

UKAEA-CCFE-PR(23)156

Daniel Padrão, David Hancock, James Paterson,  
Frank Schoofs, Christopher Tuck, Ian Maskery

# **New structure-performance relationships for surface-based lattice heat sinks**

Enquiries about copyright and reproduction should in the first instance be addressed to the UKAEA Publications Officer, Culham Science Centre, Building K1/O/83 Abingdon, Oxfordshire, OX14 3DB, UK. The United Kingdom Atomic Energy Authority is the copyright holder.

The contents of this document and all other UKAEA Preprints, Reports and Conference Papers are available to view online free at [scientific-publications.ukaea.uk/](https://scientific-publications.ukaea.uk/)

# **New structure-performance relationships for surface-based lattice heat sinks**

Daniel Padrão, David Hancock, James Paterson, Frank Schoofs,  
Christopher Tuck, Ian Maskery



# Additive Manufacturing

## New structure-performance relationships for surface-based lattice heat sinks

--Manuscript Draft--

<b>Manuscript Number:</b>	ADDMA-D-23-00577
<b>Article Type:</b>	Research Paper
<b>Keywords:</b>	Cellular structure; lattice; fluid flow; conjugate heat transfer; computational fluid dynamics (CFD)
<b>Corresponding Author:</b>	Ian Maskery  Nottingham, United Kingdom
<b>First Author:</b>	Daniel Padrão
<b>Order of Authors:</b>	Daniel Padrão David Hancock James Paterson Frank Schoofs Christopher Tuck Ian Maskery
<b>Abstract:</b>	<p>Heat sinks have manifold applications, from micro-electronics to nuclear fusion reactors. Their performance expectations will continue to increase in line with the power consumption and miniaturisation of technology. Additive manufacturing enables the creation of novel, compact heat sinks with greater surface-to-volume ratios and geometrical complexities than standard pin/fin arrays and pipes. Despite this, there has been little research into the use of high surface area lattice structures as heat sinks. Here, the hydraulic and thermal performance of five surface-based lattice structures were examined numerically. Computational fluid dynamics was used to create useful predictive models for pressure drop and volumetric heat transfer coefficients over a range of flow rates and volume fractions, which can henceforth be used by heat transfer engineers. The thermal performance of surface-based lattices was found to be heavily dependent on internal geometry, with structures capable of distributing thermal energy across the entire fluid volume having greater volumetric heat transfer coefficients than those with only localised areas of high heat transfer and low levels of fluid mixing.</p>

University of Nottingham  
Advanced Manufacturing Building  
Jubilee Campus  
Nottingham  
NG7 2GX

Dr Ryan Wicker,  
Editor-in-Chief  
Additive Manufacturing

February 13<sup>th</sup> 2023

Dear Dr Wicker,

We wish to submit an original research article entitled *New structure-performance relationships for surface-based lattice heat sinks*. We politely ask that you handle this process yourself, as discussed in our correspondence.

We confirm that this work is original and has not been published elsewhere, nor is it currently under consideration for publication elsewhere.

In this paper, we numerically examine the use of triply periodic minimal surface (TPMS) structures in heat exchangers and determine key fluid flow and heat transfer mechanisms. This is important because TPMS lattice structures have complex geometries and greater surface-to-volume ratios than standard pin/fin arrays and pipes, and may therefore outperform conventional designs in a range of heat management scenarios.

This paper is relevant to *Additive Manufacturing* as there have been previous studies on the thermal performance of additively manufactured heat exchangers (including lattices) [1-4] published by this journal. Mechanical property relationships for cellular structures have also been explored in this journal [5] which is not dissimilar to the work presented in this paper for fluid and thermal performance.

We have no conflicts of interest to disclose.

Thank you for consideration of this manuscript.

Sincerely,

A handwritten signature in black ink that reads "I. Maskery". The signature is written in a cursive style with a long, sweeping underline.

Dr Ian Maskery  
University of Nottingham

1. S. M. Thompson, Z. S. Aspin, N. Shamsaei, A. Elwany, and L. Bian, "Additive manufacturing of heat exchangers: A case study on a multi-layered Ti-6Al-4V oscillating heat pipe," *Additive Manufacturing*, vol. 8, pp. 163–174, oct 2015. [Online]. Available: <http://dx.doi.org/10.1016/j.addma.2015.09.003>
2. H. Brooks and K. Brigden, "Design of conformal cooling layers with self-supporting lattices for additively manufactured tooling," *Additive Manufacturing*, vol. 11, pp. 16–22, jul 2016. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S2214860416300409>
3. E. Huttunen, M. T. Nykänen, and J. Alexandersen, "Material extrusion additive manufacturing and experimental testing of topology-optimised passive heat sinks using a thermally-conductive plastic filament," *Additive Manufacturing*, vol. 59, p. 103123, nov 2022. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S2214860422005127>
4. J. Kelly, L. Finkenauer, P. Roy, J. Stolaroff, D. Nguyen, M. Ross, A. Hoff, and J. Haslam, "Binder jet additive manufacturing of ceramic heat exchangers for concentrating solar power applications with thermal energy storage in molten chlorides," *Additive Manufacturing*, vol. 56, p. 102937, aug 2022. [Online]. Available: <https://doi.org/10.1016/j.addma.2022.102937>
5. O. Al-Ketan, R. Rowshan, and R. K. Abu Al-Rub, "Topology-mechanical property relationship of 3D printed strut, skeletal, and sheet based periodic metallic cellular materials," *Additive Manufacturing*, vol. 19, pp. 167–183, jan 2018. [Online]. Available: <https://doi.org/10.1016/j.addma.2017.12.006>

- Triply periodic minimal surface lattices offer unique advantages for heat transfer.
- Structures with greater variance in their channel diameter exhibit localised pressure drops.
- For a directional heat input, structures which conduct heat through the walls and promote thermal mixing exhibit larger heat transfer coefficients.
- Accurate models for fluid flow and heat transfer in the examined structures are provided.

1  
2  
3  
4  
5  
6  
7  
8  
9  
10  
11  
12  
13  
14  
15  
16  
17  
18  
19  
20  
21  
22  
23  
24  
25  
26  
27  
28  
29  
30  
31  
32  
33  
34  
35  
36  
37  
38  
39  
40  
41  
42  
43  
44  
45  
46  
47  
48  
49  
50  
51  
52  
53  
54  
55  
56  
57  
58  
59  
60  
61  
62  
63  
64  
65

# New structure-performance relationships for surface-based lattice heat sinks

D. Padrão<sup>a</sup>, D. Hancock<sup>b</sup>, J. Paterson<sup>b</sup>, F. Schoofs<sup>b</sup>, C. Tuck<sup>a</sup>, I. Maskery<sup>a,\*</sup>

<sup>a</sup>Centre for Additive Manufacturing, University of Nottingham, Nottingham NG8 1BB, UK

<sup>b</sup>United Kingdom Atomic Energy Authority, Culham Science Centre, OX14 3DB, Abingdon, UK

---

## Abstract

Heat sinks have manifold applications, from micro-electronics to nuclear fusion reactors. Their performance expectations will continue to increase in line with the power consumption and miniaturisation of technology. Additive manufacturing enables the creation of novel, compact heat sinks with greater surface-to-volume ratios and geometrical complexities than standard pin/fin arrays and pipes. Despite this, there has been little research into the use of high surface area lattice structures as heat sinks. Here, the hydraulic and thermal performance of five surface-based lattice structures were examined numerically. Computational fluid dynamics was used to create useful predictive models for pressure drop and volumetric heat transfer coefficients over a range of flow rates and volume fractions, which can henceforth be used by heat transfer engineers. The thermal performance of surface-based lattices was found to be heavily dependent on internal geometry, with structures capable of distributing thermal energy across the entire fluid volume having greater volumetric heat transfer coefficients than those with only localised areas of high heat transfer and low levels of fluid mixing.

*Keywords:* Cellular structure, lattice, fluid flow, conjugate heat transfer, computational fluid dynamics (CFD)

---

## 1. Introduction

Additive manufacturing (AM) describes a range of processes which join materials to make solid parts from 3D model data, usually layer upon layer. Metal AM offers enhanced design freedom compared to conventional processes, enabling the production of near-net shape components with complex internal geometries and high customisability, such that parts can be tailored for specific applications and users [1]. This technology was initially developed for rapid prototyping, but is now used in several sectors to produce final products, such as in aerospace [2] where the GE LEAP fuel nozzle has been a notable commercial success [3].

The incorporation of 3D cellular structures is a key element of AM design. Ordered lattices (as opposed to the typically random foams that can be made with gas injection [4], for example) have received significant attention in the literature [5], and are now available in several commercial CAD packages aimed at AM. They reduce component weight, have high surface-to-volume ratios and high solid-fluid contact areas [1, 4], making metal lattices in particular ideal candidates for heat sinks [6–8]. Compared to lattices composed of interconnected struts or ‘trusses’, triply periodic minimal surface (TPMS) lattices have enclosed channels for fluid flow, greater surface areas, and are generally stiffer and stronger at equivalent weight [9]. Therefore, TPMS lattices offer unique advantages for fluid flow and heat transfer applications.

Heat sinks are common devices, with applications ranging from micro-electronics [10] to nuclear fusion [11], which typically employ channels or extended surfaces, such as pin/fin arrays, to dissipate heat. The dominant heat transfer mechanisms for these structures under laminar flow are convection in the fluid flow direction and conduction in the direction normal to that. For turbulent flow, heat transfer is driven by convection in the fluid [12]. TPMS lattice structures can be considered as an alternative to conventional geometries as they promote convective cooling due to their large surface areas. Conversely, these features also lead to greater pressure drops, and potentially lower heat sink efficiencies [13].

The need to dissipate large quantities of heat from small volumes will continue to increase in line with the increased power demands and miniaturisation of electronic devices. It is therefore necessary for heat sinks to be maximally efficient, in terms of their size and material usage. Despite this, there has been little research on the use of complex surface-based cellular structures within AM heat sinks, with much of the published work dedicated to foams [14–17], strut-based lattices [18–23], topology-optimised structures [24] or conventional pin/fin arrays and channels [21, 25–27]. Recently, heat transfer in TPMS structures has garnered some attention, with applications including heat sinks [28–33], injection mold cooling [34] and latent heat thermal energy storage systems [35, 36]. These studies characterise the thermal performance of TPMS lattice structures with varying wall thicknesses and in a range of flow conditions. These need to be developed further, so

---

\*Corresponding author

Email address: [ian.maskery@nottingham.ac.uk](mailto:ian.maskery@nottingham.ac.uk) (I. Maskery)

## Nomenclature

		$n$	Parameter correlating $Nu_{vol}$ to $Re$
$\Delta T_{LMTD}$	Logarithmic mean temperature difference ( $K$ )	$Nu_{vol}$	Volumetric Nusselt number
$\dot{m}$	Mass flow rate ( $kg\ s^{-1}$ )	$P$	Fluid pressure ( $Pa$ )
$A_\nu$	Specific surface area ( $m^{-1}$ )	$Re$	Reynolds number
$A_{w,s}$	Wetted surface area ( $m^2$ )	$T_{f,in}$	Fluid inlet temperature ( $K$ )
$c_p$	Specific heat capacity ( $J\ kg^{-1}\ K^{-1}$ )	$T_{f,out}$	Fluid outlet temperature ( $K$ )
$D_h$	Hydraulic diameter ( $m$ )	$T_h$	Heater temperature ( $K$ )
$F$	Parameter correlating $Nu_{vol}$ to $Re$	$T_s$	Average channel surface temperature ( $K$ )
$h_l$	Local wall heat transfer coefficient ( $W\ m^{-2}\ K^{-1}$ )	$u_m$	Mean channel fluid velocity ( $m\ s^{-1}$ )
$h_{m,vol}$	Volumetric heat transfer coefficient ( $W\ m^{-3}\ K^{-1}$ )	$u_s$	Superficial fluid velocity ( $m\ s^{-1}$ )
$h_m$	Global mean heat transfer coefficient ( $W\ m^{-2}\ K^{-1}$ )	$V_T$	Total volume of design space ( $m^3$ )
$K$	Darcian permeability ( $m^2$ )	$V_w$	Wetted volume ( $m^3$ )
		<b>Greek letters</b>	
$k$	Thermal conductivity ( $W\ m^{-1}\ K^{-1}$ )	$\gamma$	Volume fraction
$K_1$	Forchheimer permeability ( $m^2$ )	$\mu$	Dynamic viscosity ( $kg\ m^{-1}\ s^{-1}$ )
$K_2$	Inertial permeability ( $m$ )	$\nu$	Kinematic viscosity ( $m^2\ s^{-1}$ )
$L$	Channel length ( $m$ )	$\rho$	Density ( $kg\ m^{-3}$ )

that engineers can use well-understood structure-performance relationships when incorporating TPMS lattice geometries in heat sinks.

