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Interpretation of electromagnetic modes in the sub-TAE frequency range in JET plasmas with elevated monotonic q-profiles

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Abstract. Recent JET Deuterium experiments with an advanced internal transport barrier (ITB) scenario have been performed to clearly observe destabilised toroidicity-induced Alfvén eigenmodes (TAEs) by fast ions; interestingly, these also exhibit unstable electromagnetic (EM) perturbations in the sub-TAE frequency range. We identify such EM perturbations to be beta-induced ion temperature gradient (BTG) eigenmodes and not beta-induced Alfvén eigenmodes (BAE) nor beta-induced Alfvén acoustic eigenmodes (BAAE) which are often unstable in such high-beta plasma with high power neutral beam injection (NBI). The BTG modes are the most unstable modes due to the high thermal ion temperature gradient related to the ITB and a high ion beta regime. BTG mode experimental characteristics match analytical theory, i.e. location in the vicinity of a rational magnetic surface with a low magnetic shear, mode frequency scaling with the ion drift frequency (ω_i^*), and a coupling among Alfvén, acoustic, and drift waves. We also perform linear gyrokinetic simulations with validated plasma profiles and equilibrium, and find a mode kinetically driven by thermal ions with similar characteristics as the experimental BTG modes.

Keywords: Alfvén-acoustic-drift eigenmodes, Stability, Ion temperature gradient, Ion Landau drive/damping.

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1. Introduction

JET Deuterium experiments aiming to develop an advanced scenario to observe alpha driven toroidicity-induced Alfvén eigenmodes (TAEs) [1] in Deuterium-Tritium (DT) plasmas have been performed with an elevated monotonic safety factor (q) profile, internal transport barrier (ITB), high plasma beta (β) regime, high core thermal ion temperature compared to the thermal electron one (core $T_i \gg$ core $2 * T_e$) and high power of neutral beam injection (NBI). Demonstrating alpha particle drive of Alfvénic instabilities in the forthcoming JET DT phase is key for our understanding of the underlying physics and for the success of future tokamak operation.

During those experiments we not only observed unstable TAEs - driven by ion cyclotron resonance heating (ICRH) fast ions in the absence of DT mixture as fuel - but also electromagnetic (EM) perturbations living in a frequency range below the TAEs which is often associated with the beta-induced gap created by the coupling between acoustic and Alfvén waves. Beta-induced eigenmodes are heavily studied with both experimental and theoretical analyses since they are often considered a source of additional transport of thermal plasma and fast ions, detrimental for current and future fusion devices. Basic physics of such eigenmodes can be found in [2]. In this work we focus on three main candidates for the observed EM perturbations: beta-induced Alfvén eigenmodes (BAE) [3, 4], beta-induced Alfvén acoustic eigenmodes (BAAE) [5] and beta-induced ion temperature gradient eigenmodes (BTG) [6]. BTG mode is an electromagnetic analogue to the well-known - *electrostatic* - ion temperature gradient (ITG) instability [7, 8].

It is worth mentioning a parallel analytical theory to BTG modes by [9–11] studying ion temperature gradient driven Alfvén eigenmodes (AITG), characterising an instability from the coupling of kinetic ballooning modes (KBM) [12] and BAE. While this paper does not focus on comparing the two approaches, one can say that they differ in their treatment of the *inertial layer*. BTG mode analytical theory [6] considers that the ion dynamics can be treated with the assumption of a zero *long-scale* parallel electric field ($\delta E_{parallel}$) and neglect the $\delta E_{parallel}$ connection length fluctuations due to the vector potential; on the other hand AITG mode theory [9,10] does not make this assumption but still demonstrates its validity in the small finite ion Larmor radius (FLR) and finite drift-orbit width (FOW) limit. In [11] the authors go a step further by demonstrating the existence of AITG eigenmodes by including the full ion FLR and FOW effects. Both analytical theories agree on a few conditions of BTG/AITG mode existence such as a positive ion temperature gradient and a low magnetic shear. An important distinction, BTG mode theory has a well-defined analytical criterion on ion beta which needs to be higher than a critical threshold ($\beta_{ion} > \beta_{ic}$). AITG mode theory demonstrates a strong dependence of the AITG mode real frequency with a factor α ($= -R_0 q^2 \frac{\mathrm{d}\beta}{\mathrm{d}r}$, where R_0 is the major radius of the tokamak), i.e. on the magnetic shear and plasma beta which is then compared with the marginal stability boundary of ideal magnetohydrodynamic (MHD) ballooning modes [13]. The AITG mode real frequency scales with ω_i^* and increases when α decreases. AITG modes are also predicted to be driven by the thermal ion temperature gradient and be enhanced when β_i increases. These last two criteria are consistent with the BTG mode theory. For the purpose of this work - to understand the nature and characteristics of the observed EM perturbations and study if such modes can be predicted by analytical theory and reproduced by numerical tools - we then consider the BTG and AITG theories to agree qualitatively on the criteria of existence of beta-induced ion temperature gradient driven eigenmodes, so we focus on BTG mode theory.

Section 2 presents the experimental evidence leading us to consider the EM perturbations to be unstable BTG modes. In Section 3 we find a good agreement between BTG mode analytical theory and experimental observations. The modeling effort to find such BTG modes using linear gyrokinetic simulations with a realistic JET geometry and validated equilibrium and plasma profiles is presented in Section 4. Finally a summary is given in Section 5.

2. Experimental observations

2.1. JET pulse 92054

To study sub-TAE modes we choose JET pulse (JPN) 92054 since it displays clear unstable electromagnetic perturbations below the TAE frequency range as one can see in Fig. 1a. For this pulse and time interval the characteristic TAE frequency in the plasma frame was $f_{TAE} \in [90, 102]\ kHz$. This frequency range is calculated using on-axis values for the densities and magnetic field in $f_{TAE} = V_A/4\pi qR$ with $V_A = B/\sqrt{\mu_0 \sum n_i m_i}$ the Alfvén speed where B is the toroidal magnetic field, $\sum n_i m_i$ the mass density of the plasma and μ_0 the vacuum permeability.

Another reason is that JPN 92054 has been extensively studied in [1] as part of JET experiments to observe alpha-driven instabilities so we are confident in the equilibrium reconstruction, experimental measurements, analysis as well as plasma profiles. In Figure 5 in [1] one can see time traces of the auxiliary power, central electron and ion temperatures, toroidal rotation rate, electron density and neutron rate.

In this work we mainly focus on $6.4s$; Table 1 indicates the plasma parameters at that time slice. q_0 indicates the safety factor at the magnetic axis; one can see the q-profile at $6.4s$ from EFIT reconstruction [16] in Fig. 7 where $q = 2$ is located at $\sqrt{\psi} \sim 0.43$. Note that ion density (n_i) and temperature (T_i) are quoted as ranges instead of a single value to highlight the uncertainties in the experimental measurements which are also reproduced in the TRANSP code [17] simulations; details on the TRANSP simulations can be found in section 6 from [1]. Due to the large error bars in T_i measurements, in this paper we consider two cases: (*low*-T_i) where T_i is chosen to be the lower range of error bars of charge exchange recombination spectroscopy (CXRS) measurements and (*high*-T_i) where T_i is taken to be the experimental value. By keeping measurement uncertainties we conserve a realistic picture of the experiments; how such uncertainties influence our results is discussed throughout the paper. Figure 2 shows the thermal plasma density and temperature profiles for our two

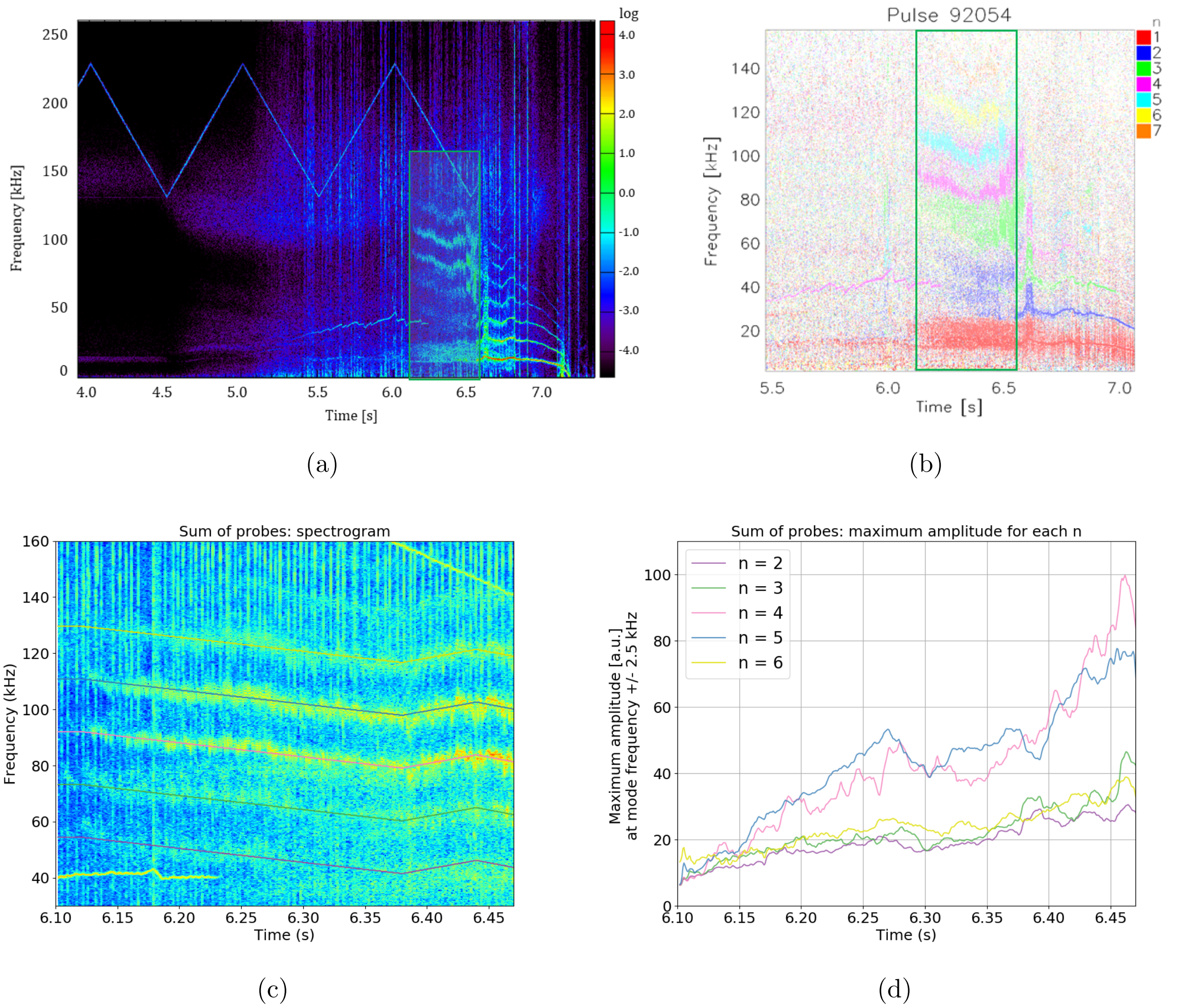

Figure 1: (a) Mirnov coil (H305) spectrogram of poloidal magnetic fluctuation frequency over time. (b) Mode analysis from a set of Mirnov coils analysing the relative phase shift of the fluctuations; the colours denote the toroidal mode numbers n. (c) Magnetic spectrogram considering all available magnetic Mirnov coils, zoomed in on the times of interest; the straight lines indicate the mean frequency (f_{mean}) used to filter and extract the amplitude information for each n. (d) Maximum amplitude in the frequency range $f_{mean} \pm 2.5kHz$ for each n in $[2, 6]$. $n = 1$ and $n = 2$ modes' identification is difficult from the spectrogram. Note that the triangular shape signal on (a) is the JET TAE antenna magnetic perturbation scanning in frequency to resonate with stable plasma modes [14, 15].

cases with the mean between the two cases for the temperature profiles. One can see that the *high-*T_i case has a higher ion temperature gradient than the *low-*T_i case. Densities and electron temperature are similar for both cases.

Another important aspect of JPN 92054 is its high-β regime [18–21] with a normalised beta $\beta_N = \beta_T B_T a / I_P \sim 4.38[\% Tm/MA]$ at $t = 6.4s$ where β_T is the total toroidal beta in percent, B_T the toroidal field, a is the horizontal minor radius in meters and I_p is the plasma

Table 1: Plasma parameters for JET pulse 92054 at 6.4s. Data in the first column is from experimental measurements, second column and beta toroidal are from TRANSP code [17] while the third column is from EFIT reconstruction [16].

