β_T) values , having the same confinement properties , at a substantially lower magnetic field and heating power.

ii) *equal major radius and aspect ratio, changing the ion mass .*

Moving from deuterium (M=2) to D-T (50%-50%) discharges , i.e. Meff=2.5, implies **increasing all the plasma parameters** , and plasma input: in particular the plasma current and magnetic field must be increased by ~19%, the input power by 43%.

2.2.Partial or weak similarity scaling for bulk plasma

The partial or weak similarity scaling can be expressed by the following statement : We assume that bulk plasma confinement can be described by the toroidal beta, collisionality, normalized ion Larmor radius and safety factor and we require that the similarity be partially fulfilled in the scaled experiment for beta and collisionality (for example) . We model the partial similarity introducing the coefficients (C_ν, C_β) for the beta and collisionality scaling, and we derive the scaling laws keeping fixed the values (q, ρ^*_T , $C_\nu\, \nu^*$, $C_\beta\, \beta_T$).

The new set of scaling laws is easily obtained and it is shown in Table III.

$n = (1/C_\beta)\, M\, R^{-2}\, A^{2}$
$T = (C_\nu/C_\beta)^{1/2}\, M^{1/2}\, R^{-1/2}\, A^{7/4}.$
$B = (C_\nu/C_\beta)^{1/4}\, M^{3/4}\, R^{-5/4}\, A^{15/8}.$
$Ip = ((C_\nu/C_\beta)^{1/4})\, M^{3/4}\, R^{-1/4}\, A^{-3/8}.$
$P_{input} = (C_\nu^{3/4}\ C_\beta^{-7/4})\, M^{7/4}\, R^{-3/4}\, A^{29/8}.$

Table III – Scaling for bulk plasma partial similarity,
(q, ρ^*_T , $C_\nu\, \nu^*$, $C_\beta\, \beta_T$) fixed.

From the Table III we can see that full similarity ($C_\nu = C_\beta = 1$) corresponds to the values of Table II. The scaled plasma parameters (B and I) are 'slowly' sensible only to the **ratio** of the partial similarity parameters. This means that if we require i) partial similarity equal for beta and collisionality (i.e. $C_\nu = C_\beta$) only the input power is affected , and ii) full similarity (C_ν=1)for collisionaliy and partial similarity (C_β <1) for beta allowing a variation of the order of 20% of the beta value (i.e. C_β =0.8) the plasma parameters (B and I) will be affected by a deviation of 5%, with respect to the exact similarity. (This approximately happens also if, instead, we require full similarity for beta C_β =1 and partial similarity for collisionality C_ν =0.8).

The input power instead will be affected by 48%. This result makes quite clear how critical is the determination of the input power in the scaled experiments.

A more interesting case is when the partial similarity is considered for ρ^*_T and beta : in this case ($C_\rho \rho^*$,$C_\beta\, \beta$, ν^*,q) are fixed . The Table IIIa shows the new partial scaling laws .

$n = C_\rho M R^{-2} A^2$

$T = (C_\rho)^{1/2} M^{1/2} R^{-1/2} A^{5/4}$.

$B = (C_\rho)^{3/4} (C_\beta)^{1/2} M^{3/4} R^{-5/4} A^{15/8}$.

$Ip = (C_\rho)^{3/4} (C_\beta)^{1/2} M^{3/4} R^{-1/4} A^{-1/8}$.

$P_{input} = (C_\rho^{7/4}) M^{7/4} R^{-3/4} A^{29/8}$.

Table IIIa – Scaling for bulk plasma partial similarity
($C_\rho \rho^*$, $C_\beta\ \beta$, ν^*, q) are fixed

The Table IIIa tells us that if we tolerate a deviation from similarity of ρ^* by 30% (C_ρ=0.7) and a quasi exact similarity in beta (C_β=0.9) , the plasma parameters (B,Ip) are affected by 30% deviation from the value defined for exact similarity.

The concept of partial similarity brings to evidence the fact that (at least for bulk plasma similarity) there is no separate sensitivity to β and ν* : in Tab.III the plasma parameters Ip, B and T depend upon the ratio (C_ν/ C_β). In practice for these parameters , the level of similarity cannot be regulated independently for β and ν*. This reveals a limit of the Kadomtsev approach : the dimensionless variables β and ν* are 'not hortogonal observables' for describing a plasma state. This limit has been discussed already in the literature, in particular in the context of the scaling laws of confinement of Spherical Tokamaks [20-23].

2.3.Determination of the heating power for bulk plasma similarity experiments using confinement scaling laws.

The derivation of the heating power scaling laws can be done taking as reference the energy confinement time scaling laws: the IPB(y,2) and the ESGB (Electrostatic-Gyrobohm) [13-14] . The ESPB is derived specifically from JET experiments while the IPB(y,2) results from the tokamak international database. The expressions taken as reference in this paper are (only the main dependences from the plasma quantities are reported, k=elongation):

$$\tau IPB(y,2) \approx Ip \quad P^{-0.69} \quad n^{2/5} \quad a^{3/5} \quad R^{7/5} \quad M^{1/5} \quad k^{4/5}. \quad B^{1/5}. \tag{1}$$

$$\tau ESGB \approx Ip^{4/5} P^{-0.55} n^{1/2} \quad a^{1/3} \quad R^{9/5} \quad M^{0.14}. k^{3/4}. B^{0.07} \tag{2}$$

The formulas (1) and (2) are obtained from the ones reported in ref. 14 rounding the exponents . The power P in (1) and (2) is the Ploss (lost power, MW) , Ip (MA) the plasma current, a (m) the minor radius, R(m) the major radius, M the ion mass, k the elongation, n) $10^{19}m^{-3}$) the plasma density, B(T) the magnetic field. In particular the eq.1 corresponds to eq.5 in ref.14, while the eq.2 corresponds to eq.12 in ref.14.

In terms of dimensionless variables the two scaling laws are given by the following expressions[14] (B is the magnetic field) :

$$B^*\tau IPB(y,2) \approx \rho^{*-2.7} \beta^{-0.90} \nu^{*-0.01} M^{0.96} q_{95}^{-3.0} A^{-0.73} k^{-2.3}. \tag{1'}$$

$$B^*\tau ESGB \approx \rho^{*-3} \beta^{0.0} \nu^{*-0.14} q_{95}^{-1.7} \tag{2'}$$

The expression (1') corresponds to eq.6 in ref.14 and to eq.21 in the ITER Physics Basis ref 6, while the eq.(2') to eq.13 in ref.14. Both expressions include the correct complete dependence on the ion mass (see also ref.6 eq.21). Including the explicit dependence upon the ion cyclotron frequency on the left hand side of (1') ad (2') , would change the dependence upon the ion mass as follows :

$$\Omega ci * \tau IPB(y,2) \approx M^{0.46}.$$

$$\Omega ci * \tau ESGB \approx M^{-1}.$$

In the context of the scaling laws for plasma confinement , specific analysis was carried out to include the spherical tokamaks(ST) data (NSTX, MAST, START) in the ITER database

[20-23] to reveal additional dependence of the confinement scaling law on the aspect ratio. The conclusion was that the dependence on the aspect ratio included in the ITER IPB(y,2) scaling (while not describing entirely the dynamics of ST) is inside the range of dependences compatible with the ST data (see Table 2b ref.23).