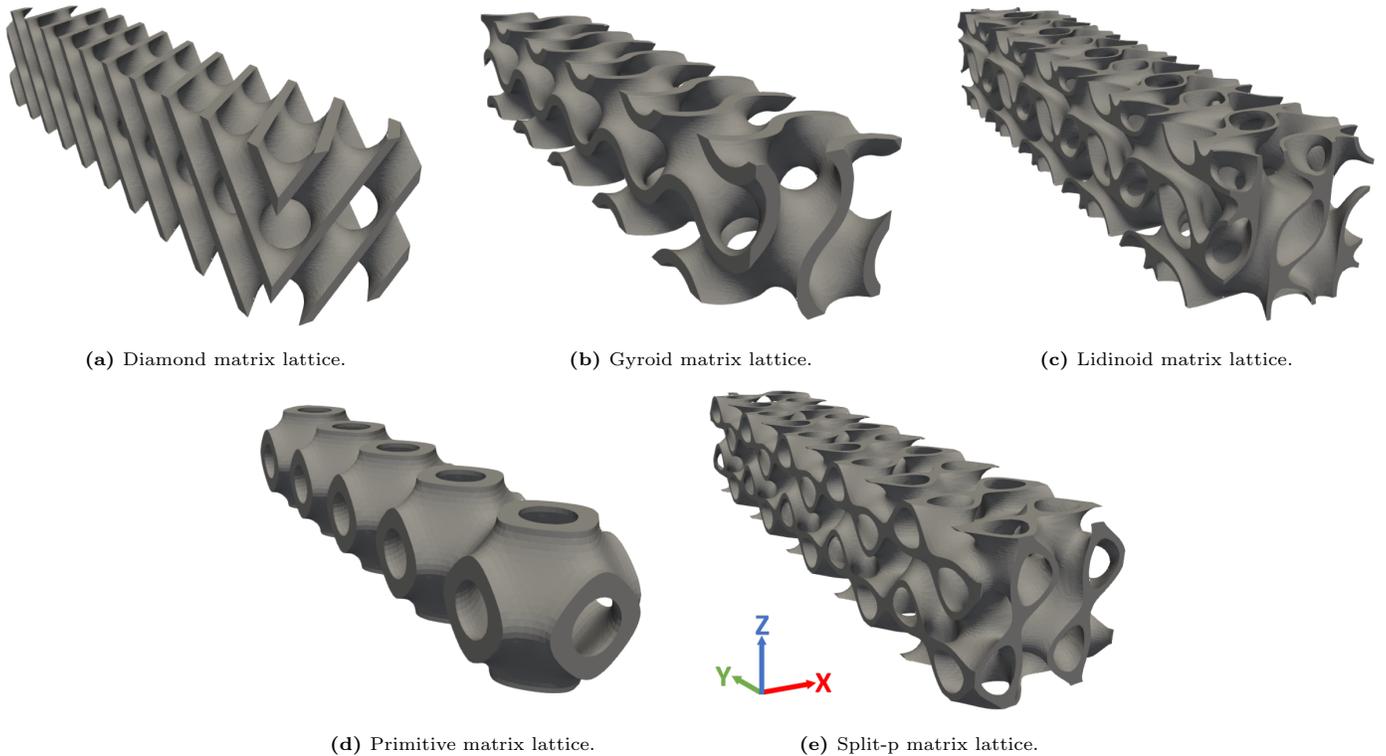
Pulvirenti *et al* [28] conducted a numerical study into the gyroid matrix lattice at low Reynolds numbers. The authors found that the lattice structure was characterised by local volumetric heat transfer coefficients similar to those of other periodic structures, such as the Kelvin geometry [16, 17]. Santos *et al* [37] examined the permeability of a range of TPMS lattice structures and found that the fluid flow was described by the Darcy-Forchheimer law, which is helpful in identifying designs for efficient lattice-based heat sinks. Concerning the permeability of foams, Della Torre *et al* [38] found an exponential dependence of the permeability on porosity, supporting the notion that porosity can be a useful design parameter for specifying flow in analogous AM lattices. Maloney *et al* [18] found the thermal conductance of a micro-strut-based lattice heat exchanger to be determined by various geometrical features such as node-to-node spacing and lattice member diameter. These studies provide an overall framework to develop structure-performance relationships for flow and heat transfer in AM cellular structures.

This paper examines the hydraulic and thermal transfer performance of TPMS-based lattice geometries over a range of fluid flow velocities and volume fractions. We establish design guides for fluid flow and heat transfer in these lattices in terms of their principal geometrical properties. Section 2 describes the methodology for our work, with sections 2.1, 2.2 and 2.3 providing details about the design of lattices and numerical modelling. Sections 3 and 4 contain the main results and discussion of this study. Concluding remarks are given in section 5.

## 2. Methodology

### 2.1. Cellular structures

Five TPMS lattice structures were chosen for this study. These were the diamond matrix (DM), gyroid matrix (GM), lidinoid matrix (LM), primitive matrix (PM) and split-p matrix (SPM) lattices. The DM, GM and PM lattices were chosen as they have received the most attention in the literature, whereas the remaining structures were chosen for their tortuous channels and high surface areas. The examined structures are illustrated in figure 1.



**Figure 1:** Examined structures (shown with a volume fraction of 0.25).

Each examined structure had dimensions of  $10 \times 50 \times 10$  mm containing  $1 \times 5 \times 1$  lattice cells. These were chosen to provide sufficient surface to allow the fluid to develop fully and to examine the evolution of mixing arising from the periodicity of the structures. To develop general structure-performance models for arbitrary lattice geometries, the fluid dynamics within the lattice cells must first be understood. For this reason, the structures examined here comprise a single unit cell in the directions normal to fluid flow.

The TPMS lattice structures were generated using FLatt Pack, a research-focused lattice design program [39]. TPMS-based lattice structures can be subdivided into ‘network’ and ‘matrix’ forms, where the matrix forms were used in this study as they possess greater surface areas per unit volume [40]. A network phase lattice consists of two continuous regions, one solid and one void. A matrix phase lattice has three continuous regions, two of which are void with equivalent geometries, with the other being a solid separating wall. For heat exchange applications, network lattices can exchange heat between a solid and a fluid while matrix lattices can exchange heat between two fluids across a solid barrier. One of the key geometrical properties of TPMS lattice structures is volume fraction,  $\gamma$ , defined as the ratio between solid volume and design space volume. This can be controlled, for TPMS matrix lattices, by modifying the thickness of the walls. The volume fraction of the examined structures in this study ranged from  $\gamma = 0.15 - 0.4$ .

## 2.2. Computational Method

Computational fluid dynamics (CFD) was used here to model fluid flow and conjugate heat transfer. Numerical results were obtained using OpenFOAM v1812 [41], an open-source CFD software written in C++. A conjugate heat transfer solver, *chtMultiRegionSimpleFoam*, was used to model incompressible, steady-state fluid and heat transfer between different bodies, where the solid region is modelled with the heat conduction equation and the fluid-solid solutions are coupled at the common boundaries by imposing continuity of temperature and heat flux.

Our CFD models included a fluid domain of dimensions  $10 \times 90 \times 10$  mm, which encapsulate the solid lattice test structure. Inlet and outlet pipes were used, 20 mm in length each. This was sufficient for the flow to develop and transition in to the structures and to prevent the propagation of any divergent results upstream from the outlet. The *snappyHexMesh* utility was used to import STL representations of the lattice structures into the computational domain to provide CFD meshes. Finally, heat was applied to the lattice structure via a constant-temperature ‘heater’ (dimensions  $10 \times 50 \times 2$  mm) at 323 K at the base of the modelled domain. Heating was applied in one direction in order to be more closely analogous to real applications (e.g., liquid cooling of a CPU or GPU) and to examine the impact of a directional heat source. The fluid-solid boundary was modelled as a smooth interface. This was chosen instead of a rough interface, which may be more reflective of AM components generally, in order to obtain useful structure-performance relationships applicable across a range of manufacturing and materials scenarios. The schematics of the computational domain of a simple circular channel model (which was used for numerical validation) are given in figure 2.

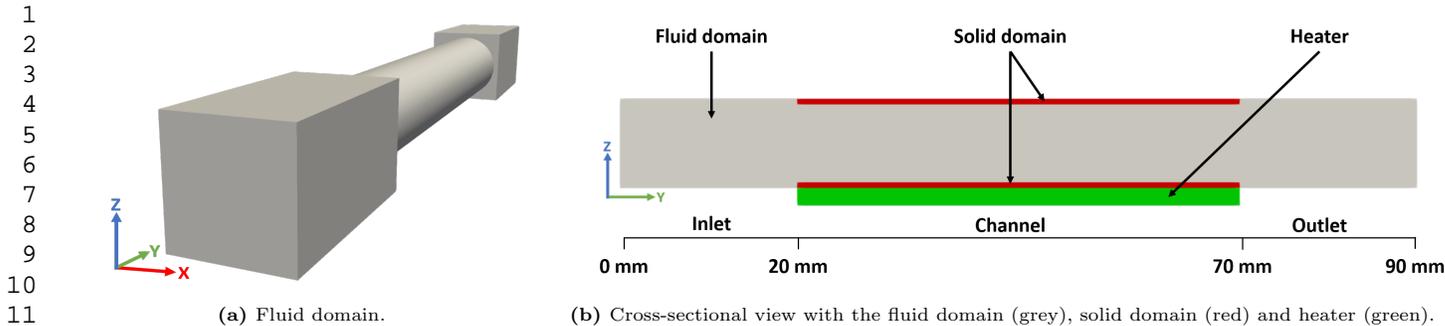


Figure 2: Schematics for the computational domain of a simple circular channel.

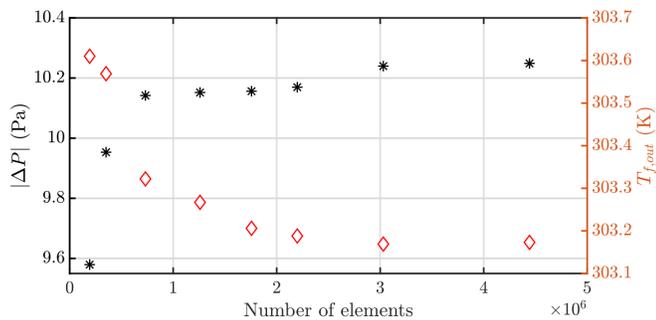


Figure 3: Pressure drop (black star) and outlet fluid temperature (red diamond) mesh convergence analysis for a gyroid matrix lattice (superficial fluid velocity =  $6 \times 10^{-3} \text{ m s}^{-1}$ , volume fraction = 0.4).