Plasma parameters at 6.4s					
$I_{\rm p}$ (MA)	2.67	$n_{\rm e0}$ (10^{19} m^{-3})	5.43	B_0 (T)	3.44
$R_{\rm NT}$ (10^{16} s^{-1})	1.44	$T_{\rm e0}$ (keV)	5.4	q_0	1.86
$P_{\rm NBI}$ (MW)	25.1	$n_{\rm i0}$ (10^{19} m^{-3})	4.80-4.84	R_0 (m)	3.03
$P_{\rm ICRH}$ (MW)	0.00	$T_{\rm i0}$ (keV)	8.9-13.7	V_A (10^6 m.s^{-1})	7.06
Beta Toroidal					
$\beta_T(\%) = 3.67$ — $\beta_{ion}(\%) = 2.00$ — $\beta_{electron}(\%) = 0.95$ — $\beta_{beam}(\%) = 0.72$					

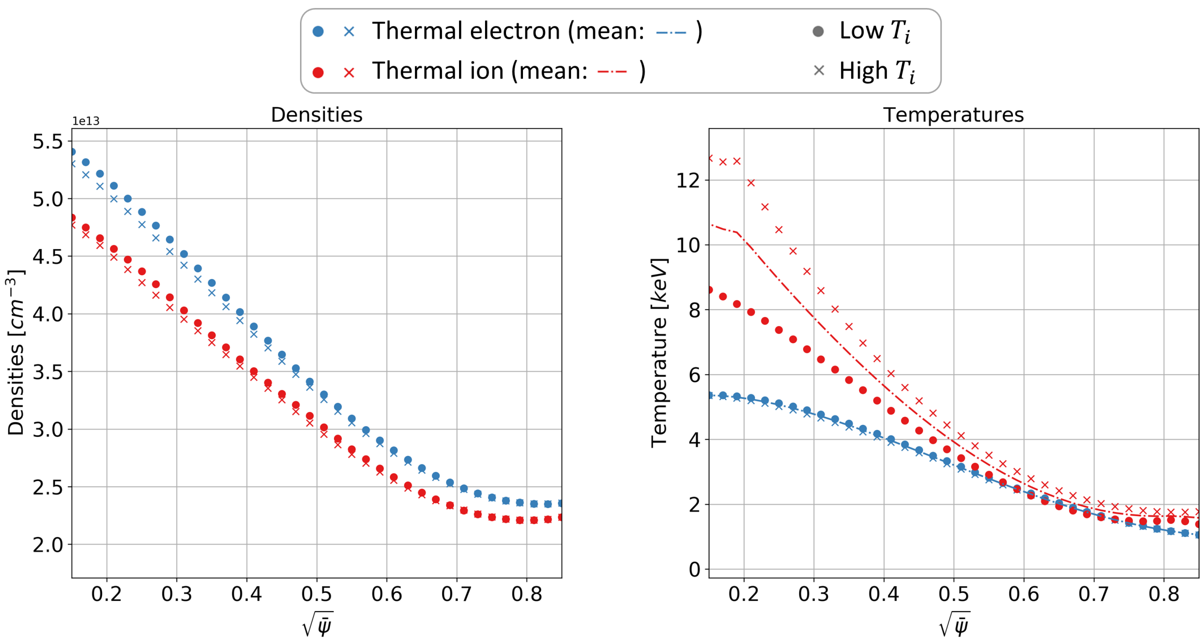

Figure 2: x-axis is the square root of the normalised toroidal flux ($\sqrt{\psi}$). TRANSP thermal plasma profiles: densities and temperatures. (*low-T_i*) is for T_i chosen to be the lower range of error bars of charge exchange recombination spectroscopy (CXRS) measurements ($T_{\rm i0} \sim 8.9keV$) and (*high-T_i*) where T_i is taken to be the experimental value ($T_{\rm i0} \sim 13.0keV$). The dashed line represents the mean between the two cases. Densities and electron temperature are similar for both cases.

current in MA§. Such a regime gives conditions for beta-induced modes to exist such as BAE, BAAE and BTG, candidates for our modes of interest.

JPN 92054 also features a clear internal transport barrier (ITB) associated with the $q = 2$ magnetic surface; both electron and ion temperature profiles exhibit high gradients, ∇T_e and ∇T_i respectively (see section 3 in [1]). The onset time of the modes of interest is near the ITB observation leading us to consider that such temperature gradients could be

§ See Table 1 for actual values

the driving source of these EM modes; this would mean that we are observing unstable BTG modes. The following subsections - Section 2.2 and Section 2.3 - confirm this conjecture.

2.2. Electromagnetic perturbation evidence

Figure 1a represents the magnetic perturbations of the plasma measured at the wall by Mirnov pick-up coils. One can see modes being destabilised between 6.1 and 6.5s from ~ 10 to $\sim 140kHz$ in the lab frame (within the green square). On Fig. 1b the toroidal mode numbers (n) are obtained by making a time-windowed Fourier decomposition of the signals of a set of toroidally separated Mirnov coils and analysing the relative phase shift of the fluctuations; this technique allows to differentiate positive and negative n. $n = 1$ and $n = 2$ modes' identification is difficult from the spectrogram so our study will mainly focus on $n \in [3, 6]$ modes. We cannot extract radial information from Magnetic signals, but this information is obtainable by analysing interferometry, Soft X-Ray (SXR) and/or reflectometry measurements on JET ‖. Unfortunately the last was not used during this JPN 92054, but interferometry and SXR acquired data which show similar perturbations in time and frequencies as the Mirnov coils. Note that both SXR and interferometer diagnostics on JET provide line-integrated data with rather limited radial resolution. The interferometer on JET has four lines of sight, two of them close to the magnetic axis and the two others at the plasma edge (see Figure 1 from [22]); only the two channels looking at the plasma core measured density fluctuations related to the modes of interest. These two channels are on a different side of the magnetic axis, and comparison of the modes signals from these positions shows that the modes are neither ballooning nor anti-ballooning; modes with a single dominant poloidal mode number m are good candidates. SXR has seventeen lines of sights from bottom to top of the plasma; the mode location is crudely estimated to be between $R[m] \in [2.2, 3.8]$ or $\sqrt{\overline{\psi}} \in [0, 0.8]$, with R the major radius and $\overline{\psi}$ the normalised toroidal flux.

To refine the estimation of the modes' location one can compare mode frequencies in the plasma frame with the ones measured in the lab frame by adding the Doppler shift from the toroidal plasma rotation frequency (f_{rot}) such as $f_{lab} = f_{plasma} + nf_{rot}$. To evaluate f_{rot} we used the TRANSP toroidal plasma rotation profile¶; the experimental uncertainties are $\pm\ 0.18kHz$ when calculating the sum of squared differences (SSD) divided by the number of data points. The plasma frame frequencies can be estimated with a linear analytical dispersion relation depending on the nature of the mode. For such EM modes in the sub-TAE frequency range in a high-β plasma, good candidates are BTG, BAE and BAAE.
BTG mode frequency scales with the ion diamagnetic drift frequency (ω_i^*), reported by [23]

‖ On JET, the reflectometer (KG8C) has the highest radial resolution followed by SXR (KJ5) and then the far infrared interferometer (KG1V).

¶ Only the toroidal rotation has been used to estimate the modes' location, considering the poloidal rotation negligible, yet adding some uncertainties to f_{rot}. At $q = 2$: $f_{rot|q=2} \sim 15.38\ kHz$.

as

$$\omega_j^* = -\frac{m}{r}\ \frac{m_j}{\omega_{cj}}\ \frac{T_j}{P_j}\ \frac{dP_j}{dr} = -\frac{nq}{r}\ \frac{c}{Z_j\ e\ B}\ \frac{T_j}{P_j}\ \frac{dP_j}{dr} \quad (1)$$
$$with\ j = i\ for\ ions\ and\ e\ for\ electrons.$$

where m is the poloidal mode number ($m = nq$), r is the minor radius, c the speed of light, T_j the species temperature, Z_j the species charge state, e the elementary charge, B the magnetic field on-axis, ω_{cj} is the species gyrofrequency ($\omega_{cj} = Z_j eB/c\, m_j$), P_j the species pressure and dP_j/dr the species pressure radial gradient.
The frequency of BAE modes follows the frequency of geodesic acoustic modes (GAMs) [24] which is calculated with

$$f_{GAM} = \frac{1}{4\pi^2}\left[\frac{2}{m_i R^2}\left(T_e + \frac{7}{4}T_i\right)\left(1 + \frac{1}{2q^2}\right)\right]\frac{2}{\kappa^2+1} \quad (2)$$

where κ is the plasma flux surface elongation.
The nature of BAAE mode and its frequency in the plasma frame are still discussed, and one can find a clear review of BAAE observations and interpretations in [25]. For the purpose of this work we focus on the BAAE described by MHD [5] which has a frequency following the GAM frequency but shifted downwards by $1/2(q^2+1)$,

$$f_{BAAE|MHD} = \frac{1}{2(q^2+1)}\ f_{GAM} \quad (3)$$

We also compare with recent DIII-D experimental “BAAE” [26] - now called low-frequency modes (LFM) [25] - which scale with diamagnetic drift frequencies with a strong dependence on electrons’ parameters hence the electron diamagnetic drift frequency (ω_e^*, Eq. (1) with $j = e$).

Figure 3 represents the characteristic frequencies of the beta-induced modes previously mentioned. We choose to only use $n = \pm 4$ for diamagnetic frequencies not to overwhelm the figure, but one should remember that modes following diamagnetic frequencies have a toroidal mode number (n) dependence. These frequencies are calculated using mean values of TRANSP profiles (see dashed line in Fig. 2) while error bars represent the experimental uncertainties of thermal plasma densities and temperatures reproduced in TRANSP profiles.

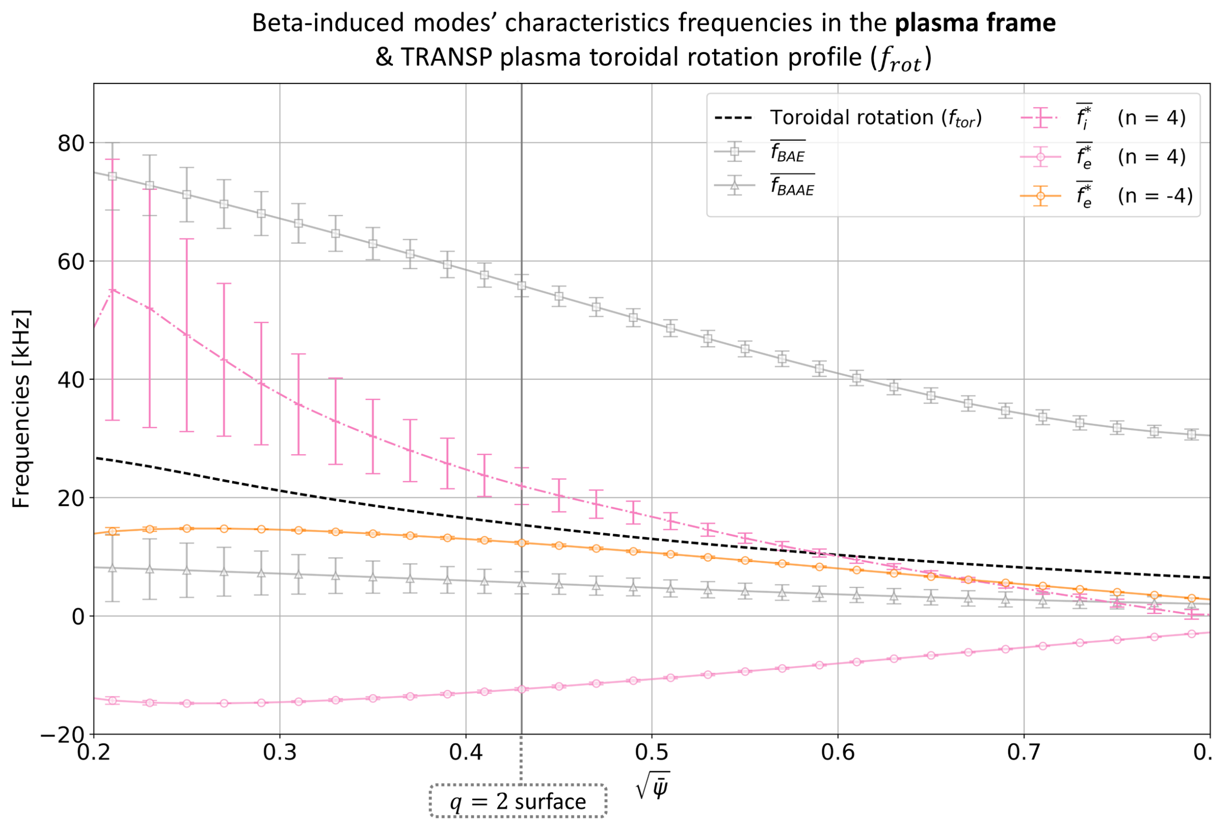

Figure 3: $\sqrt{\bar{\psi}}$ the square root of the normalised toroidal flux. Plasma frame characteristic frequencies with ion diamagnetic frequency (f_i^*) for $n = 4$, BAE/GAM frequency (f_{BAE}), BAAE frequency (f_{BAAE}) and electron diamagnetic frequency (f_e^*) for $n = 4$ and $n = -4$. Toroidal plasma rotation (f_{rot}) profile from TRANSP represented with the black dashed line. Error bars represent the experimental uncertainties on thermal densities and temperatures reproduced in TRANSP profiles. At $q = 2$, $\omega^*_{i\,|n=4} \sim 21.9 \pm 3.1kHz \sim 0.058 \pm 0.008[V_A/R_0]$, $f_{GAM} = f_{BAE} \sim 55.8 \pm 1.9kHz \sim 0.148 \pm 0.005[V_A/R_0]$, $f_{BAAE|MHD} \sim 5.6 \pm 0.2kHz \sim 0.015 \pm 0.001[V_A/R_0]$ and $\omega^*_{e\,|n=4} \sim -12.4 \pm 0.2kHz \sim -0.033 \pm 0.001[V_A/R_0]$.