The additional heating scaling laws for bulk plasma similarity can be obtained noting that (see sec.1) Pheating = plasma energy(Wth)/confinement time(τE) ≈ Ploss = plasma losses . Inserting the scaling laws given in Table II in the expression of the heating power derived using the formula $Wth/\tau E = n\ T\ R^3\ A^{-2}\ k\ /\tau E$, we get the following scaling laws for the heating:

$$P_{heatingIPB(y,2)}=R^{-0.64}\ A^{4.19}\ M^{0}\ \ k^{\ 0.645} \quad (3)$$

$$P_{heatingESGB}=R^{-1.08}A^{2.33}M^{0.43}k^{\ 0.55}. \quad (4)$$

Inspecting the formulas (3) and (4) a dependence of $P_{heatingESGB}$ upon the major radius and ion mass stronger than in the $P_{heatingIPB(y,2)}$ is found. *The different strong dependences of the heating powers upon the aspect ratio can also be noted.*In practice at the same major radius , ion mass, and plasma elongation, the ratio $P_{heating\ IPB}(y,2)/P_{heatingESGB}$ is given by:

$P_{heating\ IPB}(y,2)/P_{heatingESGB} = A^{\ 1.86}$.

the heating needed for the similarity experiments is relatively lower using the ESGB scaling law with respect to IPB(y,2) when we move from low to high aspect ratio.

It is interesting to compare the formulas (3) and (4) with the P_{input} given in Table II : the result can give a measurement of the effect of the physics of confinement on the energy flux crossing the separatrix, (the $P_{heating}$ has been evaluated supposing $P_{heating}$~P_{loss}).

For example the ratio $P_{heatingIPB(y,2)}/P_{input}$.= $R^{0.1}\ A^{0.56}\ M^{-7/4}\ k^{0.36}$. exhibits an opposite trend (with respect to the dependence upon the aspect ratio A) of the ratio $P_{heatingESGB}/P_{input}$.=$R^{-0.32}$ $A^{-1.29}\ M^{-0.366}\ k^{-0.54}$. This means that, from the IPB(y,2) scaling law of confinement, the lost power increases moderately while increasing the aspect ratio, while the *contrary* happens from the ESGB point of view.

2.4. Scaling laws for pedestal and ELM dynamics(hyp2)

The pedestal width (Δw) has been recently characterized [9] by the scaling with the beta poloidal ($\Delta w \approx \beta_P^{1/2}$). The bootstrap current fraction Ibs/Ip depends on the beta poloidal , Ibs/Ip$\approx$ $A^{-1/2}$ β_P, while the ELM dynamics has a strong dependence upon the pedestal collisionality[10]. The MHD stability of the pedestal is regulated by the stability of peeling-ballooning modes. In the (s,α) diagram [17] the stability of ballooning modes is regulated by the parameter A q . This suggests of taking as dimensionless parameters for pedestal similarity the beta poloidal, the poloidal Larmor radius, the pedestal collisionality as well as the A q : (β_p, ν^*, ρ^*_P, A q, fixed). The derived scaling laws are given in Table II(Hyp2).

Comparing the scalings in Tables II(Hyp2) and II(Hyp1) we see a different behaviour in the aspect ratio for the plasma current and magnetic field :i) the plasma current in the scaled experiment for pedestal ($I \sim A^{5/8}$) is more sensitive to the aspect ratio with respect to the bulk plasma scaled one($I \sim A^{-1/8}$) ; ii) the magnetic field in the scaled experiment for pedestal ($B \sim A^{13/8}$) is less sensitive to the aspect ratio with respect to the bulk plasma scaled one ($B \sim A^{15/8}$).

2.5. *Scaling law for MHD stability and beta limit(hyp3).*

The beta limit and MHD stability can be characterized by the beta toroidal or normal and by the poloidal normalized Larmor radius which is a scale length linked to the pedestal pressure gradient and its stability. Taking the set (q,β_T,ρ^*_P,ν^*) as the set of dimensionless parameters to be fixed, the scaling obtained is shown in Table II(Hyp3). Using this scheme, to test the MHD stability and beta limit, the magnetic field and plasma current must be increased by $\Delta B/B$=18.7% and $\Delta Ip/Ip$=-1% for an increase of aspect ratio of $\Delta A/A$=10% (at fixed major radius and isotopic mass).

3.Edge similarity

Following ref.[5] the plasma edge region differs from core since atomic physics effects play an important role. Therefore assuming that binary collisions are dominant, the temperature(T) can be assumed as an important parameter for similarity while beta being quite low can be neglected. In this case the set of parameters kept constant in the similarity can be (ρ^*_T,T, ν^*,q). The scaling obtained is shown in Table II. In this case the $P_{input}=P_{heatsep}\approx nT^{3/2} R^2 A^{-1}$ is the heating flux through the separatrix: to be noted that the similarity parameter $P_{heatsep}/R$ has some dependence upon the aspect ratio.The Greenwald density nG= $I/a^2 \approx A M^{1/2} R^{-2}$ and the Power threshold for L-H transition[11] ($P_{thrL\text{-}H} \approx n^{3/4} BR^2$) scales as $P_{thrL\text{-}H} \approx R^{1/4} M^{0.5} A^{-0.125}$.

3.1.Derivation of scaling laws including Mach number.

Introducing the Mach number (Mc) as an observable describing the plasma state can be interesting because its dependence on plasma parameters is similar to ρ* . In this view the change of the scaling laws can be of limited effects . We would like to derive the plasma parameters for a scaled plasma taking fixed the four parameters (ρ^*_P,Mc, ν*,q) .
The dependences of Mc from the plasma parameters is Mc= A $(M\ T)^{1/2}$ /(R B).
The following scaling laws are obtained :

$n = R^{-2}\ M$

$T=R^{-1/2}\ A^{3/4}.\ M^{1/2}.$

$I=R^{-1/4}\ A^{-5/8}\ M^{3/4}.$

$B=R^{-5/4}\ A^{11/8}\ M^{3/4}.$

Table IV-Scaling laws for Mach number similarity.

The scaling laws shown in TableIV are similar to that obtained for hyp2 (see Tab.II) at least as the mass dependence is concerned, the dependences on the aspect ratio are specific to this hypothesis.

4. General trends detected in the scaling laws .

Strong dependence upon the aspect ratio is derived on similarity for bulk , pedestal , MHD stability and edge plasmas (see Table II). Regarding aspect ratio, equal trends are found between hyp1 and hyp3, while remarkable differences between hyp1 and hyp2 are shown in fig.1: the largest effect is found on the plasma current . In relation to the ion isotopic mass strong differences are found between hyp1 and hyp3, see fig.2: in the context of hyp3 moving from deuterium to tritium the plasma current and magnetic field must be increased by 50%. Regarding the dependence of plasma current on the major radius, the hyp1 and hyp3 show opposite behaviour, while for the magnetic field there is an expected decrease with major radius which is stronger for hyp3, see fig.3. Similarity experiments between devices with equal major radius, at fixed ion mass , must be planned carefully , because moving from low to high A, the heating power must be increased substantially(see Table VIII, formulas (3) and (4) and fig.4). For edge similarity a comparison of the behaviour versus aspect ratio (at fixed R and M) between hyp1 and edge is given in fig.5.