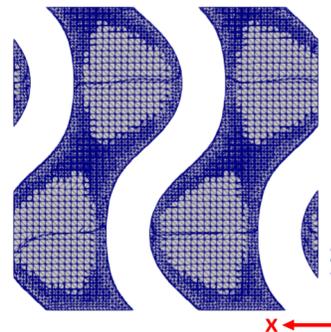


Figure 4: Mesh elements in the fluid domain for a gyroid matrix lattice with a volume fraction of 0.4 at a position  $y=0.025 \text{ m}$  after the inlet.

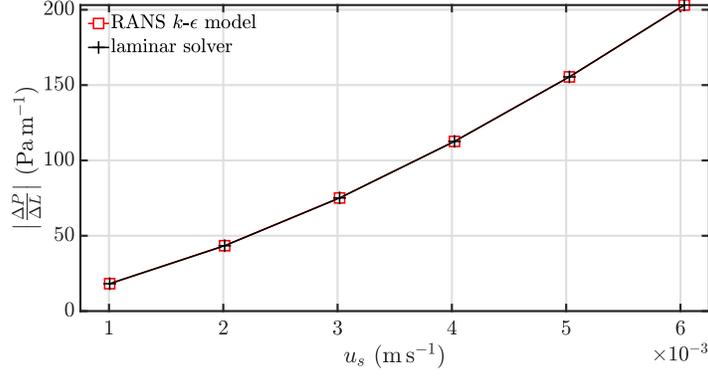
Water was modelled in the fluid domain with a density,  $\rho$ , of  $1,000 \text{ kg m}^{-3}$ , a kinematic viscosity,  $\nu$ , of  $8.9 \times 10^{-7} \text{ m}^2 \text{ s}^{-1}$  and assumed to be incompressible. The fluid travelled in the  $y$ -direction, with inlet flow rates of  $u_s = 0.8 \times 10^{-3} - 6 \times 10^{-3} \text{ m s}^{-1}$ , corresponding to  $Re = 3.2 - 62.5$ . This flow regime was examined to ensure that there would be significant differences between results at different flow rates and such that the performance of these structures can be meaningfully compared with other work in the literature, such as that of Pulvirenti *et al* [28] and Santos *et al* [37].

Flow in our CFD models was defined by the *noSlip* boundary condition (BC) at the walls and fluid boundaries, forcing the fluid velocity to zero, and the *pressureInletOutletVelocity* BC at the outlet, where a zero-gradient condition was applied for outflow. The inlet flow was defined by a *fixedValue* BC, which fixes the velocity to a specified value. The pressure at the outlet was defined by a *fixedValue* BC and the inlet was defined by a *fixedFluxPressure* BC, which sets the pressure gradient such that the flux is specified by the fluid velocity BC. The temperature of the fluid domain was defined by an *inletOutlet* BC of 293 K applied to the inlet fluid and a *zeroGradient* (adiabatic) BC at the outlet and outer walls. A *zeroGradient* BC was applied to the outlet because the profile of the outlet temperature was not known and to prevent error propagation upstream. The solid domain was modelled as Inconel-718 (which has seen extensive use in heat sinks in the aerospace industry [2]), with a density of  $8,190 \text{ kg m}^{-3}$ , specific heat capacity of  $435 \text{ J kg}^{-1} \text{ K}^{-1}$  and a thermal conductivity of  $11.4 \text{ W m}^{-1} \text{ K}^{-1}$ . These boundary conditions are well-established for finite-volume modelling and have been used to accurately predict fluid flow and heat transfer [17, 29, 38].

A CFD mesh convergence study was performed to determine a suitable mesh element density for accurate fluid flow and conjugate heat transfer predictions. This was performed for a GM lattice structure ( $\gamma = 0.4$ ) at a volumetric flow rate of  $6 \times 10^{-7} \text{ m}^3 \text{ s}^{-1}$ . The pressure drop and outlet fluid temperature were found to be well converged at around 1.8 million elements, as shown in figure 3, for an unstructured mesh featuring refined polyhedral elements at the fluid-solid boundaries and hexahedral elements elsewhere (see figure 4). The models used throughout this work therefore feature similar meshes.

A convergence study was also performed for a GM lattice structure ( $\gamma=0.4$ ) to determine whether a turbulence model was necessary to model the fluid accurately. This was done because the tortuous channels of TPMS structures may promote turbulence and, as will be discussed in section 2.3, the examined flow range is in the laminar-turbulent transition region for a porous structure [42]. The pressure drop for the RANS (Reynolds-averaged Navier-Stokes)  $k-\epsilon$  turbulence model agreed with the results from a laminar solver, as shown in figure 5, so a turbulence model was not deemed necessary for this study.

Our CFD model was first validated against the numerical results presented by Pulvirenti *et al* [28], where we obtained differences of 1.4% and 0.01% for the pressure drop and fluid exit temperature, respectively, for an equivalent GM structure.



**Figure 5:** Comparison of pressure drop results from the RANS  $k - \epsilon$  model and a laminar solver for a gyroid matrix with a volume fraction of 0.4.

For completeness, we used our CFD model to predict fluid flow in a simple circular channel. This was validated against the Hagen-Poiseuille law, with our numerical model predicting pressure drops within 1% of the analytical solution.

### 2.3. Theoretical background and method

Pressure drop,  $\Delta P$ , across the test structures was examined to determine the hydraulic performance of each lattice type. In addition, the fluid dynamics and variation of fluid pressure within the structures were examined to understand the impact of different lattice geometries at equivalent volume fractions.

Darcy's law describes pressure drop across a porous medium for slow, viscous flow [37]

$$\left| \frac{\Delta P}{\Delta L} \right| = \frac{\mu}{K} u_s, \quad (1)$$

where  $\Delta P/\Delta L$  is the pressure drop per unit length,  $\mu$  is the dynamic viscosity,  $K$  is the Darcian permeability constant and  $u_s$  is the superficial fluid velocity. At high flow rates, where the flow is no longer in the Darcy regime, a non-linear term is added to account for inertial effects. This is known as the Forchheimer term [43]. We have

$$\left| \frac{\Delta P}{\Delta L} \right| = \frac{\mu}{K_1} u_s + \frac{\rho}{K_2} u_s^2, \quad (2)$$

where  $K_1$  is the Forchheimer permeability constant and  $K_2$  is the inertial permeability constant. The permeability constants are in general associated with the geometry of the porous medium, where  $K$  and  $K_1$  represent the viscous drag and  $K_2$  is linked to the blockage of the internal geometry [43]. It is important to note that  $K$  and  $K_1$  are not the same. This is because transitioning from a Darcian to a Forchheimer flow regime implies changes to the viscous and inertial drags [43, 44]. It is vital to know which regime applies to the flow in a particular structure, in order to use the appropriate model. This was achieved by rearranging equation 2 to obtain

$$\left| \frac{\Delta P}{\Delta L u_s} \right| = \frac{\mu}{K_1} + \frac{\rho}{K_2} u_s, \quad (3)$$

which was then used to fit pressure drop data [43]; any part which is linear with  $u_s$  is Forchheimer flow.

Reynolds numbers are also quoted in this work as they provide more general descriptions of fluid flow and can be compared to other studies, which may use different initial conditions and geometries. The Reynolds number for a porous structure is [15]

$$Re = \frac{u_s D_h}{\nu (1 - \gamma)}, \quad (4)$$

where  $D_h$  is the hydraulic diameter. The transition region between laminar and turbulent flow for porous structures exists for  $10 < Re < 2000$  [42].

$D_h$  for porous structures was calculated using [15]

$$D_h = 4 \frac{V_w}{A_{w,s}}, \quad (5)$$

where  $V_w$  is the wetted volume and  $A_{w,s}$  is the wetted surface area, which was extracted from the CAD representations of the lattice structures. This approach was used to estimate the hydraulic diameter as it takes into account the complexity

of different lattice types across the entire fluid domain. Different lattice types therefore experience different Reynolds numbers at equivalent inlet flow rates, which more accurately reflects the fluid flow compared to the assumption of equal  $Re$ .

Heat transfer performance was examined through mass flow rate weighted averages of heat transfer coefficients and Nusselt numbers. Two different heat transfer coefficients were used, the first being a local wall heat transfer coefficient,  $h_l$ . This was calculated directly in OpenFOAM using the Reynolds analogy model, which relates the wall shear stress to heat transfer [45, 46]. Due to it being a local variable, it can be used to determine points of high and low heat transfer within the examined structures.

The global, mean heat transfer coefficient,  $h_m$ , was also examined. It is given by [20]

$$h_m = \frac{\dot{m}c_p(T_{f,out} - T_{f,in})}{A_{w,s}\Delta T_{LMTD}}, \quad (6)$$

where  $\dot{m}$  is the fluid mass flow rate,  $c_p$  is the fluid specific heat capacity,  $T_{f,out}$  and  $T_{f,in}$  are the fluid outlet and inlet temperatures and  $\Delta T_{LMTD}$  is the logarithmic mean temperature difference.  $\Delta T_{LMTD}$  was given by

$$\Delta T_{LMTD} = \frac{T_{f,out} - T_{f,in}}{\ln\left(\frac{T_h - T_{f,in}}{T_h - T_{f,out}}\right)}, \quad (7)$$

where  $T_h$  is the heater temperature. This definition of  $\Delta T_{LMTD}$  has been used previously by Dixit *et al.* [20], but an alternative definition uses the average channel surface temperature,  $T_s$ , in place of  $T_h$  [25].  $T_h$  was used here because  $T_h - T_{f,in}$  gives the initial temperature difference in the structures [36], whereas using  $T_s$  gives the heat transfer over the entire fluid-solid interface, the size of which varies significantly between lattice designs.  $T_s$  is also not representative of the large distribution of surface temperatures present in TPMS lattice structures, as observed by Al-Ketan *et al* [30].

In this study, the volumetric heat transfer coefficient,  $h_{m,vol}$ , was used instead of  $h_m$  as it is independent of the surface area (which differs for different lattice structures at equivalent volume fraction). This was obtained from [16]

$$h_{m,vol} = h_m A_\nu, \quad (8)$$

where  $A_\nu$  is the specific surface area (ratio of wetted surface area to design space volume).

The volumetric Nusselt number gives the ratio of convective to conductive heat transfer for a fluid and is an alternative way to express heat transfer performance. This was defined by [16]

$$Nu_{vol} = \frac{h_{m,vol} D_h^2}{k}, \quad (9)$$

where  $k$  is the thermal conductivity of the fluid. This is a dimensionless quantity and can be used alongside  $Re$  to compare structures under different flow conditions.