To compare these frequencies with experimental measurements (Fig. 1b) we then applied the Doppler shift correction using TRANSP toroidal plasma rotation profile, i.e. $f_{lab} = f_{plasma} + nf_{rot}$. The frequencies in the lab frame are then compared with the frequency range for each n obtained from the time-windowed Fourier decomposition of the signals of a set of toroidally separated Mirnov coils. Figure 4 shows the best match with experiment which is for BTG modes; for $n \in [1, 6]$ we plot $f_i^* + nf_{rot}$ with the experimental frequency ranges represented by the shaded horizontal areas, where colors match the respective single n values. Error bars for the frequencies in the lab frame also include the uncertainties from toroidal plasma rotation measurements. The BAE/GAM frequency is too high while the MHD BAAE frequency is too low. The electron diamagnetic drift frequency (ω_e^*) must all have negative n which is against experimental observation of magnetic perturbations with positive n ($n \in [1, 6]$).

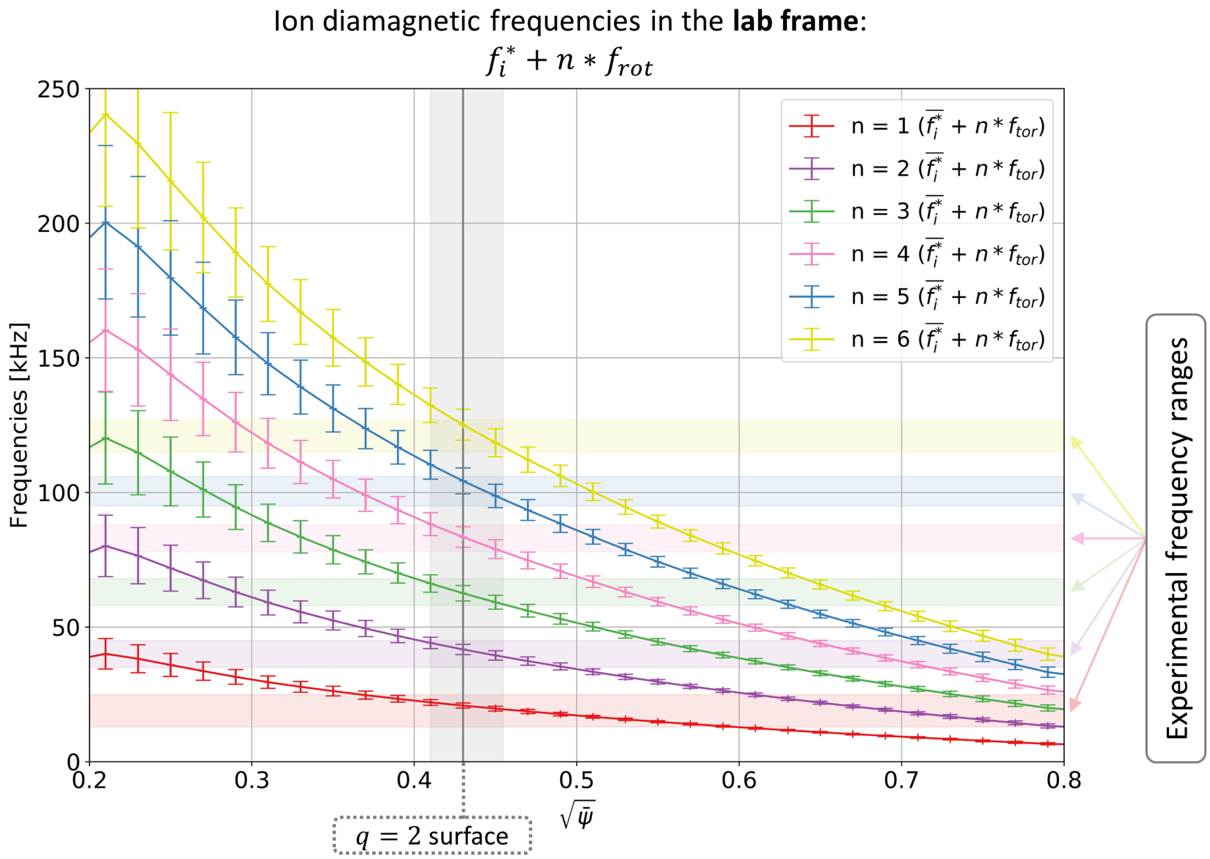

Figure 4: $\sqrt{\psi}$ the square root of the normalised toroidal flux. Ion diamagnetic frequencies (f_i^*) for $n \in [1,6]$ in the lab frame when Doppler shift is taken into account, i.e. $f_i^* + nf_{rot}$ with f_{rot} the toroidal plasma rotation from TRANSP. The shaded horizontal areas represent the experimental frequency range measured from the time-windowed Fourier decomposition of the signals of a toroidal set of Mirnov coils (see Fig. 1b). Note that the frequency ranges are large due to the very few number of coils available and signals' noise. Error bars represent the experimental uncertainties on thermal densities and temperatures reproduced in TRANSP profiles

From this analysis, we can also estimate the BTG modes' location to be around the $q = 2$ surface ($\sqrt{\psi} \in [0.41, 0.45]$) which is consistent with the ITB's formation when this magnetic surface appears (see Section 2.1).

2.3. Electromagnetic perturbation dependencies

To get a better understanding of the nature of the destabilised electromagnetic modes we compared the time evolution of the mode frequency ($\in [6.1, 6.5]s$) with several plasma parameters: n_e, T_e, n_i, T_i, ∇T_i, NBI fast ion density (n_{fi}) and temperature (T_{fi}), plasma pressure (p) and its gradient (p'), poloidal current function (f) and its gradient (f') as well as the Alfvén frequency on axis (f_{Alfven}). For each parameter we investigated the time evolution at the following radial locations: on-axis (q_0) and at $q = 2, 9/4, 10/4, 11/4, 3$ rational surfaces. Good qualitative correlations are found with $1/\sqrt{n_e}$, $1/\sqrt{n_i}$, $1/\sqrt{T_i}$ and $1/\nabla T_i$ at the $q = 2$

surface (Fig. 5) adding more confidence on the modes' location. $1/\sqrt{n_e}$ dependence means an Alfvénic nature of the destabilised modes while $1/\sqrt{T_i}$ is linked to ion sound scaling [27]; these two dependencies are expected for beta-induced modes characterised by a coupling between Alfvén and acoustic waves. We do find a stronger dependence on thermal ions' parameters compared to electrons', and we also find a good qualitative correlation with $1/\nabla T_i$ meaning that the destabilised modes could indeed be driven by the thermal ion temperature gradient, hence being BTG modes.

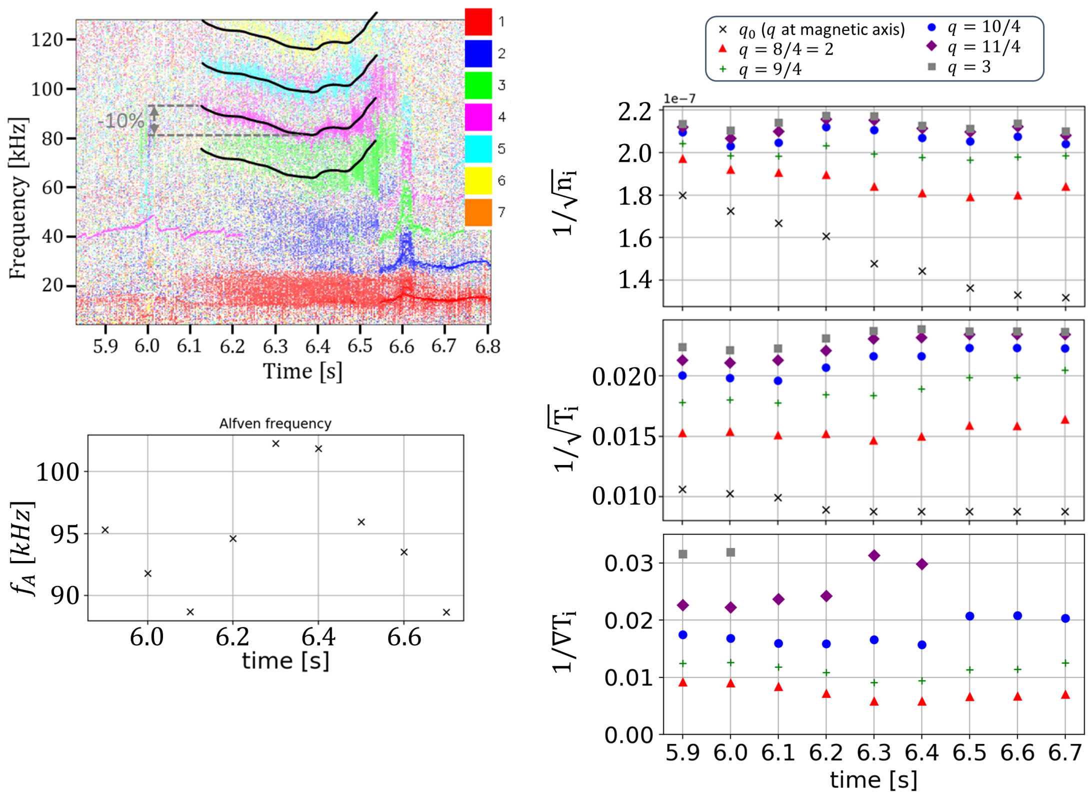

Figure 5: Time traces of the destabilised modes (top left) compared with time traces of $1/\sqrt{n_i}$, $1/\sqrt{T_i}$, f_A and $1/\nabla T_i$ for several rational surfaces (q_0 and $q = 2, 9/4, 10/4, 11/4, 3$). The EM perturbations' frequencies decrease by $\sim 10.1\%$ from $6.1s$ to its minimum at $6.4s$. Qualitative time evolution of the modes match the ones from the ion density and temperature at the $q = 2$ magnetic surface. Between the same times, $1/\sqrt{n_e}$, $1/\sqrt{n_i}$, $1/\sqrt{T_i}$ and $1/\nabla T_i$ at the $q = 2$ surface decrease by $\sim 3.4\%$, $\sim 4.5\%$, $\sim 1.3\%$ and $\sim 18.1\%$ respectively.

The next section focuses on comparing our experimental results with the theoretical conditions for BTG mode to exist and become unstable.

3. Beta-induced ion temperature gradient driven eigenmodes

MHD and kinetic theories of BTG modes have been presented in [28] and [29] respectively; the purpose of this section is **not** to reproduce these analytical theories but to compare them with our experimental observations of JPN 92054 to confirm the correlation between the observed unstable EM perturbations between ~ 6.1 and $6.5s$ and the analytical BTG mode conditions of existence. BTG mode theories predict that above a certain ion beta threshold the drift effects due to the ion temperature gradient can lead to an appearance of unstable coupled Alfvén-acoustic-drift eigenmodes, called BTG, which are localised in the vicinity of a rational magnetic surface ($q(r) = m/n$). Three well-defined conditions need to be fulfilled for BTG mode to exist. We first present these conditions for one time slice, $6.4s$, and for the *high-*T_i profile before focusing on other time slices, within and outside the times of interest ($[6.1, 6.5]s$).
The first BTG mode condition (i) is to have a positive ion temperature gradient:

$$\frac{\partial \ln T_i}{\partial \ln n_i} = \eta_{ion} > 0 \tag{4}$$

Figure 6 shows η for thermal ions and electrons where one can see that condition (i) is verified for $\sqrt{\psi} \in [0.15, 0.90]$.