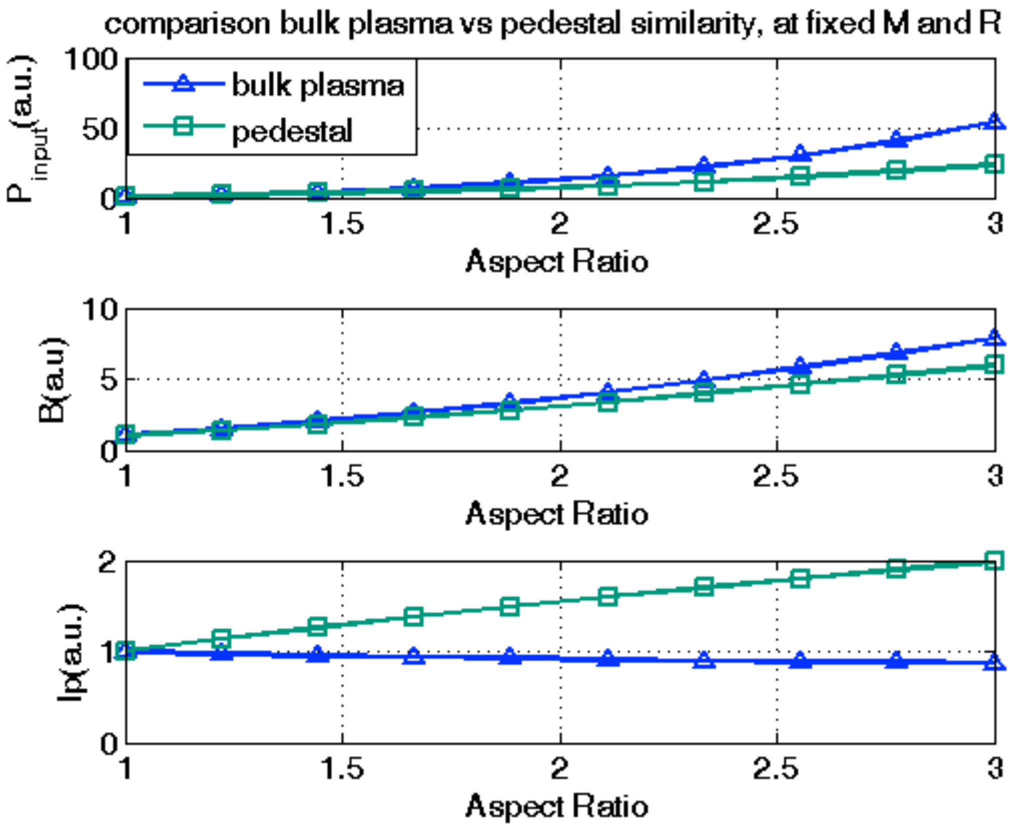

Fig.1. Aspect ratio dependences of main plasma parameters for bulk plasma and pedestal similarity .

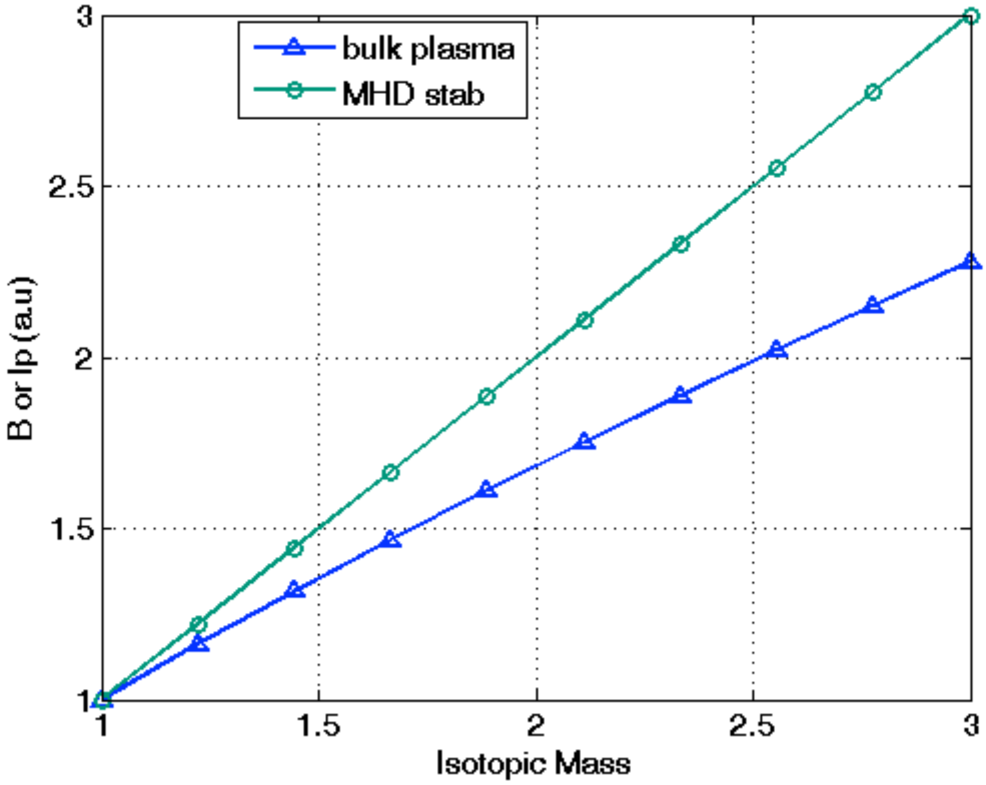

Fig.2. Plasma parameters (B and Ip) vs the isotopic mass for bulk plasma and MHD stability similarity.

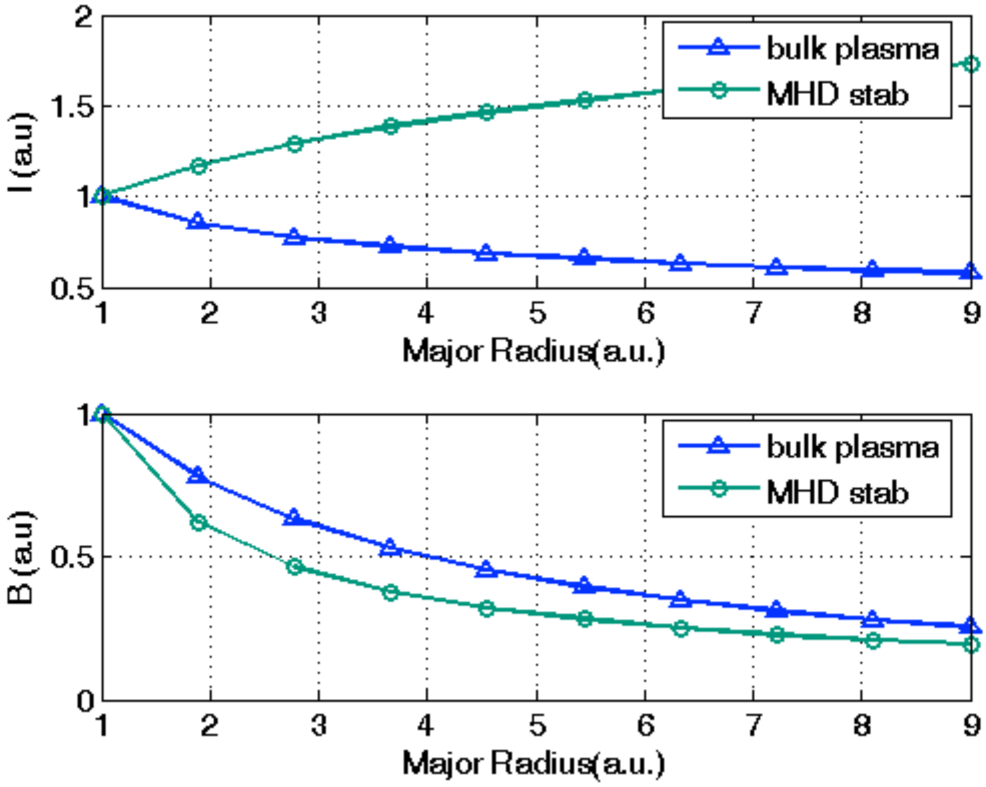

Fig.3 Plasma current and magnetic field vs major radius for bulk plasma and MHD stability.