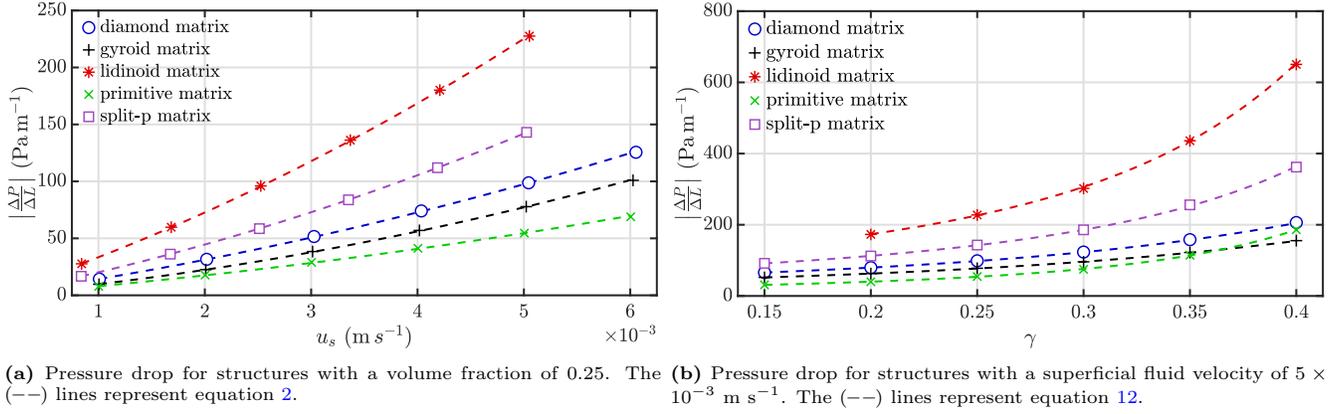
### 3. Results

#### 3.1. Hydraulic Performance

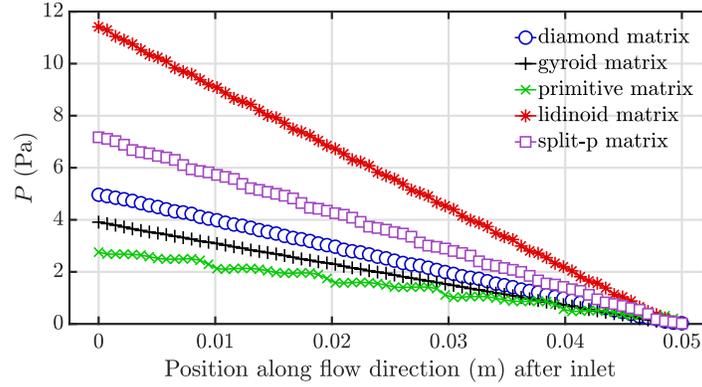
Pressure drop for a range of volume fractions and flow rates are presented in figure 6, where  $\Delta P$  is calculated by finding the difference between inlet and outlet average pressure. Figure 6 shows that  $\Delta P$  increases non-linearly with both  $u_s$  and  $\gamma$  and that the LM lattice exhibits the greatest pressure drop across the examined ranges while the PM lattice exhibits the lowest pressure drop in most cases. At low volume fractions the GM lattice exhibits greater pressure drop than the PM structure. This behaviour switches as volume fraction increases, indicating that a particular lattice geometry may not be treated as inherently more efficient than others, with performance also being dependent on fluid flow conditions.

Examining the evolution of fluid pressure, taken as a cross-sectional average (figure 7), we see that pressure decreases linearly along the flow direction in the GM, DM and LM structures, despite the tortuous nature of the channels. This is not replicated in the PM or SPM structures, which instead exhibit step-like pressure drops.

This can be explained by examining the flow within the structures, as shown in figure 8. Regarding the PM structure, the majority of the fluid passes through a central volume or ‘channel’. However, as the channel diameter decreases at the cell boundary, some fluid is recirculated in the characteristic chambers of the PM lattice, appearing as eddies. The PM geometry therefore acts as a series of bottlenecks, providing sharp pressure drops within the structure. This can also be observed in the SPM lattice, but to a lesser degree. Flow is not periodically impeded in the remaining structures because their internal geometry does not possess such large variations in channel diameter, minimising fluid recirculation. The dominant factor behind pressure drop for TPMS structures is therefore the channel diameter, where smaller channels lead to larger pressure drops, shown in figure 9, and where changes in the diameter lead to localised pressure drops.



**Figure 6:** Pressure drop for different examined geometries.



**Figure 7:** Evolution of pressure within the examined structures (superficial fluid velocity =  $5 \times 10^{-3} \text{ m s}^{-1}$ , volume fraction = 0.25).

Before calculating the permeability constants, the flow regime must be determined. We found  $\Delta P/(\Delta L u_s)$  to increase linearly with  $u_s$  for each lattice over the range of examined volume fractions, indicating that the flow is in the Forchheimer regime.  $\Delta P/\Delta L$  were therefore fit with equation 2 to determine the permeability constants for each lattice structure, which are plotted in figure 10.  $K_2$  are four orders of magnitude larger than  $K_1$ , with both constants decreasing as volume fraction increases. Figure 10 shows that at low volume fractions, the PM structure exhibits larger  $K_1$  and  $K_2$  than the GM structure. This changes at a volume fraction of  $\gamma = 0.310$  for  $K_1$  and we observe the approach of this change at  $\gamma = 0.4$  for  $K_2$ . Therefore, the volume fraction at which the pressure drop intersects for the GM and PM structures in figure 6(b) will be in the range of 0.310 - 0.4, irrespective of fluid velocity.

For each examined lattice type, the following equations

$$K_1 = A_1 \gamma^2 + B_1 \gamma + C_1, \quad (10)$$

$$K_2 = A_2 \gamma^2 + B_2 \gamma + C_2 \quad (11)$$

were used to relate  $K_1$  and  $K_2$  to the volume fraction,  $\gamma$ , where  $A_{1,2}$ ,  $B_{1,2}$  and  $C_{1,2}$  are fit parameters. Equation 2 can then be expressed as

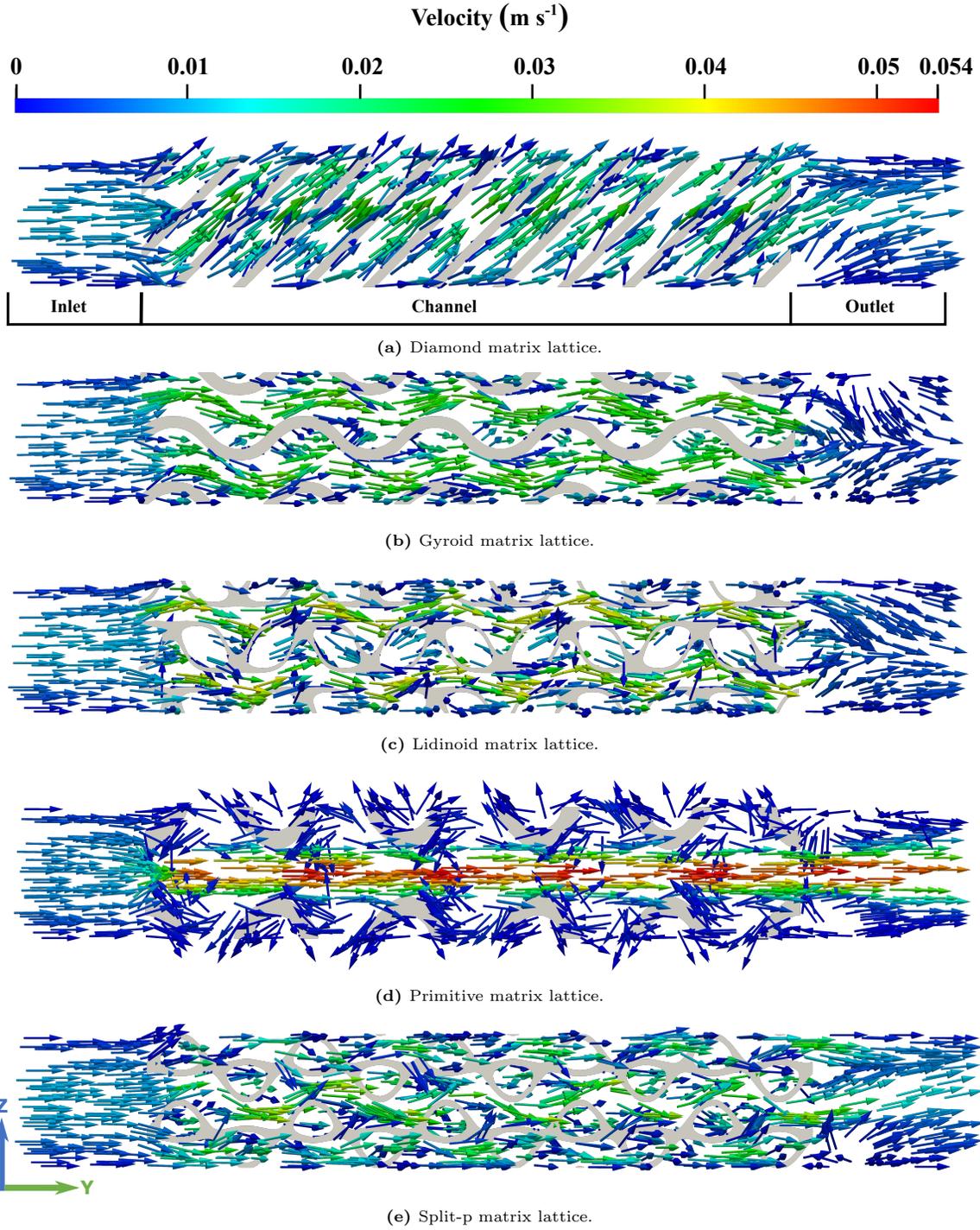
$$\left| \frac{\Delta P}{\Delta L} \right| = \frac{\mu u_s}{A_1 \gamma^2 + B_1 \gamma + C_1} + \frac{\rho u_s^2}{A_2 \gamma^2 + B_2 \gamma + C_2}, \quad (12)$$

which can be used to predict the pressure drop exhibited by each structure over a range of volume fractions and superficial fluid velocities. Equation 12 describes a surface, where figure 11 gives the hydraulic performance of the GM lattice. Fit values for the parameters are given in table 1, which can henceforth be used to specify the volume fraction for the examined TPMS structures to provide a pressure drop for a known flow rate.

### 3.2. Thermal performance

Volumetric Nusselt numbers are presented in figure 12. Correlations of the form

$$Nu_{vol} = F Re^n \quad (13)$$



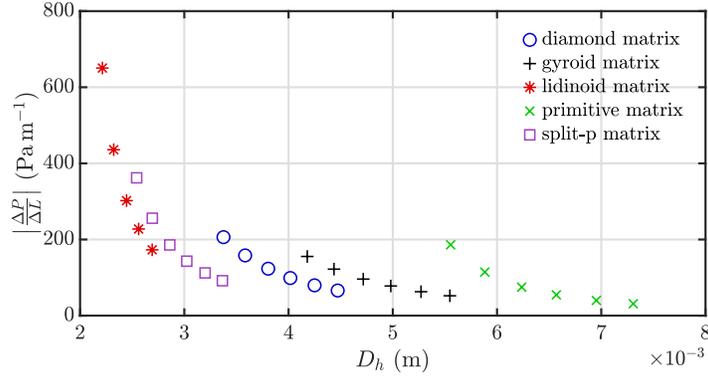
**Figure 8:** Fluid velocity vectors across the  $x = 0.005$  m plane for structures with a volume fraction of 0.25 and a superficial fluid velocity of  $5 \times 10^{-3} \text{ m s}^{-1}$ .

were sought, where Fu *et al* [16] stated that the parameters  $n$  and  $F$  are related to the geometrical features of the structure, with  $F$  also containing the Prandtl number. Figure 12 shows that this relationship describes the data accurately. Equations 9 and 13 are then combined to provide