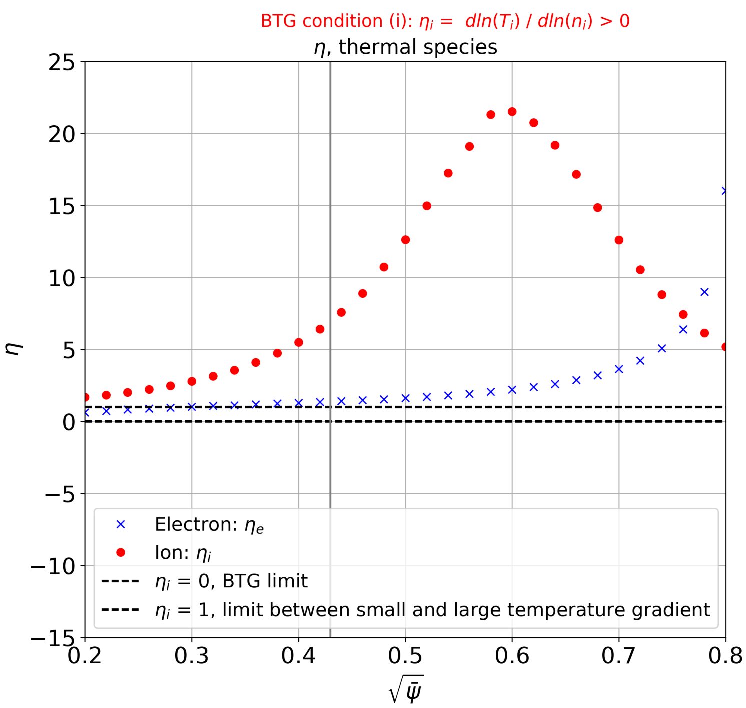

Figure 6: TRANSP thermal species temperature gradient profiles: $\eta = \partial \ln T / \partial \ln n$. For $\sqrt{\psi} \in [0.15, 0.90]$ BTG mode condition (i) ($\eta_{ion} > 0$) is verified. The vertical grey line indicates the position of the $q = 2$ surface.

The second BTG condition (ii) is that ion beta ($\beta_{ion} \cong 8\pi n_i T_i / B_0^2$) overcomes an analytical threshold value (β_{ic}) defined by

$$\beta_{ion} > \beta_{ic} = \frac{9}{2}\frac{q^2 S^2 L_{T_i}^2}{R^2} \tag{5}$$

where S is the magnetic shear, R the major radius of the tokamak and L_{T_i} is the characteristic scale length of the thermal ion temperature inhomogeneity ($L_{T_i} = T_i/\nabla T_i$). Figure 7 shows the ion beta (β_{ion}) versus the threshold value (β_{ic}): the condition (ii) is verified when $\sqrt{\psi} < 0.57$.

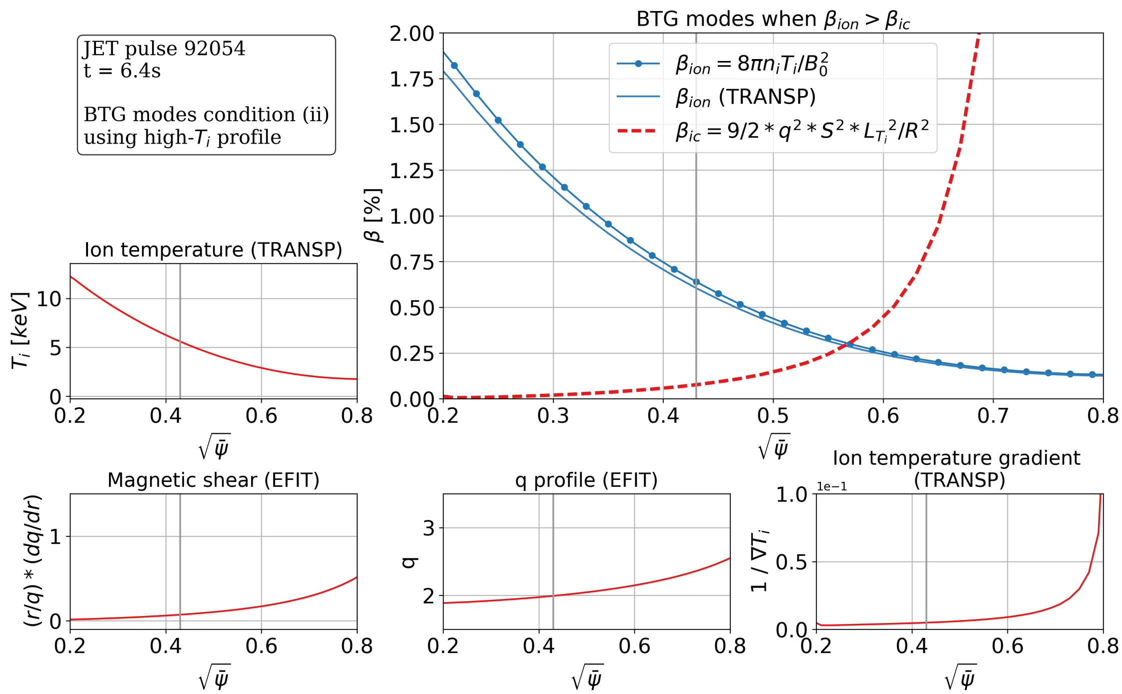

Figure 7: TRANSP ion temperature (T_i) and inverse ion temperature gradient ($1/\nabla T_i$) profiles with EFIT magnetic shear and q profiles are used to evaluate BTG mode condition (ii): $\beta_{ion} > \beta_{ic} = 9q^2S^2L_{T_i}^2/2R^2$. BTG mode could exist for $\sqrt{\psi} < 0.57$. The vertical grey lines indicate the position of the $q = 2$ surface.

The third BTG condition (iii) means to have a low magnetic shear and is defined by

$$U_0 < 2 \text{ with} : U_0 = -\frac{8\pi r p_0^{'}}{S^2 B_0^2}(q^2 - 1) \tag{6}$$

where $p_0^{'}$ is the pressure gradient and B_0 toroidal magnetic field on-axis.

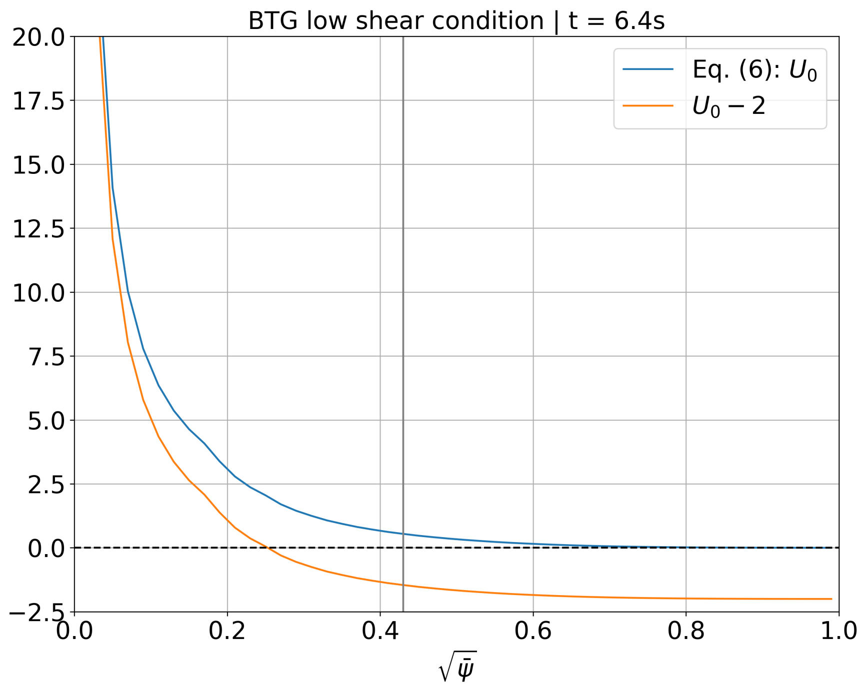

Figure 8: BTG mode condition (iii) on low magnetic shear ($U_0 < 2$) is verified for $\sqrt{\bar{\psi}} > 0.25$.

Figure 8 shows U_0 and $U_0 - 2$ calculated from TRANSP profiles where one can see that the condition (iii) is verified for $\sqrt{\bar{\psi}} \in [0.25, 1.0]$.
If we now consider the three BTG conditions together (i) + (ii) + (iii), BTG mode could exist for $\sqrt{\bar{\psi}} \in [0.25, 0.57]$ which is consistent with Section 2.2, and this range includes the $q = 2$ magnetic surface. For the *low-*T_i case we have $\sqrt{\bar{\psi}} \in [0.25, 0.56]$, almost identical to the *high-*T_i case.

MHD [28] and kinetic [29] theories of BTG mode also present the characteristic frequency range of these modes. In our case, with $1 \ll \eta_{ion}$, we have $\omega_i \ll \omega \ll \omega^*_{Ti}$ where $\omega_i = V_{Ti}/qR_0$ is the ion transit frequency with V_{Ti} the ion thermal velocity ($V_{Ti} = \sqrt{T_i/m_i}$), and ω^*_{Ti} is the temperature-gradient ion drift frequency [11] defined by

$$\omega^*_{Ti} = \frac{c\ T_i}{Z_i\ e\ B}\ \frac{1}{L_{T_i}} \tag{7}$$

Figure 9 shows characteristic frequencies using $n = 4$ for those with toroidal mode number n dependency, i.e. the ion diamagnetic frequency (ω^*_i, Eq. (1)) and the drift frequency (ω^*) which is defined by

$$\omega^* = k_y V^* = \frac{m}{r}\ \frac{c\ T_i}{Z_i\ e\ B}\ \frac{1}{L_{T_i}} = \frac{nq}{r}\ \frac{c\ T_i}{Z_i\ e\ B}\ \frac{1}{L_{T_i}} \tag{8}$$

In Fig. 9 we also plot the frequency of the cylindrical Alfvén continuum spectrum [28], which represents the lower limit for a mode existing from the coupling among Alfvén, acoustic, and drift waves, i.e. BTG mode should have a frequency larger than such a limit:

$$\omega_{BTG} \geq V_A k_{\parallel}^{'} x^* \tag{9}$$

with V_A is the Alfvén speed, $k_{\parallel}^{'} = dk_{\parallel}/dr$, $k_{\parallel}$ is the wave vector along the equilibrium magnetic field and x^* is the characteristic scale length of the coupled Alfvén and drift-acoustic waves ($x^* = (3/2)q^2\rho_i$, with ρ_i the ion Larmor radius).

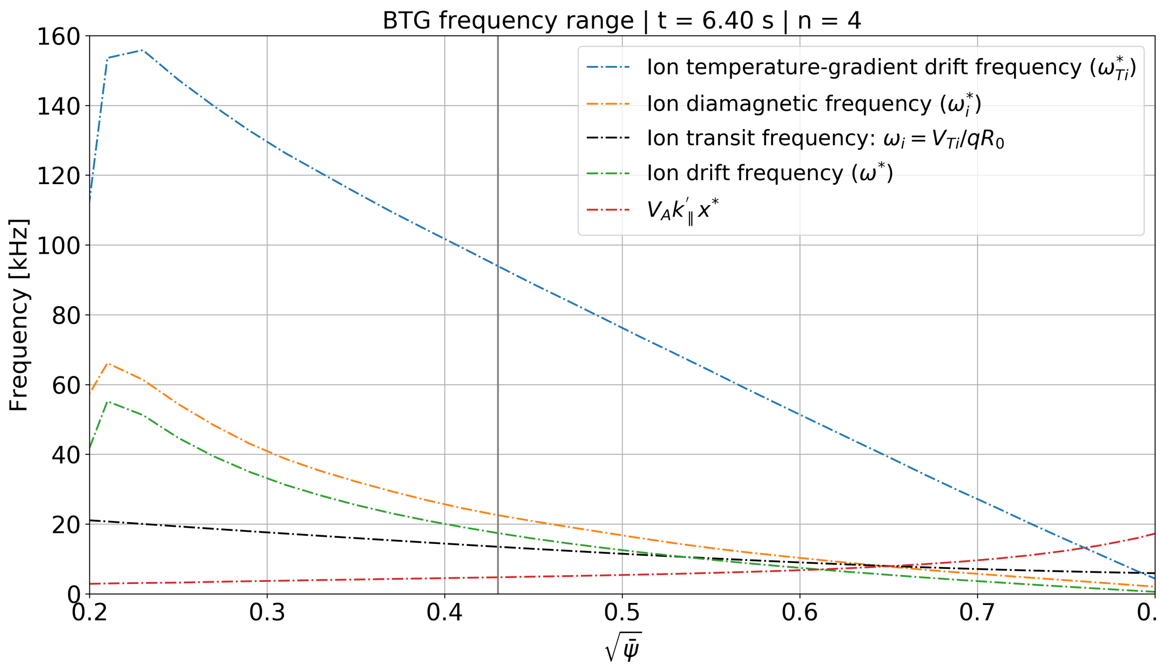

Figure 9: x-axis is the square root of the normalised toroidal flux ($\sqrt{\bar{\psi}}$). BTG mode frequency range expressed by [29]: for $0 < \eta_{ion} \ll 1$ the BTG frequency range is between $V_A k_{\parallel}^{'} x^*$ and ω^*_{Ti} while for $1 \ll \eta_{ion}$ the BTG frequency range is between ω_i and ω^*_{Ti}. The latter is of interest for JET 92054 during our times of interest. The $q = 2$ magnetic surface is indicated by the vertical grey line.