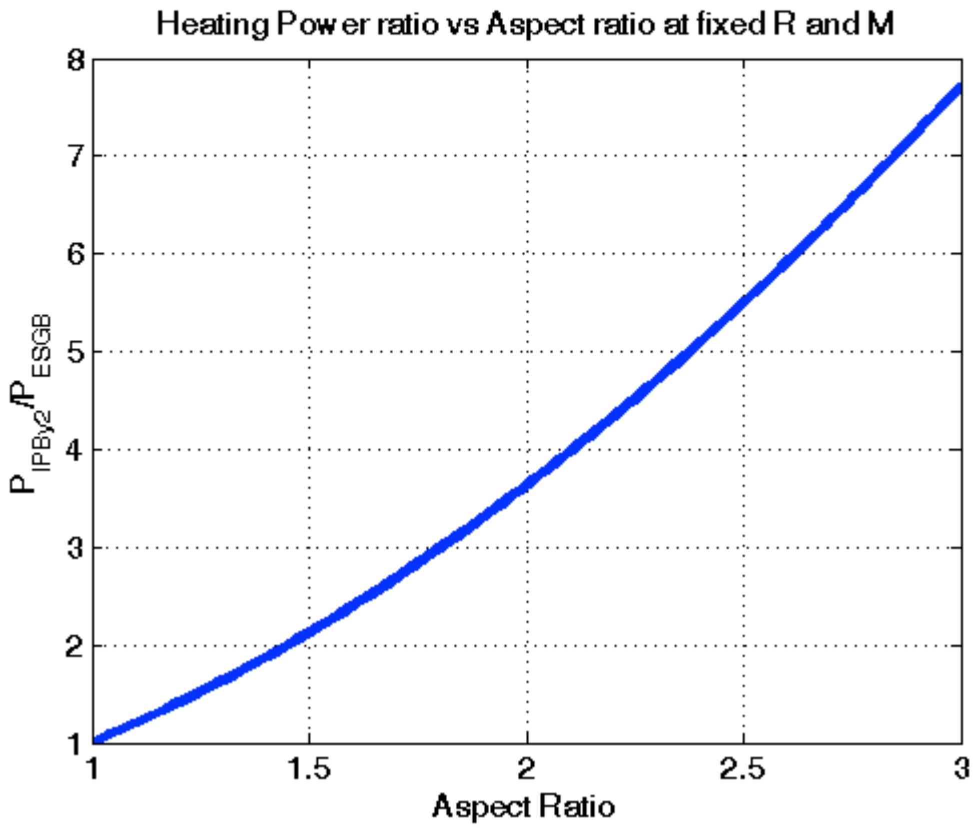

Fig.4 The Ratio $_{PIPB(y,2)}/P_{ESGB}$ vs Aspect ratio at fixed major radius and isotopic mass.

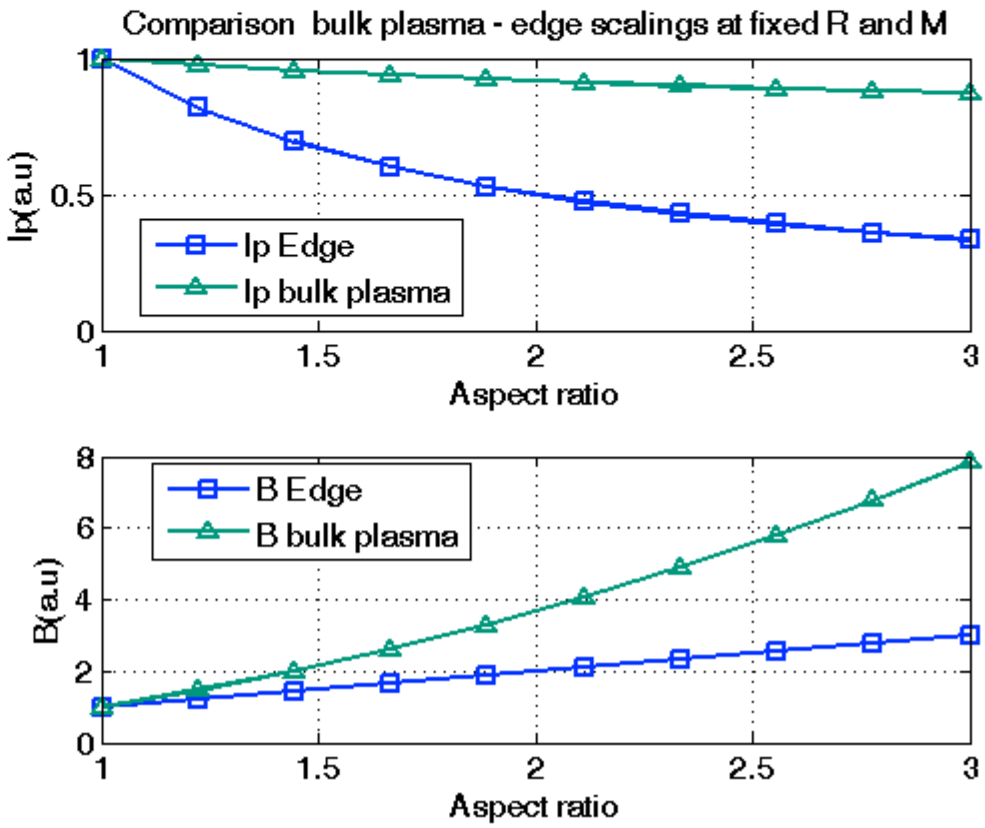

Fig.5 Plasma parameters (Ip and B) vs Aspect ratio for Bulk plasma and Edge similarity

5.Physics based scaling laws for tokamak fusion reactors .

The extension of the methodology outlined in the previous sections to burning plasmas[16] is very interesting because the alpha particles power (P_α) must be considered in the power balance as dominant heating; the power gain factor Q=Pfus/Pin and the alpha (classical) slowing down time (τ_{SD}.) must be introduced also as parameters defining the plasma state. The parameters specific of the alpha particles physics related to the ion transport and interaction with MHD modes and turbulence are $\beta_\alpha \sim n_\alpha E_\alpha/B^2$, and the ratio V_α/V_{Alfven}.

The Kadomtsev scheme is derived under the hypothesis that alpha particle and atomic physics effects are negligible: of course **this is not the case of fusion reactor plasmas** where alpha particle effects are dominant.

We can consider two sets of conditions :

i) first set : $Q=Q_0$ fixed (~30) , $\tau_{SD}.\sim\Lambda_{SD}\ \tau_E$ ($\Lambda_{SD}<<1$) , $P\alpha=\Lambda_{th}\ P_{LH}$ ($\Lambda_{th}>\sim1.5$)

where τ_E is the confinement time, P_{LH} the L-H threshold power , and Λ_{SD} and Λ_{th} are numbers used to define order of magnitudes ;

ii) second set : $Q=Q_0$ fixed , $\tau_{SD}.\sim\Lambda_{SD}\ \tau_E$ ($\Lambda_{SD}<<1$) , $P\alpha\sim Prad$ (>>PLH)

where Prad is (mainly) the Bremsstrahlung radiation.