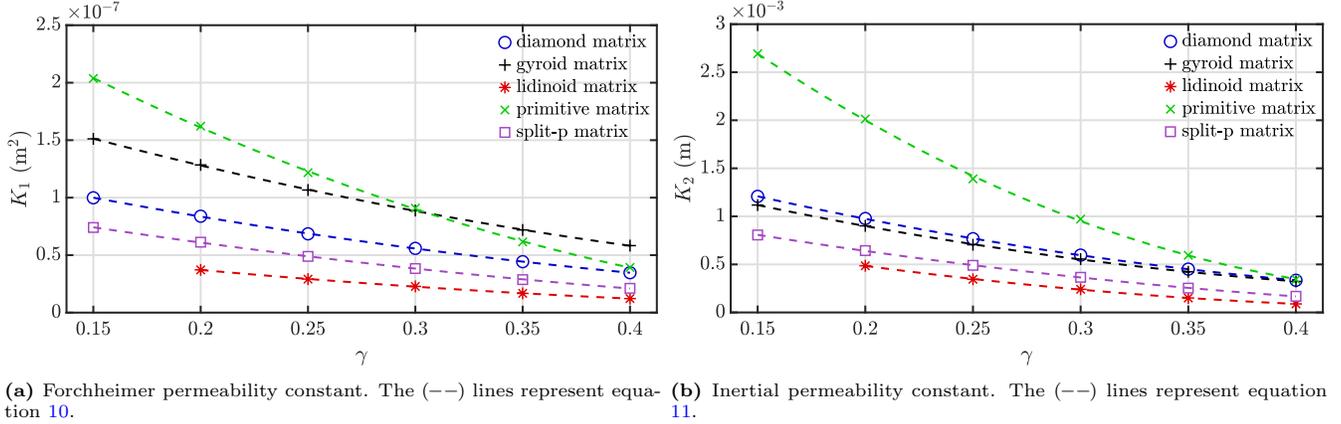
$$h_{m,vol} = \frac{F k Re^n}{D_h^2}, \quad (14)$$

which is valuable because it can be expressed in terms of  $u_s$  and  $\gamma$ . This is done using equations 4 and 5, while  $D_h$  can also be defined as

$$D_h = 4 \frac{V_w}{A_{w,s}} = 4 \frac{(V_w/V_T)}{(A_{w,s}/V_T)} = \frac{4}{A_\nu} (1 - \gamma), \quad (15)$$



**Figure 9:** Pressure drop for structures with a superficial fluid velocity of  $5 \times 10^{-3} \text{ m s}^{-1}$ .



**(a)** Forchheimer permeability constant. The (---) lines represent equation 10. **(b)** Inertial permeability constant. The (---) lines represent equation 11.

**Figure 10:** Permeability constants as a function of volume fraction for different lattice geometries.

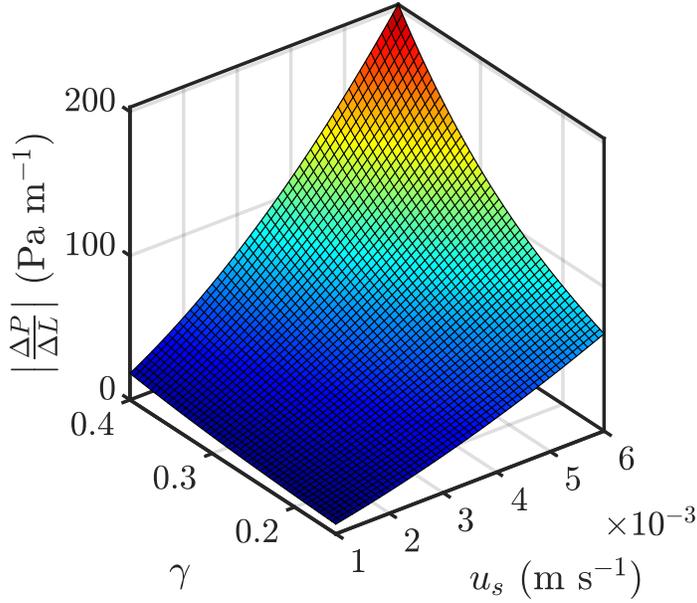
Lattice type	Fit parameter $\times 10^{-7} \text{ (m}^2\text{)}$				Fit parameter $\times 10^{-3} \text{ (m)}$			
	$A_1$	$B_1$	$C_1$	Adjusted $R^2$	$A_2$	$B_2$	$C_2$	Adjusted $R^2$
DM	3.4	-4.5	1.59	0.9999	5.9	-6.8	2.09	0.9999
GM	4.7	-6.3	2.35	0.9998	6.0	-6.5	1.95	0.9998
LM	2.1	-2.5	0.79	0.9995	4.9	-4.9	1.27	0.9994
PM	10.1	-12.1	3.63	0.9997	22.3	-21.6	5.44	0.9994
SPM	2.6	-3.6	1.22	0.9998	3.9	-4.7	1.42	0.9998

**Table 1:** Determined fit parameters for equation 12.

where  $V_T$  is the total volume of the design space. Equation 14 can therefore be expressed as

$$\begin{aligned}
 h_{m,vol} &= F k D_h^{n-2} \left( \frac{u_s}{\nu (1-\gamma)} \right)^n, \\
 &= F k \left( \frac{4}{A_\nu} \right)^{n-2} (1-\gamma)^{n-2} \left( \frac{u_s}{\nu (1-\gamma)} \right)^n, \\
 &= F k \left( \frac{4}{A_\nu} \right)^{n-2} \left( \frac{u_s}{\nu} \right)^n (1-\gamma)^{-2}.
 \end{aligned} \tag{16}$$

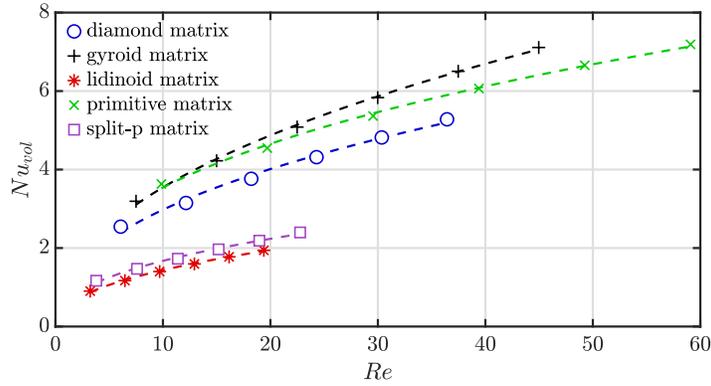
To use equation 16 as a predictive model for  $h_{m,vol}$  over a range of fluid velocities and volume fractions, the dependence of  $A_\nu$ ,  $n$  and  $F$  on volume fraction must be known. It was found that  $A_\nu$  and  $n$  decrease with volume fraction, and are well described by  $A_\nu = p_1 \gamma^{p_2} + p_3$  and  $n = n_1 \gamma + n_2$ . We observed no discernible relationship between  $F$  and  $\gamma$ , but, for



**Figure 11:** Pressure drop as a function of superficial fluid velocity and volume fraction for the gyroid matrix lattice.

Lattice type	$p_1$ ( $\text{m}^{-1}$ )	$p_2$	$p_3$ ( $\text{m}^{-1}$ )	Adjusted $R^2$	$\bar{F}$	$n_1$	$n_2$	Adjusted $R^2$
DM	-405	2.13	768	0.9999	1.06	-0.277	0.510	0.8812
GM	-308	2.09	619	0.9998	1.21	-0.173	0.499	0.9738
LM	-847	1.92	1232	0.9990	0.52	-0.455	0.554	0.9987
PM	-305	2.23	471	0.9998	1.39	-0.135	0.431	0.9709
SPM	-580	2.13	1026	0.9999	0.63	-0.106	0.444	0.8733

**Table 2:** Determined fit parameters for equation 17.

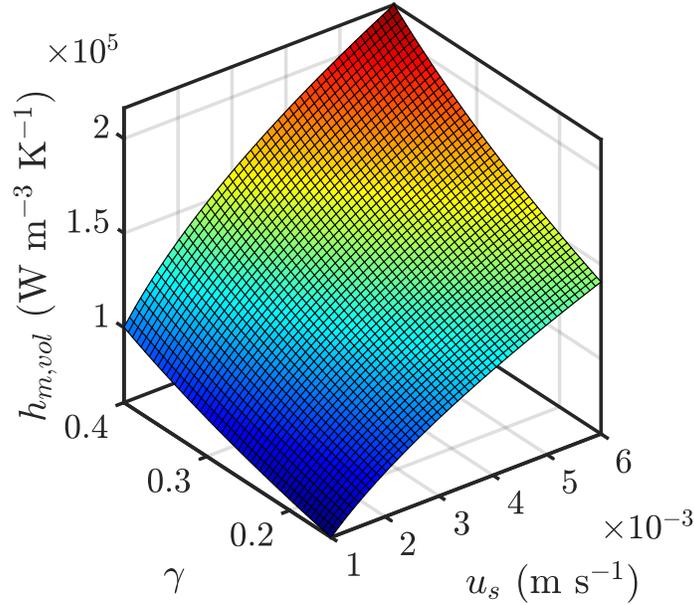


**Figure 12:** Volumetric Nusselt numbers for lattice structures with a volume fraction of 0.25. The (--) lines represent equation 13.

each lattice type, the full range of  $F$  values was seen to fall within  $\bar{F} \pm 8\%$ . Therefore,  $F$  was treated as a constant by calculating its mean value. The volumetric heat transfer coefficient can then be obtained from

$$h_{m,vol} = \frac{\bar{F} k \left( \frac{4}{p_1 \gamma^{p_2} + p_3} \right)^{n_1 \gamma + n_2 - 2} \left( \frac{u_s}{\nu} \right)^{n_1 \gamma + n_2}}{(1 - \gamma)^2}, \quad (17)$$

with  $p_{1,2,3}, \bar{F}$  and  $n_{1,2}$  given in table 2. This equation describes a surface and can be used to predict the volumetric



**Figure 13:** Volumetric heat transfer coefficient as a function of superficial fluid velocity and volume fraction for the gyroid matrix lattice.

heat transfer coefficient over a range of volume fractions and superficial fluid velocities. This model accurately predicts the  $h_{m,vol}$  from equation 8, with a maximum deviation less than 10% over the examined ranges of volume fraction and flow rate.

Figure 13 displays the volumetric heat transfer coefficient exhibited by the gyroid matrix lattice, predicted by equation 17. By comparing the predicted  $h_{m,vol}$  of the examined lattice structures, we found that the LM lattice exhibits the greatest volumetric heat transfer coefficient at low volume fractions, but at higher volume fractions the DM lattice has the highest volumetric heat transfer coefficient. The PM lattice exhibits the lowest volumetric heat transfer coefficient in this study.

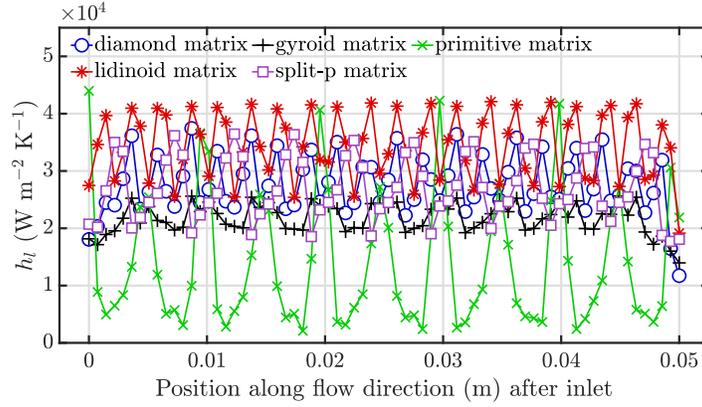
Local effects are important in determining how the lattice geometry affects heat transfer.  $h_l$  were calculated across 70 equally spaced cross-sections along the flow direction and are shown in figure 14. All of the TPMS structures exhibit periodically fluctuating  $h_l$ , with the PM lattice showing the greatest variation in  $h_l$ , from 2,100 to 44,000  $\text{W m}^{-2} \text{K}^{-1}$ . Despite having the highest  $h_l$ , the PM structure also has the lowest  $h_l$  which explains why it exhibits the lowest  $h_{m,vol}$ . The other lattice structures have much smaller variance in  $h_l$ .