BTG conditions are fullfilled at $t = 6.4s$; now we check if this is the case for other time slices to study the correlation between the observed unstable EM perturbations ($t(s) \in [6.1, 6.5]$) and analytical BTG modes. We present this analysis with yes(V)/no(X) flags, i.e. whether the BTG conditions are met or not for different time slices. Table 2 indicates time slices within and outside the time range of interest; when a BTG condition is met we indicate at which rational magnetic surface ($q = m/n$) in the corresponding cell. All three BTG conditions are met for the same magnetic surface for $t(s) \in [6.1, 6.6]$, before and after these times this is not the case. This shows a good correlation between the observed unstable EM perturbations and the analytical BTG modes.

Table 2: BTG conditions over time. "✓" means "yes" or that the condition is fullfilled while "✗" means it is not. Values in cells for BTG conditions are q values ($= m/n$). Good correlation is observed between unstable EM perturbations ($t(s) \in [6.1, 6.5]$) and analytical BTG modes. The equilibrium reconstruction is not accurate enough [1] for $t = 6.1s$ and $t = 6.2s$ with q-profiles too high to predict the $q = 2$ magnetic surface, while for $t = 6.6s$ the q-profile is too low. After $t = 6.7s$, no equilibrium reconstruction is available.

BTG conditions vs time — $n = 4$									
Time [s]	4.5-5.8	5.9-6.0	6.1	6.2	6.3	6.4	6.5	6.6	6.7
Unstable EM modes	✗	✗	✓	✓	✓	✓	✓	✗	✗
(i) $0 < \eta_{ion}$	11/4	10/4	10/4	9/4	8/4	8/4	8/4	8/4	8/4
(ii) $\beta_{ic} < \beta_{ion}$	✗	✗	10/4	9/4	8/4	8/4	8/4	8/4	8/4
(iii) $2 < U_0$	11/4	10/4	10/4	9/4	8/4	8/4	8/4	8/4	9/4
$(i) + (ii) + (iii)$	✗	✗	10/4	9/4	8/4	8/4	8/4	8/4	✗
$q = 2$ (EFIT)	✗	✗	✗	✗	✓	✓	✓	✓	✓

At $t = 6.1s$ and $t = 6.2s$ we have higher q values than for the $t(s) \in [6.3, 6.6]$; this is due to the equilibrium reconstruction which is not accurate enough [1] with q-profiles too high to predict the $q = 2$ magnetic surface. The unstable EM perturbations disappear between 6.5s and 6.6s when NBI starts to decrease and the ion cyclotron resonance heating (ICRH) system is turned on for a safe plasma termination. After $t = 6.7s$, no equilibrium reconstruction is available. All these conditions add-on to the difficulty to have a high accuracy of the equilibrium reconstruction. Here we note that the experimental observation of unstable BTG modes could be used to constrain future equilibrium reconstructions to have the correct rational magnetic surfaces during the times of such instabilities, e.g. the $q = 2$ surface probably appears between 6.1s and 6.2s instead of between 6.2s and 6.3s predicted by EFIT code.

We have experimental evidence along with MHD and kinetic theories supporting the existence of BTG modes in JPN 92054 at $t(s) \in [6.1, 6.5]$. These modes are localised in the vicinity of the $q = 2$ magnetic surface with a frequency (ω) in the plasma frame such as $\omega_i \ll \omega \ll \omega^*_{Ti}$ scaling with the ion diamagnetic frequency (at $t = 6.4s$ and $q = 2$, $\omega^*_{i\,|n=4} \sim 21.9 \pm 3.1 kHz \sim 0.058 \pm 0.008[V_A/R_0]$). Kinetic BTG theory [29] states that the drive source of BTG modes comes from inverse ion Landau damping, and the analytical dispersion relation reduces to $Re(\omega) = \omega^*_i$. The following section, Section 4, is focused on performing MHD and gyrokinetic simulations using a realistic magnetic geometry and plasma profiles from JPN 92054 at 6.4s and comparing the results with both experimental observations and analytical theories.

4. Modelling

In Fig. 1d one can see that at $t = 6.4s$ the most unstable modes from experiment are $n = 4$ and $n = 5$. Simulations run for this study use a single n and several poloidal harmonics m; we decided to focus on $n = 4$. All frequencies from simulations are in the plasma frame ($f_{TAE_{plasma}}$); lab frame frequencies ($f_{TAE_{lab}}$) are estimated by adding the plasma toroidal rotation from the Doppler shift at the mode location ($f_{tor|mode\ location}$). Mode frequencies are either expressed in kHz or normalised by V_A/R_0, with $V_A[m/s]$ the Alfvén speed and $R_0[m]$ the radius of the magnetic axis.

We started by looking at the magnetohydrodynamic (MHD) picture using the linear ideal MHD code MISHKA-1 [30] (Section 4.1) since it is well-established on JET experiments for TAE studies; MISHKA-1 finds incompressible ideal solutions which is perfectly adapted to TAE studies, but it cannot capture beta-induced modes' physics hence not BAE, BAAE nor BTG modes. So in Section 4.2 we perform linear gyrokinetic simulations using the Gyrokinetic Toroidal Code (GTC) [31], a Particle-In-Cell (PIC) code, to study such beta-induced modes in JPN 92054. GTC has been successfully used to predict beta-induced modes and their stability with an analytical equilibrium [32] and more recently with a realistic experimental equilibrium and plasma profiles on DIII-D [33]. The gyrokinetic approach allows us to treat thermal ions and electrons independently, a necessity here to study the ion temperature gradient effect correctly. To demonstrate that we are running the GTC code in a correct manner for JET equilibrium and profiles we first perform a sanity check by comparing TAE predictions with both MISHKA-1 and GTC codes.

4.1. Magnetohydrodynamic simulations

4.1.1. MISHKA-1, incompressible MHD

We perform a frequency scan looking for modes near the $q = 2$ magnetic surface with MISHKA-1, which solves the linearised ideal MHD equations in a JET toroidal geometry; it includes a vacuum region up to an ideally conducting wall. The JET equilibrium for JPN 92054 at 6.4s is calculated using the HELENA code [34] producing straight field line metric elements; the electron density profile from TRANSP is fitted with an 8^{th} order polynomial from which the coefficients are used to describe the density in MISHKA-1. As expected, we found some TAEs but no modes in the sub-TAE frequency range; Figure 10 presents a $n = 4, m = (8, 9)$ TAE mode at $\omega/\omega_0 \sim 0.241$ ($f = 91.6kHz$) with a ballooning character being mainly localised on the outboard side as one can see on the poloidal plane plot.

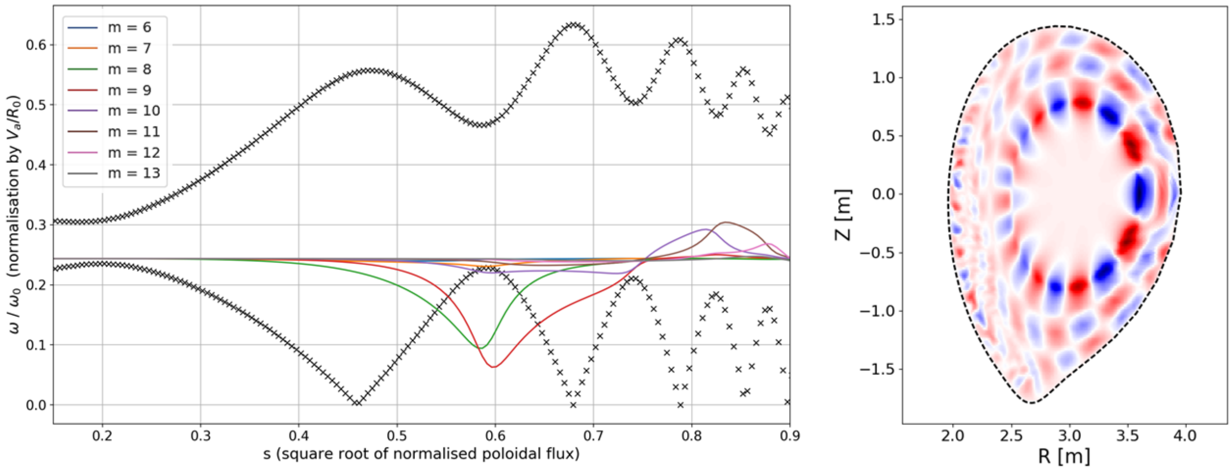

Figure 10: MISHKA-1 and CSCAS codes - (left) Alfvén continuum calculated by CSCAS code [35] of JET 92054 at 6.4s, solved for $n = 4$, $m \in [6, 26]$, 199 radial points ($\sqrt{\psi} \in [0.01, 1.0]$). Plotted on top of the Alfvén continuum is the real component of the electrostatic potential of the TAE mode at $\omega/\omega_0 \sim 0.241 \sim 91.6\ kHz$. Note the horizontal axis is the square root of the normalised **poloidal** flux. (right) is the real component of the electrostatic potential plotted on the poloidal plane. The black dashed line represents the last closed flux surface (LCFS).

4.1.2. GTC, TAE matching incompressible MHD

The same equilibrium and profiles used for MISHKA-1 simulations are used as inputs to GTC; this requires us to map the EFIT equilibrium to Boozer coordinates using a module from the ORBIT code [36] because GTC uses a field-aligned mesh in Boozer coordinates. Once this step is done, consistency of the equilibria is checked and validated to make sure simulations from different codes can indeed be compared. The GTC simulations presented in this paper are all linear electromagnetic global δf. The thermal electrons are treated as a massless fluid without kinetic effects. We neglect collisions and reduced our simulation domain to $\sqrt{\psi} \in [0.20, 0.80]$ to avoid any nonphysical effect from the lack of precision of TRANSP profiles at the edge or near the magnetic axis. We use 100x400x32 grids in radial, poloidal and parallel directions, respectively. To compare the MHD incompressible ideal solutions from MISHKA-1 (TAE mode in Fig. 10) with GTC prediction we need to only consider a single fluid of electrons keeping only the adiabatic terms in the linearized gyrokinetic equation. Figure 11 shows the mode found with GTC; it has a spatial structure and frequency similar to the MISHKA-1 eigenmode giving confidence in using the GTC code on JPN 92054 with such equilibrium and plasma profiles.

With GTC we also study the TAE stability to see if it matches the experimental observations: no unstable TAE was observed during this pulse. We now need to take into account the thermal ion population which is treated gyrokinetically, and the thermal electron population is still simulated as a massless fluid but with kinetic effects from trapped electrons only [37]. The thermal ion population is described by an initial Maxwellian distribution. To respect quasi-neutrality, the ion density is identical to the electron one when we perform simulations without fast ions. The particle number per cell is 200. We obtain a non-perturbative calculation of thermal damping of $\gamma/\omega_{TAE} \sim -2.95\%$, which includes continuum, radiative and ion Landau damping mechanisms. We also performed a GTC simulation with NBI fast ions treated similar to the thermal ions: marginal difference in the total damping rate of the TAE was found. These predictions of a damped TAE are then consistent with experimental observations. Note that such a *stable* TAE is meant to be probed by the JET TAE antenna [14, 15], but unfortunately the TAE antenna scanned too high in frequency to resonate with this mode (see the antenna signal on Fig. 1a) with an antenna frequency at $155kHz$ (at $t = 6.4s$) compared to the simulated TAE mode frequency in the lab frame of $f_{TAE_{lab}} = f_{TAE_{plasma}} + nf_{tor|mode\ location} \sim 92 + 4 * 11 \sim 136kHz$.