The first set of conditions means that the reactor works in H-mode , where we fix the fusion gain factor at a value Q_0, the alpha slowing down time much less than the confinement time , and the alpha power higher definitely than the L-H power threshold.

The condition $\tau_{SD}\sim\Lambda_{SD}\ \tau_E$ ($\Lambda_{SD}<<1$) is consistent with the usual assumption that the alpha particle beta(β_α) must be smaller than the thermal plasma beta (β_{th}). In fact if we define (n_α, E_α , P_α, alpha particle density, energy , and power respectively):

$\beta_\alpha<\beta_{th}$ ($n_\alpha E_\alpha / B^2 < nT /B^2$.) and approximate $n_\alpha E_\alpha\sim P_\alpha \tau_{SD}$ and $nT\sim P_{loss}\ \tau E$

from the condition $P_\alpha\sim P_{loss}$, we obtain $\tau_{SD}< \tau_E$.

In Appendix A1 the detailed calculations related to the first set of conditions are carried out leading to the following scaling:

$$R \sim C_R Q_0^{1.61} M^{1/2} A^{5/4} B^{-1.42} . \sim C_R Q_0^{8/5} M^{1/2} A^{5/4} B^{-7/5} \quad (5)$$

The slow dependence of gain factor on the major radius, implied by eq.5, is in some agreement with the formulas reported in ref.25.The eq.5 implies a strong dependence on the magnetic field and aspect ratio also , which is not reported in ref.25.

The similarity parameter corresponding to a fusion reactor is S_{FR}=R $B^{7/5}$ $A^{-5/4}$ which is different with respect to the Kadomtsev similarity parameter S_K=R $B^{4/5}A^{-3/2}$.(see Table II). The conditions related to the fusion reactor , i.e. burning plasma (first set of conditions), lead to a strong effect on the similarity parameter , giving more importance to the value of the magnetic field.

The second set means that the reactor is working at a fixed Q_0 gain factor, in conditions where the fusion power is of the order of the plasma radiation and still in H-mode : such condition is often considered (see ref. 19) necessary to keep the power load on the divertor lower than the limit of 10-15MW/m^2.

In Appendix A2 the calculations related to the second set of conditions is reported , the final result is the following scaling :

$$S_{HR} = R\ B\ A^{-7/4}. \qquad (6)$$

6.Discussion

6.1.Similarity parameters and ITER-similar devices

In this section few notes will be outlined on the main results of sec.6 (scaling lows for fusion reactors) , comparing them with the Kadomtsev scaling laws obtained in sec.2.

The first point to be noted is that the Kadomtsev similarity parameter given in TableII

$$S_K = R B^{4/5} M^{-3/5} A^{-3/2} \qquad (7)$$

must be replaced in the context of fusion reactor scaling laws by the new scaling parameter (see eq.A1.22):

$$S_{FR} = R B^{1.3} M^{-0.22} A^{-1.23} \qquad \sim R\ B^{4/3}\ M^{-1/5}\ A^{-6/5} \qquad (8)$$

The expression (8) includes the hypothesis that the scaling is done at fixed fusion gain Q_0 and safety factor q (see eq.A1.22).

The scaling parameter for the highly radiating fusion reactor scenario is given by the following expression:

$$S_{HR} = R\ B\ \ A^{-7/4} \qquad (9)$$

In the similarity parameters the aspect ratio plays a strong role , balancing the effect of the magnetic field and the device dimensions. In this respect the aspect ratio can be considered an additional free parameter for the design optimization of a fusion reactor.

The evaluation of the parameters of a device similar to ITER (whose parameters are R=6.2m,B=5.2T,A=3.1, M=2.5) , but working at B=6.5T and A=2.5 leads to the following values :

$$R_K = 3.756m; R_{FR} = 3.560m; R_{HR} = 3.40m \qquad (10)$$

where R_K is the major radius obtained using the Kadomtsev scaling , R_{FR} that obtained using the scaling for fusion reactor first set , and R_{HR} the major radius obtained using the second set of hypothesis for fusion reactor scaling laws.

If we take as reference R_{FR} , a device with major radius 22% longer than JT60SA (whose parameters are R=2.9m, B=2.5T,A=2.5), with a magnetic field 2.6 times higher than JT60SA is the similarity brother of ITER.

6.2.Similarity experiments between JET and JT60SA

One can ask whether the similarity parameters derived in sec.2 (i.e. the s_K) can be used to design similarity experiments between JET and JT60SA . As we've noted the aspect ratio A is a 'new' free parameter in the scaling law and the JET aspect ratio is A_JET=3.1 , while the JT60SA aspect ratio is A_SA=2.5-2.6 (depending on the scenario) .

For example is it possible to design JET experiments in hydrogen where the similarity with JT60SA can be exploited?

An answer to this question is positive provided the JET magnetic field is B_JET=1.95T , in the scenarios of JT60SA where the magnetic field is B_SA=2.28T.

So similarity experiments between JET and JT60SA can be done when JET main isotope is hydrogen and the JET magnetic field is B_JET=1.95T .

If we take as reference the JT60SA hybrid scenario (#4-2) , where the plasma parameters of JT60SA are (Ip=3.5MA,B=2.28T,R=2.93m,A=2.6), the plasma current on JET is IP_JET=2MA, to make a similarity experiment on JET in a hydrogen discharge.

Similar parameters are obtained if we want to make a similarity experiment between JET(in hydrogen) and JT60SA scenario inductive #4.1 where the plasma current if IP(JT60SA)=4.5MA: the JET plasma parameters in hydrogen would be B_JET=1.95T and IP_JET=2.6MA. A summary of the results is given in Tab.V where also the JT60SA scenario 5#2 (high β_N) is included.

	JT60SA	JT60SA	JT60SA	
	4#1	4#2	5#2	
	M=2 Deut	M=2 Deut	M=2 Deut	
IP_JET	2,68	2	1,22	M=1 Hyd
B_JET	1,95	1,95	1,35	M=1 Hyd

Table V- JET plasma parameters for Similarity experiments JET –JT60SA

6.2.1.Similarity experiments between TCV and MAST-U.

MAST-U and TCV share approximately the same major radius (see Tab VI) , but different aspect ratio. Both devices can be used to explore high beta regimes . It is natural to ask

whether similarity experiments can be done and which scaling laws can be used to link the plasma parameters .

	R0(m)	a(m)	A	B(T)	Ip(MA)
	major radius	minor radius	R0/a, aspect ratio	magnetic field on axis	plasma current
TCV	0.88	0.25	3.52	1.5	1
MAST-U	0.85	0.65	1.3	0.6	1.5

Table VI . Parameters of TCV and MAST Upgrade

The scaling law for similarity experiments at fixed (q, ρ^*_T , ν^*, β_T) are (see Table II):

$B=A^{15/8}$

$Ip=A^{-1/8}$

Therefore, the parameters of the TCV high beta experiments B_TCV=1.4T ,

Ip_TCV=150kA , should correspond to B_MAST-U=0.21T and Ip_MAST-U=170kA.

6.3. Design of Q~1 neutron sources and Q=10 ITER-like devices.

The possibility of obtaining parameters for Q=1 devices can be explored using the scaling laws (see eq.54) in the following way. We can take as reference the JET DTE1 discharges [18] and applying the scaling laws we obtain the parameters of a class of Q~1 devices, see fig.6.