The distribution of  $h_l$  on the lattice surface further explains the differences observed in figure 14. For simplicity, figure 15 displays the distribution of  $h_l$  for the PM lattice, which was chosen due to the large variation in  $h_l$ . Peak  $h_l$  are seen in regions where the central channel diameter is narrowest. Other high  $h_l$  regions are observed outside the central channel, where the diameter is at a local minima. The fluid moves faster in these sections, as shown in figure 8(d), and can therefore transport more heat from the walls there. The same effect is also observed, but to a lesser extent due to the smaller variation in channel diameter, in the other examined TPMS structures. For example,  $h_l$  for the GM lattice takes values from 14,000 - 25,000  $\text{W m}^{-2} \text{K}^{-1}$ , which is a much narrower range than that exhibited by the PM lattice (2,100 - 44,000  $\text{W m}^{-2} \text{K}^{-1}$ ). From figure 8, we can therefore deduce that  $h_l$  is driven mainly by local fluid velocity, which itself is largely determined by channel diameter.

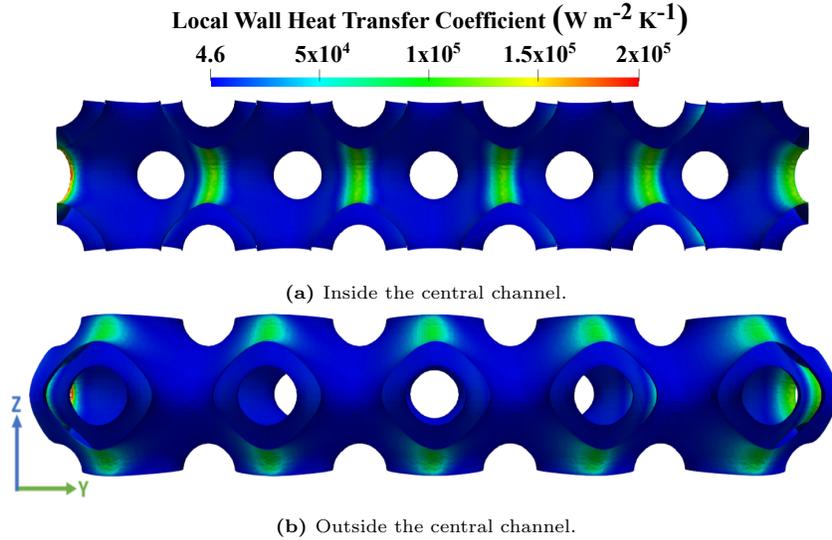
## 4. Discussion

### 4.1. Hydraulic performance

Santos *et al* [37] calculated the permeability constants for a variety of lattice structures consisting of  $4 \times 4 \times 4$  cells in a  $13 \times 13 \times 13$  mm volume over a range of different flow regimes. Compared to the structures in the present study, those lattices have greater surface area and a larger number of cells per unit volume, and we can therefore expect the structures of Santos *et al* [37] to be less permeable than the lattice structures examined here within the Forchheimer flow region. This is confirmed in figure 16, where the permeability constants,  $K_1$  and  $K_2$ , for the gyroid and primitive matrix lattices in this study are up to two orders of magnitude greater than those of Santos *et al* [37]. This highlights the challenge of developing general and practicable analytical relationships for the flow in these structures, as a range of geometrical properties, such as the number of cells and size of the design space, clearly have a large impact on the permeability. However, the work of Santos *et al* [37] confirms our observation that the primitive matrix lattice is more permeable at lower volume fractions



**Figure 14:** Evolution of local wall heat transfer coefficient within the examined structures (superficial fluid velocity =  $5 \times 10^{-3} \text{ m s}^{-1}$ , volume fraction = 0.25).

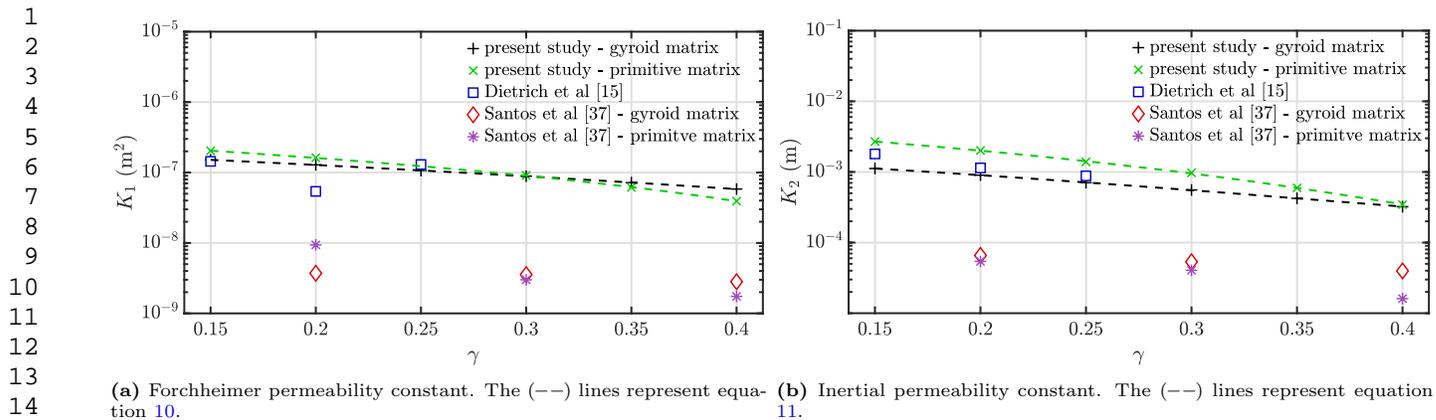


**Figure 15:** Distribution of local wall heat transfer coefficient in the primitive matrix lattice (superficial fluid velocity =  $5 \times 10^{-3} \text{ m s}^{-1}$ , volume fraction = 0.25).

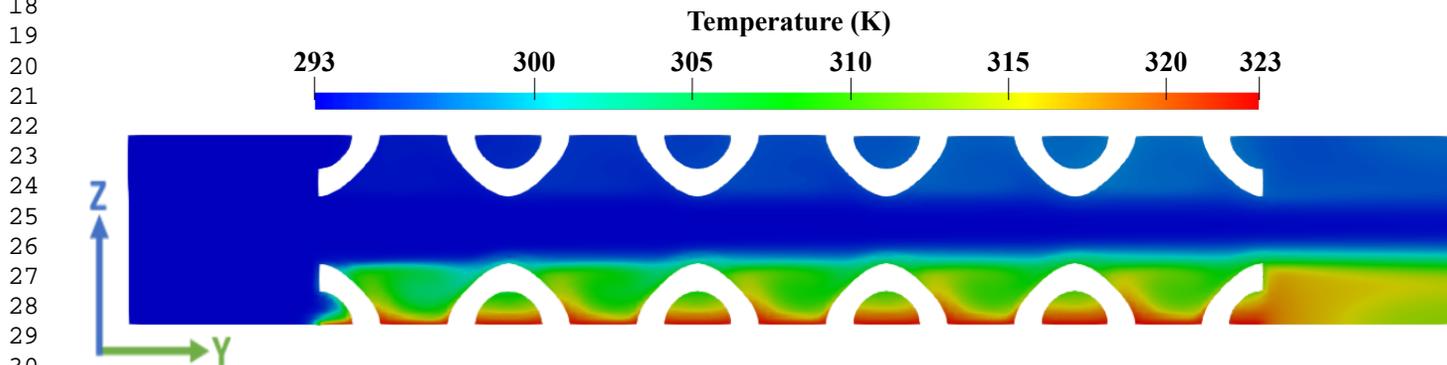
and the gyroid matrix is more permeable at larger volume fractions. This finding was further elaborated in the current study by examining the fluid dynamics (see figure 8) and evolution of pressure drop (see figure 7) within the structures. Sharp pressure drops were found in the primitive matrix lattice at the cell boundaries, while the gyroid matrix lattice does not exhibit this behaviour.

Dietrich *et al* [15] calculated the permeability constants for foams of different materials with varying pore sizes and volume fractions. A selection of those results are compared to this study in figure 16. The gyroid and primitive matrix lattices possess permeabilities similar to manufactured foam. Additionally manufactured surface-based lattices can therefore be a valid substitute for conventional foams in fluid flow applications, as they are hydraulically no less efficient and also possess a greater degree of tailorability due to their computer-based design method.

The Forchheimer and inertial permeabilities of lattice structures are dependent on the internal geometry, and therefore, quite clearly, the volume fraction. The fits used here (equations 10 and 11) are empirical, and do not account specifically for such factors as surface area or channel tortuosity, either of which may be found to have a predominant effect on fluid flow. A robust, general model will incorporate these, and other, geometrical factors into structure-performance relationships capable of accurately predicting fluid through any lattice type. We have made a contribution towards this goal with equation 12 and the parameters quoted in table 1. These can be used to predict pressure drop over a range of volume fractions and superficial fluid velocities for the examined lattice structures. This will enable designers to make informed decisions on lattice design for fluid flow applications. These fit parameters are valid only for structures with  $1 \times 5 \times 1$  cells for a design space of  $10 \times 50 \times 10 \text{ mm}$ . These results are still valuable however, since flow in larger lattice structures (i.e.,  $N_1 \times N_2 \times N_3$  cells) is determined to a large extent by the characteristic fluid dynamics in individual cells.



**Figure 16:** Permeability constants of the gyroid matrix and primitive matrix structures of the current study, gyroid and primitive matrix structures [37] and a foam structure [15].



**Figure 17:** Fluid temperature across the  $x = 0.005$  m plane for the primitive matrix lattice (superficial fluid velocity =  $5 \times 10^{-3}$  m s $^{-1}$ , volume fraction = 0.25).

#### 4.2. Thermal performance

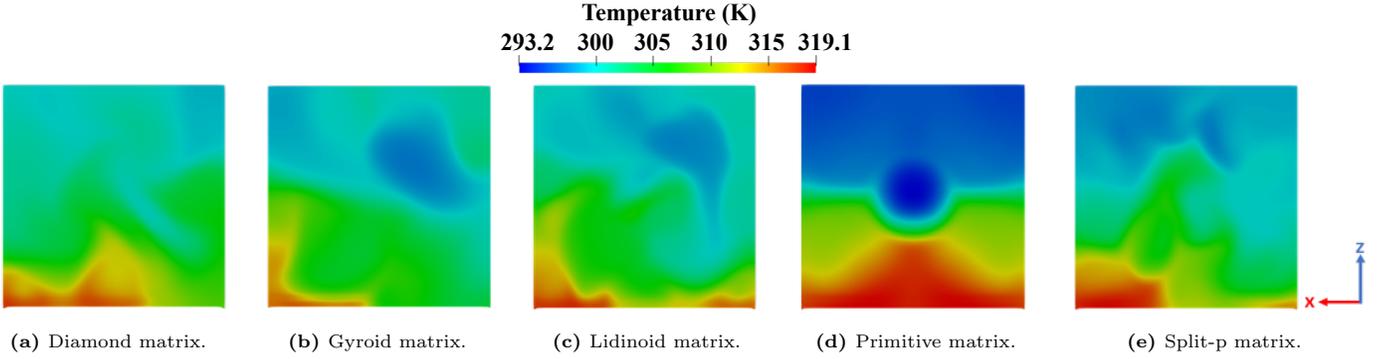
As discussed in sections 3.1 and 3.2, both the pressure drop and local wall heat transfer coefficient of surface-based lattice structures are dependent on channel diameter. We would therefore expect volumetric heat transfer coefficients to correlate with pressure drop. However, this is not always the case, as the diamond matrix lattice exhibits relatively small pressure drops but high volumetric heat transfer coefficients. Hence, engineers need to inspect both the hydraulic and thermal performance in tandem to determine appropriate surface-based lattice structures for heat transfer applications.