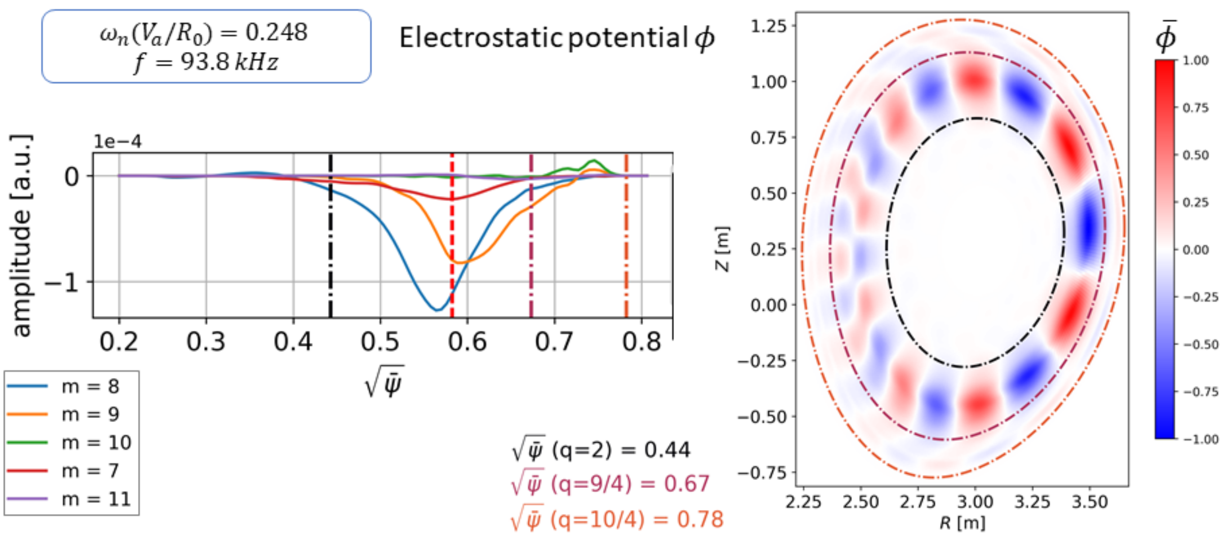

Figure 11: GTC - Real component of the electrostatic potential of the $n = 4$ TAE mode at $\omega/\omega_0 \sim 0.248$ plotted over the square root of the normalised toroidal flux (left) and poloidal plane (right). (left) The vertical red dashed line at $\sqrt{\psi} \sim 0.58$ indicates the position of the TAE gap from the coupling between the $m = 8$ and $m = 9$ poloidal harmonics. The mode spatial structure and frequency are similar to the MISHKA-1 eigenmode (Fig. 10).

4.2. Beta-induced modes in JPN 92054

Before presenting GTC simulations for modes with frequencies below the TAE frequency we first use the ALCON code [38] to solve the ideal MHD Alfvén continuum, i.e., Eq. (10) in [39] using a poloidal-spectral method described in Appendix A in [38]. Finite compressibility of the plasma is taken into account to predict MHD beta-induced gaps, the continuum is shown in Fig. 12. One can see several open gaps (TAE and BAAE gaps[+]) and the BAE accumulation point [9] which aligns well with the top of the beta-induced gap in the Alfvén continuum [40] or at the bottom of the TAE gaps. We also indicate the MHD BAAE frequency from Eq. (3) which is located near the bottom of the BAAE gaps predicted by the ALCON code. Note that

[+] Higher frequency gaps (EAE, NAE, ... [2]) are not showed here to avoid overwhelming Fig. 12.

such MHD continuum does not include ion drift effects so we do not expect to see a gap from drift and Alfvén/sound branches' coupling corresponding to a BTG mode. For consistency we also indicate the thermal ion and electron diamagnetic frequencies. The experimental estimation of the plasma frame frequency range of the observed EM perturbations is indicated by the shaded horizontal grey area, which includes the thermal ion diamagnetic frequency but excludes the MHD BAE and BAAE frequencies.

Having similar mode locations for BTG and BAE/BAAE modes makes the identification of modes from global linear gyrokinetic simulations challenging. In Section 4.2.1 and Section 4.2.2 we present our effort to clearly identify predicted modes with the GTC code to be BTG mode and not MHD BAE or BAAE mode.

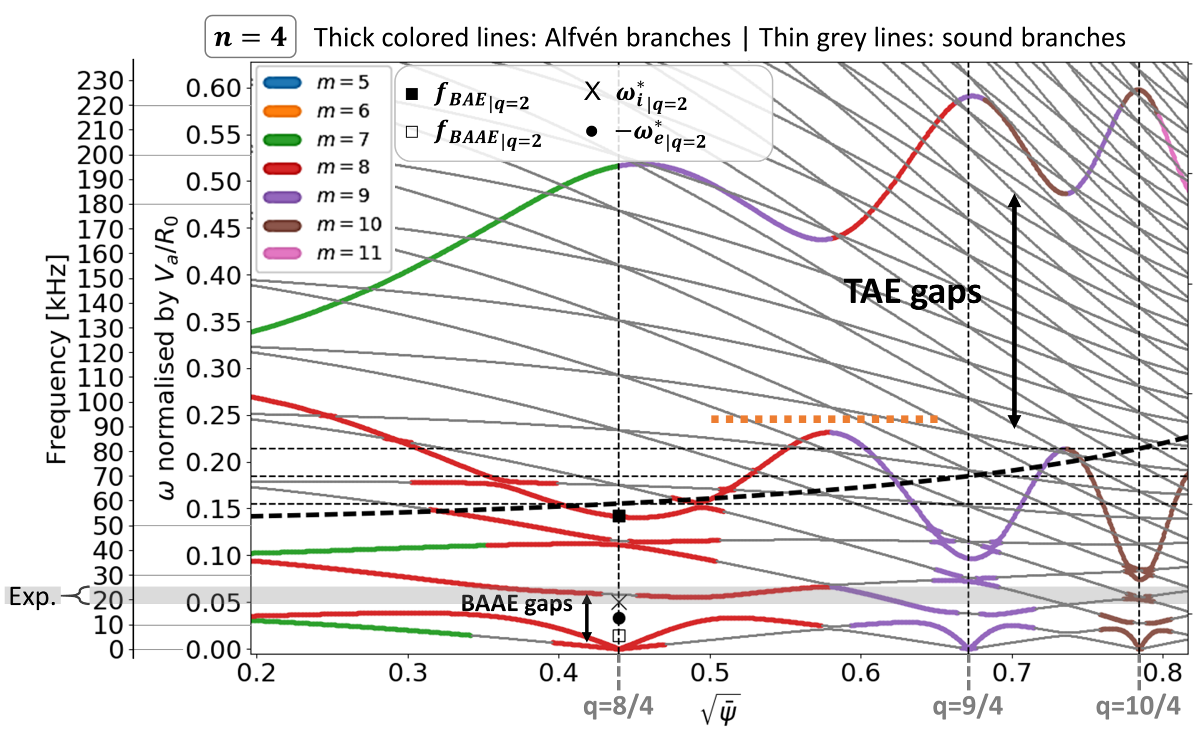

Figure 12: ALCON code [38] - Alfvén and sound continua of JET 92054 at 6.4s, solved for $n = 4$, $m \in [-20, 50]$, 2000 radial points ($\sqrt{\bar{\psi}} \in [0.01, 1.0]$). The thick colored lines are the Alfvén branches while the thin grey ones are the sound branches. The bold dashed line shows the q profile, thin ones mark rational surfaces $q = 8/4 = 2$, $q = 9/4$ and $q = 10/4$. At $q = 2$, we indicate the characteristic plasma frame frequency for MHD BAE (full black square) and BAAE (empty black square) along with the thermal ion (black cross) and electron (full black circle) diamagnetic frequencies. The orange horizontal dotted line at $\omega/\omega_0 \sim 0.25$ in the $(m = 8, m = 9)$ TAE gap indicates the TAE predicted by the GTC code (Fig. 11). Note that the difference between ω/ω_0 with Fig. 10 is due to the different definition of the effective pressure between ALCON and CSCAS codes.

To study β effects we perform full non-perturbative calculations with the GTC code. We input thermal ion temperature and density profiles (Fig. 2) extracted from TRANSP code simulations. The thermal ion population is described by an initial Maxwellian distribution while the thermal electrons are treated as a massless fluid without kinetic effects*.

4.2.1. GTC, beta-induced modes in an uniform thermal plasma

Our first step is to use a synthetic antenna in GTC to scan in frequency the linear gyrokinetic response of the uniform thermal plasma♯; using a uniform plasma does not allow us to see any thermal plasma inhomogeneity effects, such as temperature gradient effects, but rather probe resonance conditions between plasma modes and an external perturbation. We use the densities and temperatures at the $q = 2$ position to set the uniform densities and temperatures' values. The antenna is set at a fixed frequency ($f_{antenna}$) and by running multiple simulations we can perform a frequency scan probing the different modes at various frequencies. Such GTC simulations impose an antenna field or excitation in the plasma at a certain radial location over a set radial range. The GTC synthetic antenna can be a perturbation of the electrostatic potential ($\delta\phi_{ant}$) or the parallel vector potential ($\delta A_{\|,ant}$). The latter has been used throughout this work since we are looking at electromagnetic modes. The GTC synthetic antenna has been modeled with Eq. (10) where $A(\psi)$ is a Gaussian envelope peaking at the location of the mode of interest (at the $q = 2$ magnetic surface in our simulations), ψ is the poloidal magnetic flux, θ is the poloidal angle and ζ is the toroidal angle.

$$A_{\|,ant} = A(\psi)\ cos(m\theta - n\zeta)\ cos(\omega t) \tag{10}$$

For each simulation, we then (a) calculate the power spectrum using a Fourier transform of the time evolution of the plasma electrostatic potential filtered for ($n = 4,\ m = 8$), and (b) extract from the power spectrum the maximum power around the input antenna frequency index. We take the maximum between powers at $f_{antenna} \pm 0.5kHz$ and $-\ f_{antenna} \pm 0.5kHz$; positive frequency means the mode moves in the electron diamagnetic direction while negative frequency corresponds to a mode moving in the ion diamagnetic direction. Looping for each simulation we get the linear gyrokinetic response of the thermal plasma over frequency in Fig. 13, i.e. how effectively the plasma resonates with the synthetic antenna perturbation at $f_{antenna}$. A peak in such a frequency scan indicates a higher resonance condition. This method can also be used to quantify the damping rate of modes by fitting a peak with a cavity resonance function [33] or similarly appropriate resonance transfer function (TF) such as a weakly-damped harmonic oscillator TF.

We performed two frequency scans: (1) with the *low*-T_i profile and (2) with the *high*-T_i profile (see Fig. 2 for the profiles). Figure 13 shows the results from scan (1) which has a higher resolution with 17 simulations compared to 13 in the scan (2); one can see a single peak near the MHD BAAE frequency with frequencies indicating that the mode is moving in the electron diamagnetic direction. This resonance is identified as a BAAE mode weakly

* Some simulations have been run with electron kinetic effects included but only a marginal difference was found.

♯ Uniform particle marker for density and temperature and assuming equilibrium-fluctuation scale separation.

damped by ion Landau damping. This is consistent with $T_i \gg T_e$ and a recent BAAE mode study with GTC [41]. The frequency of the predicted BAAE mode is too low compared to our experimental EM perturbations. With scan (2) we also find a similar BAAE resonance. On both scans, higher frequencies do not show any other clear resonance, neither near the BAE/GAM frequency (f_{GAM}) nor near the ion diamagnetic frequency (ω_i^*) for BTG mode. This is expected since BTG mode is driven by the thermal ion temperature gradient so we need to consider thermal plasma inhomogeneity in our gyrokinetic simulations to capture diamagnetic effects.