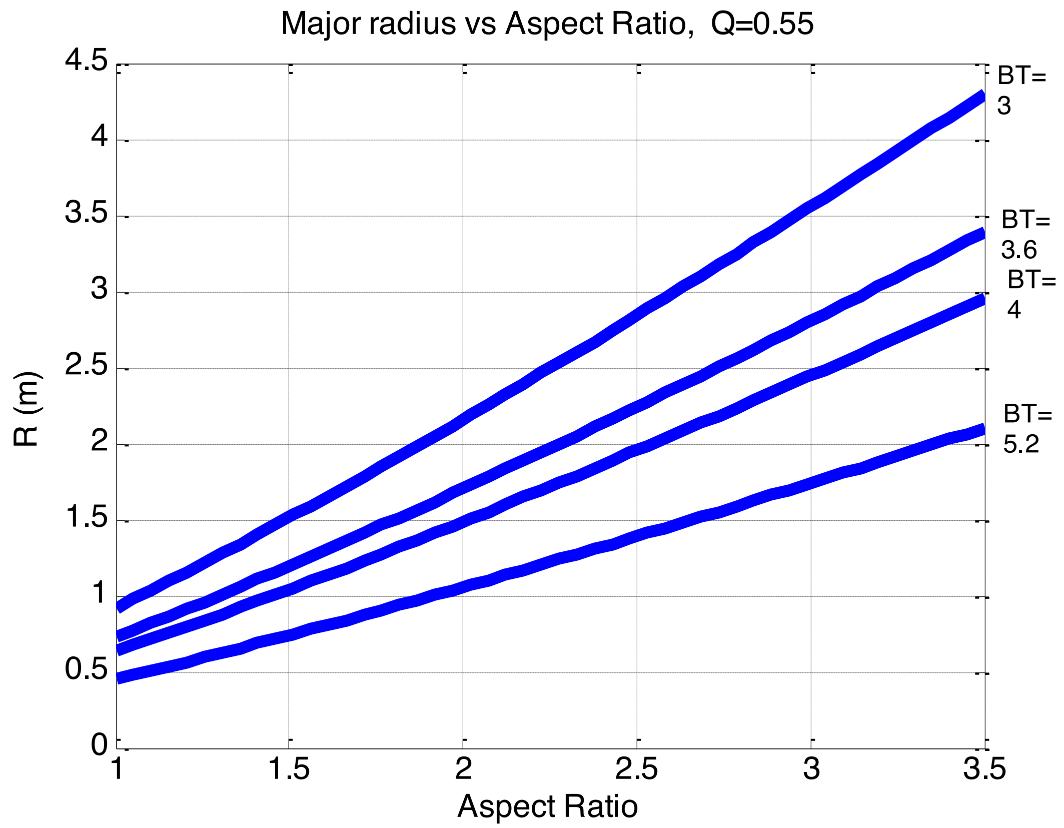

Fig.6. Design Parameters for tokamaks with Q=0.55 fusion gain .

The fig.6 shows that parameters of Q=0.55 device can realized by :

i) A standard aspect ratio tokamak R=2.9, A=3.1 at a magnetic field on axis B=3.6T (JET-DTE1)
ii) A standard aspect ratio tokamak R=1.75m , A=3 at a magnetic field on axis B=5.2T
iii) A low aspect ratio tokamak R=1.5m , A=1.5 at a magnetic field B=3.T

The same method can be applied for the determination of the ITER-like equivalent device: in this case the ITER parameters (Q_0=10) are taken as reference (see ref.6), and the parameters of devices equivalent to ITER are obtained by inspecting the fig.7. For example parameters for a ITER-like device are :

i) tokamak JT60SA-like R~4.83m, A~2.5, and B~5.2T

ii) low aspect ratio tokamak R~4.23m, A=1.5, B=3.6T.

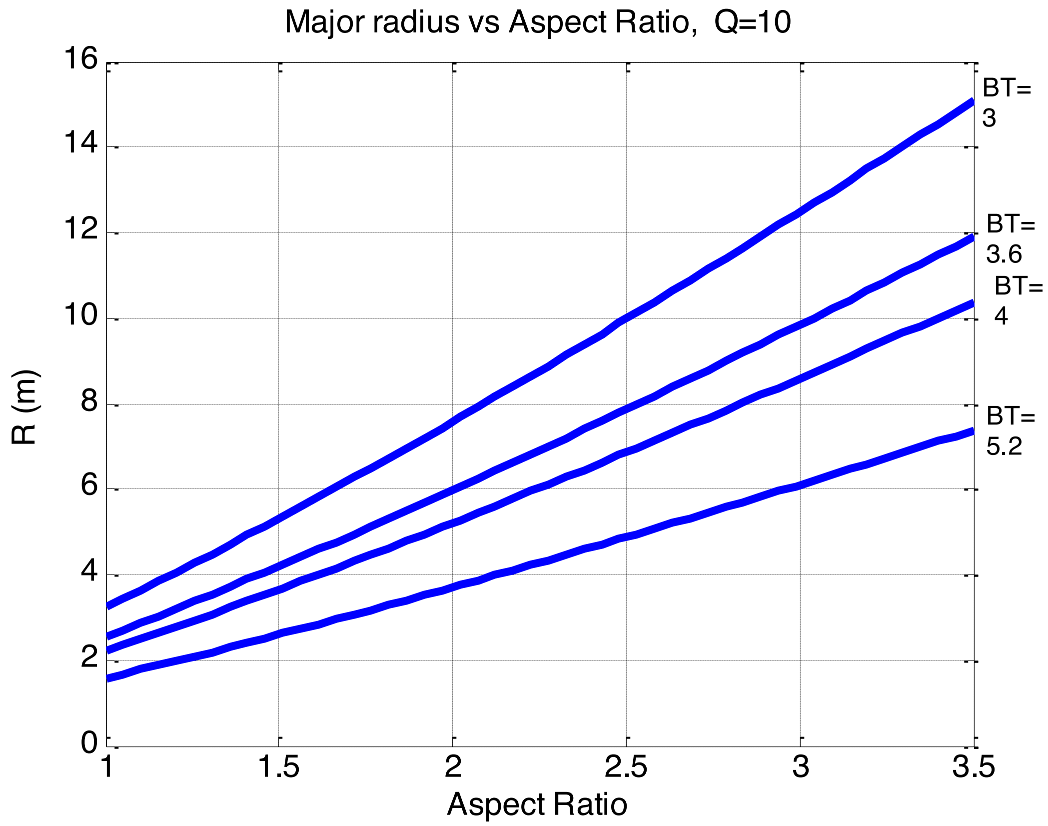

Fig.7 . Design Parameters for tokamaks with Q=10 fusion gain .

7.Conclusions .

The paper starts introducing the aspect ratio in the Kadomtsev similarity scheme, and it makes a step forward in the definition of a complete set of similarity parameters in non-burning plasmas Kadomtsev scheme. The result is that there is strong sensitivity of similarity parameters to the aspect ratio.

The second part of the paper is devoted to an extension of the similarity scheme to fusion reactor plasmas with two variants : i) fixed Q burning plasmas and ii) highly radiating , fixed Q burning plasmas.

In the case of fusion reactor plasmas the alpha particle heating is dominant , and the plasma state is defined starting from the fusion reactivity. The scaling laws for fusion reactor plasmas can be expressed in terms of dimensionless parameters as well leading to a sort of the extension of the Kadomtsev method to the fusion reactor plasmas.