By examining the distribution of fluid temperature in the primitive matrix lattice in figure 17, we see that there is minimal thermal mixing within the structure due to the high-velocity central flow channel (shown in figure 8(d)) which prevents the fluid from moving across it. It is also observed that there is minimal heat transfer taking place in the upper regions where the local wall heat transfer coefficient peaks (figure 15). This is because heat is applied to the structure only from one direction (below). Therefore, structures which (i) conduct more heat through the lattice walls far away from the heat input, and (ii) maximise thermal mixing, should boast greater volumetric heat transfer coefficients.

This theory is corroborated by the distribution of fluid outlet temperature in figure 18. Here, we observe that the primitive matrix lattice, which has the lowest volumetric heat transfer coefficient, has the least well distributed fluid outlet temperature and that the top half of the structure is rendered relatively ineffective for heat transfer. Following on from this, the gyroid matrix and split-p matrix lattices have the next highest volumetric heat transfer coefficients and more evenly distributed fluid outlet temperatures, though a discontinuity is still observed between the bottom and top halves of those structures. Finally, the diamond and lidinoid matrix lattices have the highest volumetric heat transfer coefficients in this study and have relatively well distributed fluid outlet temperatures indicating better fluid mixing.

We conclude that the thermal performance of TPMS lattice structures is heavily dependent on the internal geometry of the structure in the case of a directional heat input, where lattices that can distribute heat across the entire fluid volume exhibit greater performance. In the case of a non-directional heat input (i.e., fixed wall temperatures) the thermal performance will be largely determined by the channel diameter. This is in keeping with the emerging picture from investigations of TPMS lattices as heat sinks; their thermal performance is heavily dependent on lattice cell geometry [30].

Nusselt numbers for foams were previously examined by Wu *et al* [17] and Fu *et al* [16], who employed the relationship given in equation 13. The excellent agreement with this model for the lattice structures examined here (figure 12) confirms that these surface-based lattices can be characterised by volumetric Nusselt numbers in the same way as conventional



**Figure 18:** Distribution of outlet fluid temperature (superficial fluid velocity =  $5 \times 10^{-3} \text{ m s}^{-1}$ , volume fraction = 0.25).

foams. A robust, general model will be able to predict the thermal performance of surface-based lattices across a range of superficial fluid velocities and volume fractions. Equation 17 and the parameters quoted in table 2 contribute to this and can accurately predict the volumetric heat transfer coefficient for the examined surface-based lattice structures with  $1 \times 5 \times 1$  cells in a  $10 \times 50 \times 10 \text{ mm}$  design space. This model can be improved, and generalised to other lattice structures, by incorporating more complete descriptions of how the Nusselt parameters  $F$  and  $n$  are affected by the internal lattice geometry.

Combined with the model discussed above for hydraulic performance, surface-based lattice structures can henceforth be designed in a way which minimises their pressure drop for a given flow rate whilst achieving a specified heat transfer coefficient. This can be achieved via a simple search-based algorithm applied to their pressure drop and heat transfer relationships (i.e., the surfaces given by equations 12 and 17). The practical implication of reduced pressure drop for a given flow rate is reduced power consumption for the pumps, or fans, which move the coolant through the heat sink. Being able to design surface-based heat sinks which reduce or maintain power consumption compared to traditional designs, whilst improving heat transfer capabilities, will be important for various applications which require enhanced cooling.

## 5. Conclusions

This work examines the fluid flow and conjugate heat transfer of five surface-based lattices suitable for additively manufactured heat sinks. Surface roughness, which influences fluid flow, was not considered in this study. This was done to obtain more general and useful results, as surface roughness is determined by the nature of the additive manufacturing process (extrusion, laser-sintering, etc.) and cannot easily be controlled as a design parameter. Our results provide a meaningful comparison between a selection of lattice types considered for heat management.

Permeability constants for the lattice structures were calculated and used to create a predictive model for pressure drop over a range of fluid velocities and volume fractions. Relationships between volumetric Nusselt number and Reynolds number were also found and can be used to predict volumetric heat transfer coefficients for examined structures across a ranges of fluid velocities and volume fractions. With these models, heat sinks based on the examined lattice structures can be designed to meet pre-defined performance requirements. The investigative method used in the current study can be applied across the large, and ever-increasing, family of lattice structures for heat sinks without expensive manufacturing and testing. It is unsuitable to extrapolate these relationships outside of the examined ranges of fluid velocity and volume fraction as they could yield erroneous results. For example, turbulent flow may have a large impact on the hydraulic and thermal performance at high Reynolds numbers.

The complex internal geometries of the lattice structures cause mixing and eddy formation, meaning they can be effective heat sinks. However, analysis of flow and fluid temperature distributions indicate that the primitive matrix lattice examined here is a poorer candidate for heat management than the other four structures, as fluid mixing is impeded by the formation of a high-velocity central flow channel. Lattice structures which are able to distribute heat across the entire fluid volume, such as the diamond matrix, are much better candidates for heat management as they maximise fluid-solid thermal interactions and fluid mixing.

Structure-performance relationships of the kind uncovered here can be used in conjunction with other such rules, like the Gibson-Ashby scaling laws for stiffness or thermal conductivity [5], to design multifunctional components which, for example, provide maximal stiffness and thermal transport within a given weight restriction. New lattice structures for efficient heat sinks can be identified, or even designed from first-principles, with greater understanding of their flow and heat transfer mechanisms. Such optimised surface-based heat sinks can only be manufactured via additive manufacturing and can therefore be embedded in components of arbitrary geometry without the need for subsequent joining or assembly processes.

## Acknowledgement

This work has been part-funded by the University of Nottingham and the EPSRC Energy Programme [grant number EP/T012250/1]. We are grateful for access to the University of Nottingham’s Augusta HPC service. The authors would also like to thank Mirco Magnini (University of Nottingham) for the support provided in developing the OpenFOAM model.

## References

- [1] I. Gibson, D. Rosen, and B. Stucker, *Additive manufacturing technologies: 3D printing, rapid prototyping, and direct digital manufacturing, second edition*, 2nd ed., D. W. D. W. Rosen and B. B. Stucker, Eds. New York: Springer, 2015.
- [2] B. Blakey-Milner, P. Gradl, G. Snedden, M. Brooks, J. Pitot, E. Lopez, M. Leary, F. Berto, and A. du Plessis, “Metal additive manufacturing in aerospace: A review,” *Materials & Design*, vol. 209, p. 110008, nov 2021. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S0264127521005633>
- [3] General Electric. (2018, oct) New manufacturing milestone: 30,000 additive fuel nozzles. [Online]. Available: <https://www.ge.com/additive/stories/new-manufacturing-milestone-30000-additive-fuel-nozzles>
- [4] M. F. Ashby, *Metal foams : a design guide*, M. F. Ashby, Ed. Oxford: Butterworth-Heinemann, 2000.
- [5] L. J. Gibson, *Cellular solids : structure and properties*, 2nd ed., ser. Cambridge solid state science series, M. F. Ashby, Ed. Cambridge: Cambridge University Press, 1997.
- [6] H. N. Wadley and D. T. Queheillalt, “Thermal Applications of Cellular Lattice Structures,” *Materials Science Forum*, vol. 539-543, pp. 242–247, mar 2007. [Online]. Available: <http://www.scientific.nethttps://www.scientific.net/MSF.539-543.242>
- [7] T. Lu, H. Stone, and M. Ashby, “Heat transfer in open-cell metal foams,” *Acta Materialia*, vol. 46, no. 10, pp. 3619–3635, jun 1998. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S1359645498000317>
- [8] J. Tian, T. Lu, H. Hodson, D. Queheillalt, and H. Wadley, “Cross flow heat exchange of textile cellular metal core sandwich panels,” *International Journal of Heat and Mass Transfer*, vol. 50, no. 13-14, pp. 2521–2536, jul 2007. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S0017931007000129>
- [9] O. Al-Ketan, R. Rowshan, and R. K. Abu Al-Rub, “Topology-mechanical property relationship of 3D printed strut, skeletal, and sheet based periodic metallic cellular materials,” *Additive Manufacturing*, vol. 19, pp. 167–183, jan 2018. [Online]. Available: <https://doi.org/10.1016/j.addma.2017.12.006>
- [10] B. Jaspersen, Yongho Jeon, K. Turner, F. Pfefferkorn, and Weilin Qu, “Comparison of Micro-Pin-Fin and Microchannel Heat Sinks Considering Thermal-Hydraulic Performance and Manufacturability,” *IEEE Transactions on Components and Packaging Technologies*, vol. 33, no. 1, pp. 148–160, mar 2010. [Online]. Available: <http://ieeexplore.ieee.org/document/5286856/>
- [11] D. Hancock, D. Homfray, M. Porton, I. Todd, and B. Wynne, “Exploring complex high heat flux geometries for fusion applications enabled by additive manufacturing,” *Fusion Engineering and Design*, vol. 136, pp. 454–460, 2018.
- [12] J. H. Ferziger, M. Perić, and R. L. Street, *Computational Methods for Fluid Dynamics*, 4th ed. Cham: Springer International Publishing, 2020. [Online]. Available: <http://link.springer.com/10.1007/978-3-319-99693-6>
- [13] J. Iyer, T. Moore, D. Nguyen, P. Roy, and J. Stolaroff, “Heat transfer and pressure drop characteristics of heat exchangers based on triply periodic minimal and periodic nodal surfaces,” *Applied Thermal Engineering*, vol. 209, p. 118192, jun 2022. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S135943112200148X>
- [14] Z. Wu, C. Caliot, F. Bai, G. Flamant, Z. Wang, J. Zhang, and C. Tian, “Experimental and numerical studies of the pressure drop in ceramic foams for volumetric solar receiver applications,” *Applied Energy*, vol. 87, no. 2, pp. 504–513, feb 2010. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S0306261909003122>
- [15] B. Dietrich, W. Schabel, M. Kind, and H. Martin, “Pressure drop measurements of ceramic sponges-Determining the hydraulic diameter,” *Chemical Engineering Science*, vol. 64, pp. 3633–3640, 2009.
- [16] R. Viskanta, X.Fu and J. Gore, “Measurement and correlation of volumetric heat transfer coefficients of cellular ceramics,” *Experimental Thermal and Fluid Science*, vol. 17, no. 4, pp. 285–293, aug 1998. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S089417779810002X>