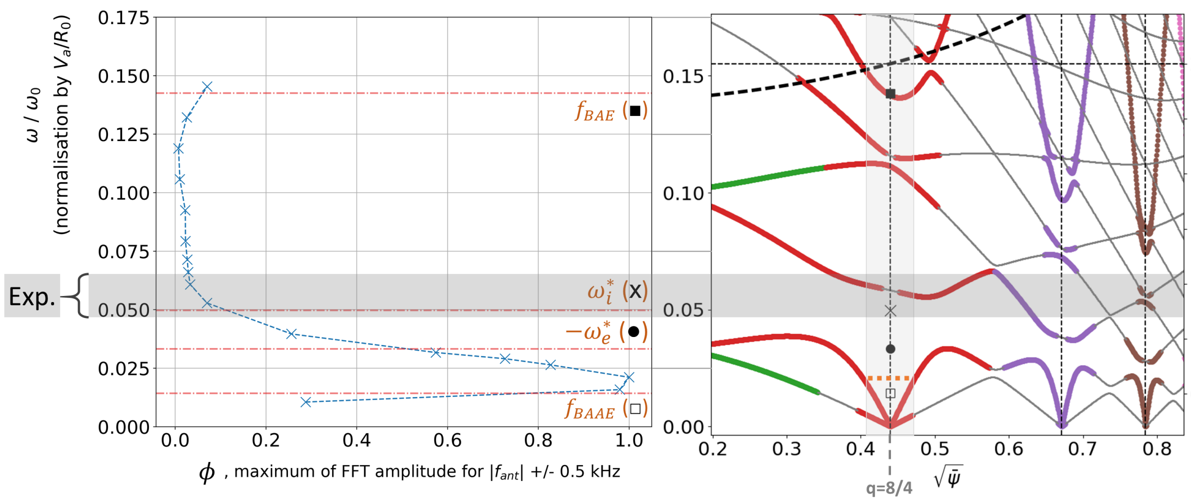

Figure 13: (left) GTC antenna frequency scan; each cross corresponds to a single simulation. The different simulations are identical except for the antenna perturbation frequency. The peak in frequency indicates the dominant resonance which corresponds to a weakly damped BAAE mode with a frequency near the MHD BAAE frequency. (right) Alfvén-acoustic continuum (same as Fig. 12) zoomed in the sub-TAE frequency range. One can see a BAAE gap with the MHD BAAE frequency (f_{BAAE}, empty black square) at the bottom while the frequency of the predicted BAAE mode is indicated by the dotted horizontal orange line. The shaded horizontal grey area shows the experimental estimation of the plasma frame frequency of the observed EM perturbations: the predicted BAAE mode frequency is too low. At $q = 2$, we also add the characteristic plasma frame frequency for MHD BAE (full black square) along with the thermal ion (black cross) and electron (full black circle) diamagnetic frequencies; there is no clear resonance for BTG nor BAE modes.

4.2.2. GTC, beta-induced modes in a non-uniform thermal plasma

We now perform simulations with non-uniform markers for density and temperature which allow gradients to be accounted for by marker weight; with such method we find a physical mode kinetically driven by the thermal plasma, without fast ions, at $\omega/\omega_0 \sim 0.11$. Figure 14 shows the characteristics of this $n = 4$ mode with a dominant $m = 8$ $(= nq = 4 * 2)$ poloidal harmonic with (a) the time evolution of the real and imaginary components of the mode's electrostatic potential - noted ϕ - and its amplitude $\|\phi\| = \sqrt{Re(\phi)^2 + Im(\phi)^2)}$. (b) is the radial mode structure while (c) is the mode structure in the poloidal plane. From (a) one can see an exponential growth of the mode amplitude, we calculate the mode growth rate ($\gamma/\omega \sim 23.8\%$) by fitting the linear growth of $log(\|\phi\|)$.

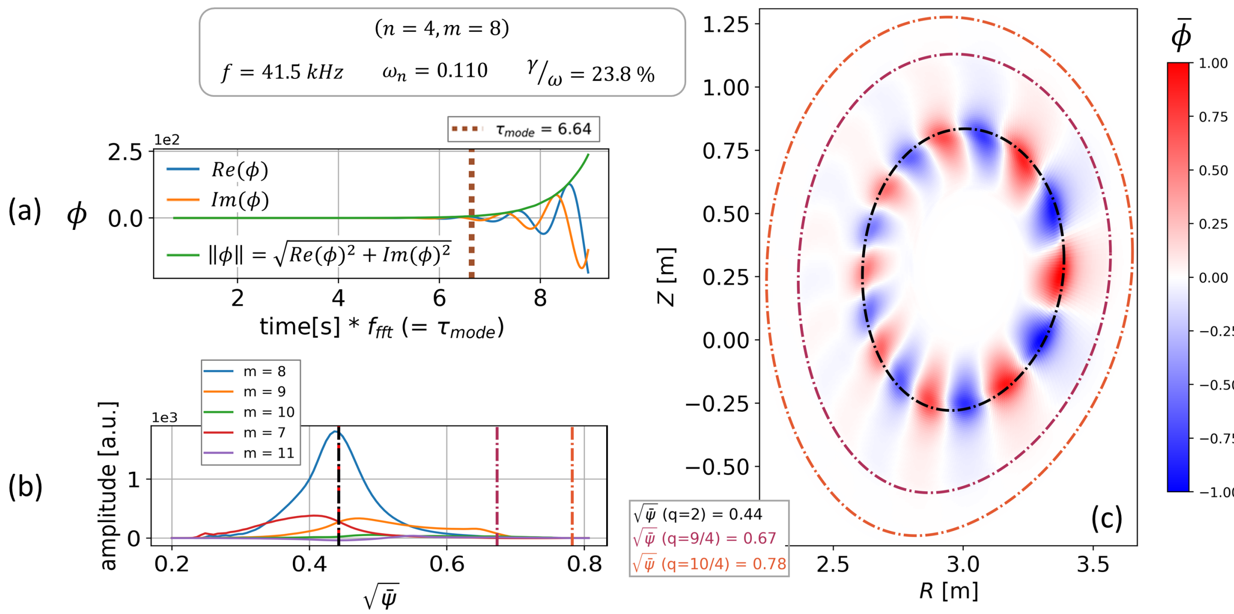

Figure 14: x-axis for (a) is the simulation time multiplied by mode ($n = 4$, $m = 8$) frequency; blue and orange lines are respectively Real and Imaginary components of the electrostatic potential (ϕ) while green is the amplitude ($\sqrt{Re(\phi)^2 + Im(\phi)^2}$) which grows exponentially ($\gamma/\omega = 23.8\%$). (c) is the radial mode structure of the kinetically driven mode with $m = 8$ the dominant poloidal harmonic. (d) is the mode structure in the poloidal plane.

This kinetically driven mode matches well the experimental observations and theoretical predictions of BTG mode, i.e. is localised at the $q = 2$ magnetic surface, has a single poloidal harmonic, is driven by thermal ions and is moving in the ion diamagnetic direction. Its frequency, $\sim 41.5kHz \sim 0.011[V_A/R_0]$, is between the ion diamagnetic frequency and the BAE frequency. It's however higher than expected from the experimental estimation by $\sim 15kHz$ (Section 2.2); such discrepancy is associated with the large quality factor of $\gamma \sim 10kHz$ and uncertainties on the thermal ion temperature profile and its gradient. Below we explore the nature of this kinetically driven mode to clearly distinguish between BTG and BAE modes.

We perform a toroidal mode number scan ($n \in [1, 6]$); BAE modes should have similar frequencies in the plasma frame (or simulated frequency) while BTG modes would have different frequencies shifted by ω_i^*. The poloidal mode numbers were changed to model modes around the $q = 2$ surface similar to the $n = 4$ reference case: we used $m = nq + [-1, 0, +1, +2, +3]$. Figure 15 presents the ($n, m = 2n$) mode frequency (kHz) and growth rate (%) for each simulation. Not shown here to avoid overfilling the paper, the mode structures are very similar with a dominant single $m = nq$ poloidal harmonic as one can see on Fig. 14. We have a significant frequency dependency on the toroidal mode number:

$\Delta f \sim 5kHz$ for n to $n \pm 1$, i.e. $\Delta f/f > 10\%$. This Δf corresponds to the ion diamagnetic frequency without n contribution from Eq. (9). These new results match what we expect from a drift-type mode, hence from a BTG mode.

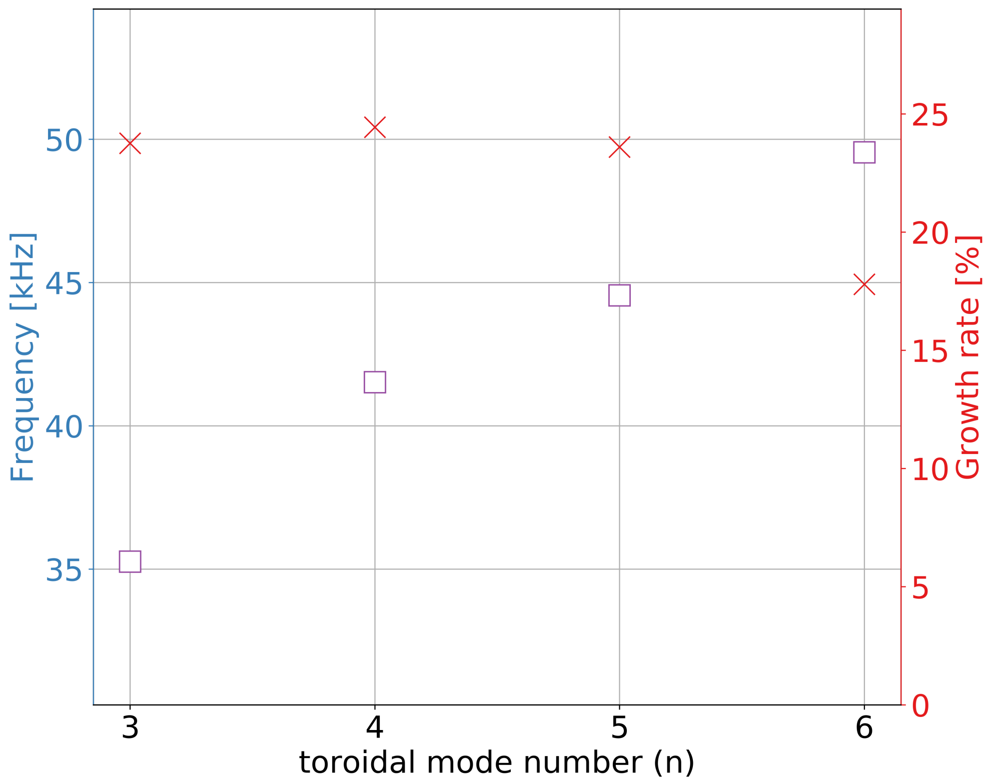

Figure 15: For $n \in [3, 6]$ and $(n, m = 2n)$: frequency (square) and growth rate (cross) of the mode for each toroidal mode number.

Now to confirm the effect of the thermal ion population on the drive of our reference mode, we perform a T_i scan while keeping the total plasma beta constant, i.e. if T_i is multiplied by a coefficient $A\%$ then T_e is multiplied by $(1 - (A\% - 1)T_i/T_e)$ since we use $n_e = n_i$ to respect quasi-neutrality. Figure 16 shows a clear effect of T_i on the stability of the mode; we can estimate a threshold from which the mode becomes unstable: $\sim 0.72 * T_i$. We also see little effect of the T_i scan on the mode frequency indicating that the mode is affected by both thermal ions and electrons. This is confirmed by a second T_i scan for which we only vary T_i while keeping T_e constant: the threshold is $\sim 0.83 * T_i$ and the mode frequency slightly decreases.

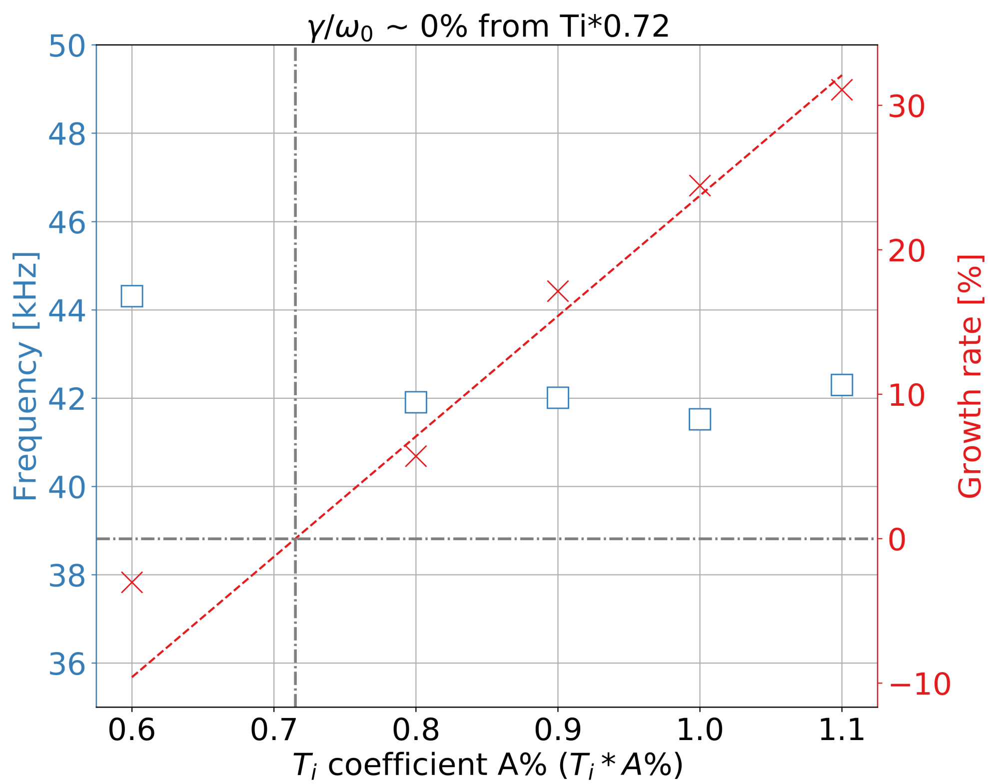

Figure 16: For $(n = 4, m = 2n = 8)$ mode frequency (square) and growth rate (cross) for several thermal plasma temperatures; when T_i is multiplied by a coefficient $A\%$, T_e is multiplied by $(1 - (A\% - 1)T_i/T_e)$. Clear effect from T_i on the stability of the reference mode. For simulations with driven mode ($T_i \precsim 0.72 * T_i$), the mode frequency is stable.