The main result consists in the fact that the similarity parameters of burning plasmas , while obtained following a path quite different from the physics point of view , are NOT strongly dissimilar from the Kadomtsev non-burning plasma similarity parameters.

The dependence of the similarity parameters from the aspect ratio allows for feasible plasma parameters similarity experiments between JET in hydrogen and JT60SA in deuterium plasma: JET plasma parameters in hydrogen are B=2T, $1.2<Ip<2.7$MA.

The possibility of determining a ITER-similar device at low aspect ratio and (not so) high magnetic field is also explored , by means of the generalized scaling parameter , leading to a device parameter set :

R=3.4-3.7m ; B=6.5T ; A=2.5

The scaling laws for fusion reactor can be used also for determining the parameters of a Q~1 neutron source . A low aspect ratio device with R=1.5m , A=1.5 , B=3.0T is derived scaling down the JET DTE1 parameters. Following the same method a ITER-like low aspect ratio tokamak R~4.2m, A=1.5, B=3.6T can be considered.

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Appendix A1.Analysis of the first set of conditions.

The confinement time is given by the ITER IPB(y,2) scaling which is expressed in terms of dimensionless variables by the expression :

$\tau E \sim \tau B\ \rho^{*\,-0.7}\ \beta^{-0.9}\ \nu^{*\,-0.01}\ M^{0.96}\ A^{-0.73}\ ka^{2.3}\ q^{-3}$.

τB is the Bohm diffusion time $\sim A^{-2}\ R^2\ B\ T^{-1} = M\ \rho^{*\,-2}\ B^{-1}$; ka the plasma elongation .

The confinement time ITER IPB(y,2) can then be put in the form :

$\tau E \sim M^{1.96}\ A^{-0.73}\ ka^{2.3}\ q^{-3}\ B^{-1}\ \beta^{-1}\ \rho^{*\,-2.7}\ \beta^{0.1}\ \nu^{*\,-0.01}$.

In the following analysis we'll use a generalized form of τE :

$\tau E \sim A_{\tau E}\ B^{-1}\ \beta^{-1}\ \rho^{*\alpha}\ \beta^{\varepsilon b}\ \nu^{*\,\varepsilon n}$. (A1.1)

The ITER IPB(y,2) scaling law for H-mode corresponds to :

$\alpha H=-2.7,\ \varepsilon bH=0.1, \varepsilon nH=-0.01$ (A1.2)

$A\tau EH = M^{1.96}\ A^{-0.73}\ ka^{2.3}\ q^{-3}$ (A1.2a)

while for the L-mode confinement scaling law we have the following parameters :

$A\tau EL = M^{1.67}\ A^{0.09}\ ka^{3.22}\ q^{-3.74.}$ (A1.3)

$\alpha L=-1.85,\ \varepsilon bL=-0.41, \varepsilon nL=0.19$

The equation $Q=Q_0$ becomes:

$Q=n\ T\ \tau E = \beta\ B^2\ A_{\tau E}\ B^{-1}\ \beta^{-1}\ \rho^{*\alpha}\ \beta^{\varepsilon b}\ \nu^{*\,\varepsilon n}. = A_{\tau E}\ B\ \rho^{*\alpha}\ \beta^{\varepsilon b}\ \nu^{*\,\varepsilon n}. =Q_0$. (A1.4)

The eq.A1.4 has the following meaning : **devices with the same Q_0 can be realized only changing scaling the $\rho^* \sim B^{1/\alpha} \sim B^{-1/3}$.** This statement is valid for H-mode and neglecting the dependences of the confinement time upon β and ν^*,(see eqs.A1.2).

Now, recalling the definition of $\rho^* \sim (T\,M)^{1/2}\ B^{-1}\ R^{-1}\ A$, this means that the relation between the major radius and magnetic field would be (keeping temperature and isotopic composition constant):

$$R=[\ (T\,M)^{1/2}\ *\ (Q_0/A_{\tau E})^{1/3}]\ A\ \ B^{-2/3} \qquad \sim A^{5/4}\ B^{-2/3}\ . \qquad (A1.5)$$

The dependence $A^{3/4}$ is valid for H-mode in the expression of $A_{\tau E}$. The same Q_0 can be realized at lower radius decreasing the aspect ratio and increasing the magnetic field, keeping fixed the temperature and ion mass, the value of q and the elongation. A device equivalent to ITER (Q_0=10) can be realized having R=4.15m , B=6.5T, Aspect ratio A=2.5 and operating in H-mode.

Considering a device operating in L-mode, the eq.A1.5 becomes:

$$R=[\ (T\,M)^{1/2}\ *\ (Q_0/A_{\tau E})^{1/3}]\ A\ \ B^{-1/2} \qquad \sim A^{\,0.97}\ B^{-1/2}\ . \qquad (A1.6)$$

and in this case(L-mode) a device equivalent to ITER (Q_0=10) can be realized increasing the major radius to R=4.54m, for a magnetic field B=6.5T and aspect ratio A=2.5.

Moving to the equation

$$\tau_{SD}.\sim\Lambda_{SD}\ \ \tau_E \qquad (A1.7)$$

we need to transform both sides of eq.16 in function of the dimensionless parameters .

The slowing down time is

$$\tau_{SD}.\sim A_{st}\ T^{\,3/2}\ /n\ \ . \qquad (A1.8)$$

From the definitions of β and ν^* we can write the temperature and density in function of these quantities, and recalling the condition $\tau_{SD}.\sim\Lambda_{SD}\ \ \tau_E$ (Λ_{SD}<<1, a number) , we obtain :

$$A_{st}\,\beta^{-1/6}\ \nu^{*\,-5/6}\,B^{-1/3}\,A^{\,5/4}\,(q\,R)^{\,5/6} = \Lambda_{sd}\,A_{E\tau}\,B^{-1}\,\beta^{-1}\,\rho^{*\,\alpha}\,\beta^{\,\varepsilon b}\,\nu^{*\,\varepsilon n} \qquad (A1.9)$$

Now inserting (A1.4) in (A1.9) we obtain the following expression :

$$\frac{\beta}{\nu^*} = \left[\frac{\Lambda_{sd}}{A_{st}\,Q_0}\right]^{6/5} A^{-3/2}\ q^{-1}\ \frac{1}{R\,B^2} \qquad (A1.10)$$

We now consider the condition

$P_\alpha = \Lambda_{th}\, P_{LH}$ (Λ_{th}>~1.5) (A1.11)

This condition leads to a decoupling of β and ν*. We now develop the expression (A1.11).

The threshold L-H power scaling law[24] is

$$P_{LH} = A_{lh}\, B\, n^{3/4}\, R^2 \qquad \text{(A1.12)}$$

The expression for P_{LH} can be derived in function of β and ν*. Inserting the expression obtained in this way , in eq.A1.11 (alpha power higher than the L-H power threshold) we obtain:

$$P_\alpha = f_\alpha\, \beta^2\, B^4\, R^3 / A^2 = \Lambda_{lh}\, P_{LH} = \Lambda_{lh}\, A_{lh}\, B^2\, R^{7/4}\, q^{-1/4} A^{-3/8}\, \nu*^{1/4}\, \beta^{1/2} \qquad \text{(A1.13)}$$

f_α includes the plasma dilution and some geometry.