- 1  
2 [17] Z. Wu, C. Caliot, G. Flamant, and Z. Wang, "Numerical simulation of convective heat transfer  
3 between air flow and ceramic foams to optimise volumetric solar air receiver performances," *International*  
4 *Journal of Heat and Mass Transfer*, vol. 54, no. 7-8, pp. 1527–1537, mar 2011. [Online]. Available:  
5 <https://linkinghub.elsevier.com/retrieve/pii/S0017931010006514>
- 6  
7 [18] K. J. Maloney, K. D. Fink, T. A. Schaedler, J. A. Kolodziejska, A. J. Jacobsen, and C. S. Roper,  
8 "Multifunctional heat exchangers derived from three-dimensional micro-lattice structures," *International*  
9 *Journal of Heat and Mass Transfer*, vol. 55, no. 9-10, pp. 2486–2493, apr 2012. [Online]. Available:  
10 <https://linkinghub.elsevier.com/retrieve/pii/S0017931012000129>
- 11  
12 [19] J. Ho, K. Leong, and T. Wong, "Additively-manufactured metallic porous lattice heat exchangers for air-side heat  
13 transfer enhancement," *International Journal of Heat and Mass Transfer*, vol. 150, p. 119262, apr 2020. [Online].  
14 Available: <https://linkinghub.elsevier.com/retrieve/pii/S0017931019355140>
- 15  
16 [20] T. Dixit, P. Nithiarasu, and S. Kumar, "Numerical evaluation of additively manufactured lattice architectures for  
17 heat sink applications," *International Journal of Thermal Sciences*, vol. 159, p. 106607, jan 2021. [Online]. Available:  
18 <https://linkinghub.elsevier.com/retrieve/pii/S1290072920310577>
- 19  
20 [21] M. Wong, I. Owen, C. Sutcliffe, and A. Puri, "Convective heat transfer and pressure losses across novel heat sinks  
21 fabricated by Selective Laser Melting," *International Journal of Heat and Mass Transfer*, vol. 52, no. 1-2, pp.  
22 281–288, jan 2009. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S0017931008003311>
- 23  
24 [22] S. M. Thompson, Z. S. Aspin, N. Shamsaei, A. Elwany, and L. Bian, "Additive manufacturing of heat exchangers:  
25 A case study on a multi-layered Ti–6Al–4V oscillating heat pipe," *Additive Manufacturing*, vol. 8, pp. 163–174, oct  
26 2015. [Online]. Available: <http://dx.doi.org/10.1016/j.addma.2015.09.003>
- 27  
28 [23] H. Brooks and K. Brigden, "Design of conformal cooling layers with self-supporting lattices for additively  
29 manufactured tooling," *Additive Manufacturing*, vol. 11, pp. 16–22, jul 2016. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S2214860416300409>
- 30  
31 [24] E. Huttunen, M. T. Nykänen, and J. Alexandersen, "Material extrusion additive manufacturing and experimental  
32 testing of topology-optimised passive heat sinks using a thermally-conductive plastic filament," *Additive*  
33 *Manufacturing*, vol. 59, p. 103123, nov 2022. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S2214860422005127>
- 34  
35 [25] C. K. Stimpson, J. C. Snyder, K. A. Thole, and D. Mongillo, "Roughness Effects on Flow and Heat Transfer for  
36 Additively Manufactured Channels," *Journal of Turbomachinery*, vol. 138, no. 5, may 2016. [Online]. Available:  
37 [https://asmedigitalcollection.asme.org/turbomachinery/article/doi/10.1115/1.4032167/378649/Roughness-Effects-](https://asmedigitalcollection.asme.org/turbomachinery/article/doi/10.1115/1.4032167/378649/Roughness-Effects-on-Flow-and-Heat-Transfer-for)  
38 [on-Flow-and-Heat-Transfer-for](https://asmedigitalcollection.asme.org/turbomachinery/article/doi/10.1115/1.4032167/378649/Roughness-Effects-on-Flow-and-Heat-Transfer-for)
- 39  
40 [26] K. L. Kirsch and K. A. Thole, "Pressure loss and heat transfer performance for additively and conventionally  
41 manufactured pin fin arrays," *International Journal of Heat and Mass Transfer*, vol. 108, pp. 2502–2513, may 2017.  
42 [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S0017931016337243>
- 43  
44 [27] A. J. Wildgoose, K. A. Thole, P. Sanders, and L. Wang, "Impact of Additive Manufacturing on Internal Cooling  
45 Channels With Varying Diameters and Build Directions," *Journal of Turbomachinery*, vol. 143, no. 7, jul 2021.  
46 [Online]. Available: [https://asmedigitalcollection.asme.org/turbomachinery/article/143/7/071003/1101298/Impact-](https://asmedigitalcollection.asme.org/turbomachinery/article/143/7/071003/1101298/Impact-of-Additive-Manufacturing-on-Internal)  
47 [of-Additive-Manufacturing-on-Internal](https://asmedigitalcollection.asme.org/turbomachinery/article/143/7/071003/1101298/Impact-of-Additive-Manufacturing-on-Internal)
- 48  
49 [28] B. Pulvirenti, M. Celli, and A. Barletta, "Flow and Convection in Metal Foams: A Survey and New CFD Results,"  
50 *Fluids*, vol. 5, no. 3, p. 155, sep 2020. [Online]. Available: <https://www.mdpi.com/2311-5521/5/3/155>
- 51  
52 [29] R. Attarzadeh, M. Rovira, and C. Duwig, "Design analysis of the "Schwartz D" based heat exchanger: A numerical  
53 study," *International Journal of Heat and Mass Transfer*, vol. 177, p. 121415, oct 2021. [Online]. Available:  
54 <https://linkinghub.elsevier.com/retrieve/pii/S0017931021005184>
- 55  
56 [30] O. Al-Ketan, M. Ali, M. Khalil, R. Rowshan, K. A. Khan, and R. K. Abu Al-Rub, "Forced Convection  
57 Computational Fluid Dynamics Analysis of Architected and Three-Dimensional Printable Heat Sinks Based on  
58 Triply Periodic Minimal Surfaces," *Journal of Thermal Science and Engineering Applications*, vol. 13, no. 2,  
59 apr 2021. [Online]. Available: [https://asmedigitalcollection.asme.org/thermalscienceapplication/article/doi/10.1115/](https://asmedigitalcollection.asme.org/thermalscienceapplication/article/doi/10.1115/1.4047385/1084162/Forced-Convection-Computational-Fluid-Dynamics)  
60 [1.4047385/1084162/Forced-Convection-Computational-Fluid-Dynamics](https://asmedigitalcollection.asme.org/thermalscienceapplication/article/doi/10.1115/1.4047385/1084162/Forced-Convection-Computational-Fluid-Dynamics)
- 61  
62 [31] Z. Cheng, R. Xu, and P.-X. Jiang, "Morphology, flow and heat transfer in triply periodic minimal surface based  
63 porous structures," *International Journal of Heat and Mass Transfer*, vol. 170, p. 120902, may 2021. [Online].  
64 Available: <https://doi.org/10.1016/j.ijheatmasstransfer.2021.120902>
- 65

- [32] J. Kelly, L. Finkenauer, P. Roy, J. Stolaroff, D. Nguyen, M. Ross, A. Hoff, and J. Haslam, “Binder jet additive manufacturing of ceramic heat exchangers for concentrating solar power applications with thermal energy storage in molten chlorides,” *Additive Manufacturing*, vol. 56, p. 102937, aug 2022. [Online]. Available: <https://doi.org/10.1016/j.addma.2022.102937>
- [33] I. Kaur and P. Singh, “Flow and thermal transport characteristics of Triply-Periodic Minimal Surface (TPMS)-based gyroid and Schwarz-P cellular materials,” *Numerical Heat Transfer, Part A: Applications*, vol. 79, no. 8, pp. 553–569, apr 2021. [Online]. Available: <https://www.tandfonline.com/doi/abs/10.1080/10407782.2021.1872260>
- [34] S.-H. Oh, J.-W. Ha, and K. Park, “Adaptive Conformal Cooling of Injection Molds Using Additively Manufactured TPMS Structures,” *Polymers*, vol. 14, no. 1, p. 181, jan 2022. [Online]. Available: <https://www.mdpi.com/2073-4360/14/1/181>
- [35] Z. A. Qureshi, S. A. B. Al Omari, E. Elnajjar, F. Mahmoud, O. Al-Ketan, and R. A. Al-Rub, “Thermal characterization of 3D-Printed lattices based on triply periodic minimal surfaces embedded with organic phase change material,” *Case Studies in Thermal Engineering*, vol. 27, p. 101315, oct 2021. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S2214157X21004780>
- [36] Z. A. Qureshi, E. Elnajjar, O. Al-Ketan, R. A. Al-Rub, and S. B. Al-Omari, “Heat transfer performance of a finned metal foam-phase change material (FMF-PCM) system incorporating triply periodic minimal surfaces (TPMS),” *International Journal of Heat and Mass Transfer*, vol. 170, p. 121001, may 2021. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S0017931021001046>
- [37] J. Santos, T. Pires, B. P. Gouveia, A. P. Castro, and P. R. Fernandes, “On the permeability of TPMS scaffolds,” *Journal of the Mechanical Behavior of Biomedical Materials*, vol. 110, p. 103932, oct 2020. [Online]. Available: <https://doi.org/10.1016/j.jmbbm.2020.103932>
- [38] A. Della Torre, G. Montenegro, G. Tabor, and M. Wears, “CFD characterization of flow regimes inside open cell foam substrates,” *International Journal of Heat and Fluid Flow*, vol. 50, pp. 72–82, dec 2014. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S0142727X14000642>
- [39] I. Maskery, L. Parry, D. Padrão, R. Hague, and I. Ashcroft, “FLatt Pack: A research-focused lattice design program,” *Additive Manufacturing*, vol. 49, p. 102510, jan 2022. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S2214860421006576>
- [40] O. Al-Ketan, D.-W. Lee, R. Rowshan, and R. K. Abu Al-Rub, “Functionally graded and multi-morphology sheet TPMS lattices: Design, manufacturing, and mechanical properties,” *Journal of the Mechanical Behavior of Biomedical Materials*, vol. 102, p. 103520, feb 2020. [Online]. Available: <https://linkinghub.elsevier.com/retrieve/pii/S1751616119312482>
- [41] OpenFOAM. [Online]. Available: <https://www.openfoam.com/>
- [42] M. J. M. J. Rhodes, *Introduction to particle technology*, 2nd ed., ProQuest (Firm), Ed. Chichester, England ; Hoboken, NJ: Wiley, 2008.
- [43] N. Dukhan and C. A. Minjaur, “A two-permeability approach for assessing flow properties in metal foam,” *Journal of Porous Materials*, vol. 18, no. 4, pp. 417–424, aug 2011. [Online]. Available: <http://link.springer.com/10.1007/s10934-010-9393-1>
- [44] S. Ergun, “Fluid flow through packed columns,” *Chem. Eng. Progress.*, vol. 48, pp. 89–94, 1952.
- [45] F. P. Incropera and D. P. DeWitt, *Fundamentals of Heat and Mass Transfer*, 6th ed. Hoboken, N.J. : Wiley, 2007.
- [46] S. P. Mahulikar and H. Herwig, “Fluid friction in incompressible laminar convection: Reynolds’ analogy revisited for variable fluid properties,” *The European Physical Journal B*, vol. 62, no. 1, pp. 77–86, mar 2008. [Online]. Available: <http://link.springer.com/10.1140/epjb/e2008-00115-0>

**Declaration of interests**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

The authors declare the following financial interests/personal relationships which may be considered as potential competing interests:

**Daniel Padrao:** Conceptualization, Methodology, Software, Validation, Formal analysis, Investigation, Data Curation, Writing - Original Draft, Writing - Review & Editing, Visualization.

**David Hancock:** Writing - Review & Editing, Supervision, Project administration, Funding acquisition.

**James Paterson:** Supervision.

**Frank Schoofs:** Writing - Review & Editing, Supervision, Project administration.

**Christopher Tuck:** Supervision, Project administration.

**Ian Maskery:** Conceptualization, Software, Resources, Writing - Review & Editing, Supervision, Project administration, Funding acquisition.

This is our original work and has not been submitted elsewhere.