To complement the analysis of our reference case, we also performed the following simulations:

- Mode polarisation ††:
  - Electric field polarisation: we analyse the ratio between the parallel electric field and its electrostatic component ($E_\parallel/E_{\parallel,ES}$) defined by

$$E_\parallel = -\mathbf{b_0} \cdot \nabla \delta\phi - \frac{1}{c}\frac{\partial \delta A_\parallel}{\partial t}$$
$$E_{\parallel,ES} = -\mathbf{b_0} \cdot \nabla \delta\phi \tag{11}$$

where $\mathbf{b_0}$ represents the equilibrium magnetic field direction, $\delta\phi$ is the electrostatic potential and $\delta A_\parallel$ is the parallel vector potential. $E_\parallel = 0$ for an Alfvénic wave, and $E_\parallel = E_{\parallel,ES}$ for an ion acoustic wave and drift wave. For our reference mode (Fig. 17) we get $E_\parallel/E_{\parallel,ES} \sim 0.1$ using volume-average of square of $E_\parallel$ and $E_{\parallel,ES}$ indicating a dominant Alfvénic character which is consistent with BTG theory [6].

††Details of the calculation can be found in [33]

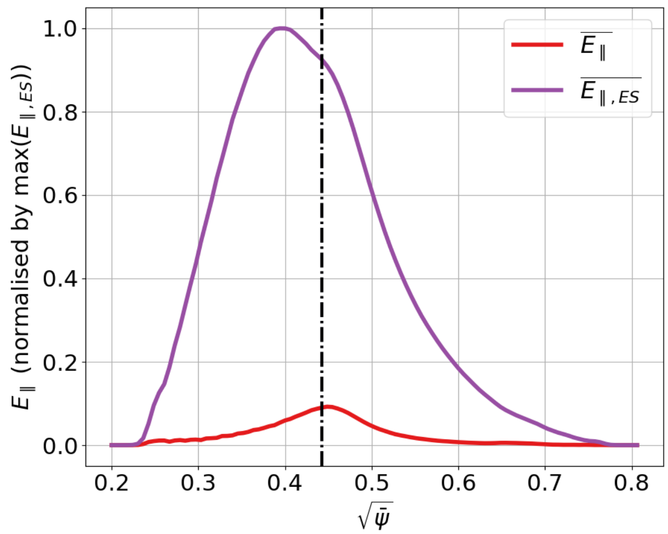

Figure 17: Reference mode's (Fig. 14) normalised radial profile of parallel electric field $E_\parallel$ and its electrostatic part $E_{\parallel,ES}$.

– Magnetic perturbation polarisation: we analyse the ratio between the parallel and the perpendicular magnetic perturbations ($\delta B_\parallel/\delta B_\perp$). $\delta B_\parallel/\delta B_\perp = 0$ for a shear Alfvénic wave, and $\delta B_\parallel/\delta B_\perp$ is finite for sound and drift waves. For our reference mode (Fig. 18) we get $\delta B_\parallel/\delta B_\perp \sim 0.45$ using volume-average of square of $\delta B_\parallel$ and $\delta B_\perp$ confirming the nature of the mode being a coupling between Alfvén and acoustic/drift waves.

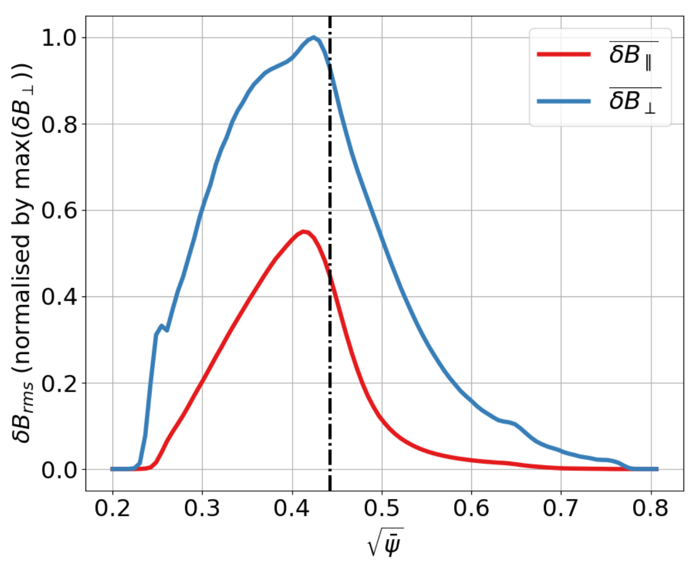

Figure 18: Reference mode's (Fig. 14) normalised radial profiles of flux surface-averaged perpendicular ($\delta B_\perp$) and parallel ($\delta B_\parallel$) perturbed magnetic field B_{rms}.

- Wave-particle energy exchange ††: we analyse the direct energy exchange between the thermal plasma and the wave/mode. We use the power of work done on the thermal ion particles by the wave/mode to calculate the time rate of change of the wave/mode energy density (δW) [32, 33]:

$$\frac{\mathrm{d}\delta W}{\mathrm{d}t} = \langle -Z\mathbf{v}_\perp \cdot \mathbf{E}_\perp - Z\nu_\parallel E_\parallel \rangle \tag{12}$$

where Z is the particle charge, $\mathbf{v}_\perp$ is the guiding center Grad-B ($\boldsymbol{v}_{\boldsymbol{\nabla B}}$) and curvature ($\boldsymbol{v}_{\boldsymbol{R}}$) drifts, $\mathbf{E}_\perp = -\nabla_\perp \delta\phi$ is the perpendicular electric field and $\nu_\parallel$ is the guiding center parallel velocity. The brackets denote a flux-surface averaging along with a gyrocenter velocity space integral weighted by the perturbed distribution function. Note that both the perpendicular and parallel energy transfers include the non-resonant (fluid) as well as the resonant (kinetic) energy exchanges. The interchange drive represents only the fluid parts of the energy exchange rate. In Fig. 19 one can see that the perpendicular energy exchange is the source of the drive of the wave/mode while the interchange drive is low, which indicates a dominant perpendicular energy transfer from the thermal ions to the wave/mode coming mostly from the resonant (kinetic) energy exchange. This analysis confirms that the reference mode is kinetically driven by thermal ions.

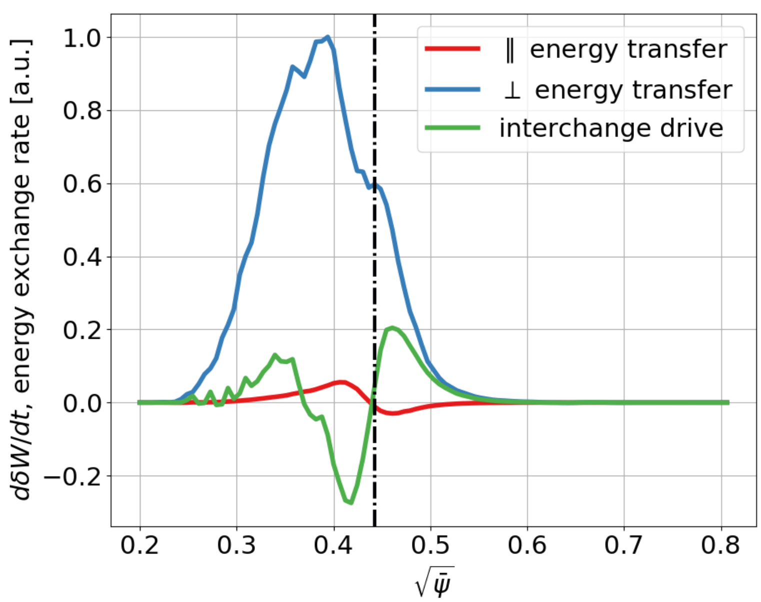

Figure 19: Reference mode's (Fig. 14) time rate of change of the wave/mode energy density. Normalised radial profiles of the parallel and perpendicular energy exchange rates as well as the interchange (non-resonant) part. The perpendicular energy transfer is the source of the drive of the wave/mode.

- Incompressible MHD simulation: reference mode was not found, so it confirms that our reference mode is not an interchange-type mode which is consistent with the wave-particle energy exchange analysis.

- Including kinetic effects from trapped electrons using a fluid-kinetic hybrid electron model has an insignificant effect on the reference mode characteristics confirming the strong dependence of the reference mode on the thermal ion population.
- Electrostatic simulation: reference mode was not found which indicates that our reference mode is not an electrostatic mode; this is consistent with the electromagnetic nature of the BTG modes.
- Without $\delta B_{\parallel}$ we see similar characteristics of the reference mode but with a significant reduction of its growth rate indicating an important effect from $\delta B_{\parallel}$ often neglected in gyrokinetic simulations.

Many features in our reference case obtained with GTC is therefore consistent with experimental observations and analytical theories of beta-induced ion temperature gradient driven eigenmodes, i.e. a strong thermal ion dependence especially with the significant thermal ion temperature gradient; a propagation in the ion diamagnetic direction; a localisation near a rational magnetic surface ($q = 2$) with a low magnetic shear; a coupling among Alfvén, acoustic and drift waves with a dominant Alfvén polarisation; a single dominant poloidal harmonic and a frequency scaling with the ion diamagnetic frequency. The simulated mode frequency is however over-estimated by $\sim 15kHz$ compare to the frequency estimated from the experiment †. A few factors can contribute to such uncertainty: the large quality factor of the simulated mode ($\gamma \sim 10kHz$), the uncertainties on the thermal ion temperature measurements/profiles as stressed by Fig. 2 and its (very) high ion-temperature gradient as one can see with Eq. (7) in Fig. 9, and the GTC local Maxwellian distribution function.

† $f_{GTC} \sim 41.5kHz \sim 0.110[V_A/R_0]$ compared to the expected EM modes estimated to be around $\omega^*_{i\,|n=4} \sim 21.9 \pm 3.1kHz \sim 0.059[V_A/R_0]$ from the experiment.

5. Summary

JET pulse #92054, a hot ion JET plasma with elevated monotonic q profile and clear ITB, exhibits unstable electromagnetic perturbations with frequencies below the TAE frequency which have been identified as beta-induced ion temperature gradient (BTG) eigenmodes. Experimental investigations show that the BTG modes have a strong dependence on the thermal ions, particularly on the thermal ion temperature gradient, are localised near the $q = 2$ magnetic surface related to the ITB and scale with the ion drift frequency (ω_i^*). These experimental characteristics are in good agreement with BTG mode analytical theories [6,29]. Such theories also predict three well-defined conditions for BTG mode to exist which are fullfilled by the JPN 92054 plasma; i.e. positive relative ion temperature gradient, ion beta higher than a critical value, a low magnetic shear and BTG mode analytical dispersion relation reducing to $Re(\omega) = \omega_i^*$. Ref. [29] predicts that a BTG mode is a coupling among Alfvén, acoustic and drift waves as well as that it is driven by inverse ion Landau damping due to the high ion temperature gradient. Many of these BTG mode experimental and theoretical features are consistent with gyrokinetic simulations using the code GTC [31] with a realistic magnetic geometry and plasma profiles: we find a mode kinetically driven by thermal ions localised near the $q = 2$ magnetic surface with an Alfvén-acoustic polarisation and a frequency scaling with the ion drift frequency (ω_i^*).

BTG modes are also observed in *recent* JET plasmas during energetic particle scenario experiments aiming to study alpha driven AEs, performed in JET 2019/2020 Deuterium campaigns. Reflectometer diagnostic data is available for some of these pulses and confirms the mode location being near the $q = 2$ magnetic surface. We also observe a correlation between the BTG modes stability and the neutron rate roll-over, but this study is beyond the scope of this work; it will be discussed in future publications.

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