From the eq.A1.13 we can deduce a formula linking β and ν*. Now using the eq.A1.10, we obtain an expression for ν*:

$$\nu* = \left[\frac{A_{lh}\,\Lambda_{lh}}{f_\alpha}\right]^{4/5} \left(\frac{A_{st}\,Q_0}{\Lambda_{sd}}\right)^{6/5} B^{4/5}\, R^{1/5}\, A^{31/10}\, q \qquad \text{(A1.14)}$$

and a formula for β:

$$\beta = \left[\frac{\Lambda_{sd}}{A_{st}\,Q_0}\right]^{1/5} \left(\frac{A_{lh}\,\Lambda_{lh}}{f_\alpha}\right)^{4/5} A^{16/10}\, R^{-4/5}\, B^{-6/5} \qquad \text{(A1.15)}$$

The previous expressions are valid for a device operating in H-mode(ITER IPB(y,2) scaling) and depend upon: i) the form of the scaling law assumed for the threshold power for the L-H transition; ii) the conditions (A1.4) and (A1.7); iii) there is no assumption on the confinement time scaling law.

To get the ρ*, we need to resume the condition (A1.4) :

$$\rho* = \left(\frac{Q_0}{A_{\tau E}}\right)^{1/\alpha} \beta^{-\varepsilon b/\alpha}\, \nu*^{-\varepsilon n/\alpha}\, B^{-1/\alpha} \qquad \text{(A1.16)}$$

Inserting the expressions (A1.14) and (A1.15) in the eq.A1.16 we obtain the formula for ρ*:

$$\rho* = A_\rho\, q^{\varepsilon q}\, A^{\varepsilon a}\, R^{\varepsilon R}\, B^{\varepsilon B} \qquad \text{(A1.17)}$$

where

$$\varepsilon q = \frac{\varepsilon b - 4\varepsilon n}{5\alpha} \; ; \; \varepsilon a = -\frac{13\varepsilon b + 28\varepsilon n}{10\alpha} \; ; \; \varepsilon R = -\frac{\varepsilon n - 4\varepsilon b}{5\alpha} \; ; \; \varepsilon B = \frac{-4\varepsilon n + 9\varepsilon b - 5}{5\alpha} \qquad \text{(A1.18)}$$

The values of the exponents in (A1.18), for the ITER IPB(y,2) scaling law of confinement time are given by :

$\varepsilon q=-0.02$; $\varepsilon a= -0.04$; $\varepsilon R=-0.03$; $\varepsilon B=0.3$ (A1.19)

Inserting the values (A1.19) in the formula (A1.17) we get an expression for ρ*:

$\rho^* = A_\rho \; B^{0.3}$ (A1.20)

$A_\rho=Q_0^{-1/2.7} \; A_{\tau E}^{\;1/2.7}$. (A1.21)

$A_{\tau E}$ is given in eq.(A1.2a). The expression (A1.20) depends upon the condition (A1.4), i.e. fixed fusion gain factor and ITER IPB(y,2) H-mode scaling law. On the other side ρ* can be expressed using eq.A1.10 and its definition (see Tab.I) ,

$$\rho^* = \left[\frac{\beta}{\nu^*}\right]^{1/6} M^{1/2} A^{5/4} q^{1/6} (B R^{5/4})^{-2/3} = \left(\frac{\Lambda_{sd}}{A_{st}\, Q_0}\right)^{1/5} A M^{1/2} \frac{1}{B R} \qquad \text{(A1.21)}$$

Using eqs.(A1.20) and (A1.21), we obtain the main dependence of the major radius upon the magnetic field :

$$R = \left(\frac{\Lambda_{sd}}{A_{st}}\right)^{1/5} Q_0^{\;8/15} M^{-0.22} ka^{-0.85} A^{1.23} \; q^{1.13} \; B^{-1.3} \qquad \text{(A1.22)}$$

Comparing the expression (A1.22) with the formula (A1.5), we observe that the dependence upon the magnetic field is stronger in eq.A1.22 with respect to eq.A1.5. This means that a device similar to ITER at low aspect ratio A=2.5 and higher magnetic field B=6.5T will have a major radius R=3.7m , at fixed Q_0 , q and elongation ka (instead of R=4.1m as evaluated using (A1.5)).

Appendix A2.Analysis of the second set of conditions.

The second set of conditions is :

$Q = Q_0$

$\tau_{SD} = A_{SD}\,\tau_E$

$$P_\alpha = P_{Br} >> P_{LH} \;;\quad \frac{P_\alpha}{P_{LH}} >> 1 \;;\; \frac{P_{Br}}{P_{LH}} >> 1 \qquad \text{(A2.1)}$$

$$P_\alpha - P_{Br} = \Lambda P_{LH}\;; \qquad \Lambda = 1.5$$

The alpha power is of the order of the plasma radiation , and alpha power and plasma radiation power are definitely higher than the L-H power threshold. The plasma is operating in H-mode, while it is radiating a power of the order of a significant part of fusion power.

The first two eqs. in (A2.1) are already known and elaborated in the Appendix A1.

We start developing the ratios included in eq.A2.1, using for the plasma radiation the Bremsstrahlung expression :

$P_{Br} = f_{Br}\, Z_{eff}\; \beta^2\, B^4\, T^{-3/2}\, R^3\, A^{-2}$. (A2.2)

Inserting the eq.A2.2 (where the plasma temperature is expressed in terms of dimensionless variables) in the fourth eq. in A2.1 and applying the conditions $P_\alpha/P_{LH}>>1$, which means that

$$\frac{P_\alpha}{\Lambda P_{LH}} \approx \frac{P_\alpha}{\Lambda P_{LH}} \qquad \text{(A2.3)}$$

we get

$$\frac{f_{Br}\, Zeff}{f_\alpha} = q^{1/2}\, A^{3/4}\, B\, \beta^{5/2}\, \nu *^{1/2} \qquad \text{(A2.4)}$$

The eq.A2.4 gives an important result : if we fix β , ν* and q , and the aspect ratio A , the magnetic field is in practice fixed by the values of the dimensionless variables; as it is shown by the following expression , derived from the eq.A2.4:

$$B \approx \frac{fBr\,Zeff}{f\alpha}\, q^{-1/2}\ \beta^{5/2}\ \nu^{*-1/2}\ A^{-3/4} \qquad \text{(A2.5)}$$

Relaxing the constraint on the aspect ratio, *we can lower the magnetic field increasing the aspect ratio A of the reactor.*

This means that a plasma scenario with high radiation has strict operating windows determined by the values of the dimensionless parameters and by the dilution and impurities. The scaling variable for the high radiation scenario (ν^* and β are fixed) can be considered :

$$B\ A^{3/4} = \text{constant} \qquad \text{(A2.6)}$$

Inserting the eq.A2.6 in the ratio $P\alpha /(\Lambda\, P_{LH})$, we obtain an explicit expression linking the major radius and the dimensionless parameters :

$$P\alpha /(\Lambda\, P_{LH}) \sim q^{-1}\ A^{-(25/8)}\ \nu^{*-(5/4)}\ \beta^{13/2}\ R^{5/4}. \qquad \text{(A2.7)}$$

From the expression A2.7 we can extract the scaling of the major radius upon the aspect ratio (the value of the ratio $P\alpha /(\Lambda\, P_{LH})$, ν^*, β , q are fixed) :

$$R \sim A^{5/2}\ . \qquad \text{(A2.8)}$$

Collecting together the scalings A2.8 and A2.6 we obtain the following scaling for the high radiation scenario:

$$S_{HR} = B\ R\ A^{-7/4}. \qquad \text{(A2.9)}$$