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Abstract

The theory of neoclassical transport in an impure, toroidal plasma is extended to allow for steeper pressure and temperature gradients than are usually considered. It is found that the ion particle flux can be a non-monotonic function of these gradients for plasma parameters typical of the tokamak edge. A sudden transition between states of low and high confinement is therefore possible.

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I Introduction

In well confined tokamak plasmas, the ion particle transport can be comparable to the neoclassical prediction. On the other hand, the pressure and temperature profiles are often very steep, especially near the plasma edge. It is well known that the observed gradients are frequently too large for conventional neoclassical theory [1, 2] to be valid, and the need to extend the theory in this direction is widely recognized [3, 4, 5, 6, 7]. Since the conventional neoclassical transport fluxes are proportional to the gradients but cannot increase indefinitely, one might expect that some saturation of the transport should occur when the profiles become very steep. As we shall see in the present paper, it is also possible that the plasma transport becomes *qualitatively* different under such conditions.

The essential difficulty in generalizing neoclassical transport theory to steep gradients lies in the use of the expansion parameter

$$\delta \equiv \frac{\rho_\theta}{L_\perp},$$

where ρ_θ is the poloidal ion gyroradius and $L_\perp$ the radial scale length associated with the density and temperature profiles. Neoclassical theory requires $\delta \ll 1$, and it is very difficult to envisage constructing a tractable transport theory when this assumption does not hold. There is then no separation of scales, so the transport fluxes become non-local and depend not only on the local gradients but on the entire density and temperature profiles.

While constructing transport theory in the regime $\delta = O(1)$ is fundamentally difficult, it is nonetheless possible to allow for steeper gradients than are admissible in conventional neoclassical theory while still assuming $\delta \ll 1$. The point is that not only is δ assumed to be small in conventional theory, but it is also effectively taken to be the smallest of all parameters of the transport problem. This has the consequence of making all densities and temperatures flux functions, essentially because the system of lowest-order drift kinetic equations

$$v_{\|}\nabla_{\|}f_a = \sum_b C_{ab}(f_a, f_b)$$

for all species a only has solutions that are Maxwellian and are constant on flux surfaces [1]. The only two-dimensional feature of the plasma that survives in this ordering is the magnetic field inhomogeneity, which is what gives rise to the neoclassial enhancement of the transport over the classical level.

When δ is made larger, poloidal asymmetries become possible. Typically the first plasma parameter to develop a poloidal variation is the density, n_z, of highly charged impurity ions [8, 9], whose poloidal modulation is of the order [10]

$$\frac{\tilde{n}_z}{n_z} \sim \Delta \equiv \delta \hat{\nu}_{ii} z^2, \tag{1}$$

where $\hat{\nu}_{ii} \equiv L_{||}/\lambda_{ii}$ is the collisionality, with λ_{ii} the mean-free path for the bulk ions and $L_{||}$ the connection length. If the plasma is not deep into the collisionless regime, Δ can easily be of order unity while δ remains small.

Here, we study neoclassical transport in an impure plasma with steep profiles, with the ordering

$$\delta \ll 1, \qquad \Delta = O(1), \tag{2}$$

enabling a non-uniform distribution of impurities over each flux surface. For simplicity, we restrict our attention to the case of a hydrogen plasma with a single species of highly charged ($z \gg 1$) impurity ions. The electrons (e) and H ions (i) are taken to be collisionless while the impurities are assumed to be collisional, as is typical of a tokamak plasma somewhat inside the last closed flux surface. The analysis presented here complements earlier work by Hsu and Sigmar [10], who considered a collisional, isothermal plasma in a torus with large aspect ratio and circular cross section.

In order to calculate the poloidal distribution of impurities, it is necessary to solve their parallel momentum equation. This is accomplished in Sec II, where it is shown that the impurity density generally has both up-down and in-out asymmetry. This has surprising implications for the neoclassical transport, which is evaluated in Sec III. When the gradients are sufficiently steep the confinement is improved, and the radial ion particle flux can even be a *non-monotonic* function of the gradients for plasma parameters typical of the tokamak edge. Sudden transistions between states with low and high particle confinement are therefore possible. The conclusions are summarized in the last section.

II Parallel dynamics

We begin by estimating the time scales associated with the transport along and across the magnetic field. The frequency of collisions of one particle species (a) with another (b) is, generally [11]

$$\frac{1}{\tau_{ab}} \sim \frac{n_b e_a^2 e_b^2 \ln \Lambda}{(4\pi\epsilon_0)^2 m_a^2 v_{T>} v_{Ta}^2},$$

where $v_{T>}$ is the larger of the thermal velocities v_{Ta} and v_{Tb}. In a tokamak, the impurity density is usually large enough to make the ion-impurity and ion-ion collision frequencies comparable, so we assume

$$Z_{\text{eff}} - 1 = n_z z^2 / n_i = O(1). \tag{3}$$

The time scale on which a parallel equilibrium is established for the impurities is

$$\tau_{\|} \sim \frac{L_{\|}^2}{v_{Tz}^2 \tau_{zz}},$$

where $L_{\|}$ is the connection length, and the time scale associated with the cross-field particle transport is

$$\tau_{\perp} \sim \frac{L_{\perp}^2}{\rho_z^2 / \tau_{zi}},$$

with $L_{\perp}$ the perpendicular scale length and $\rho_z = v_{Tz}/\Omega_z$ the impurity gyroradius. The ratio between these time scales can be written as

$$\frac{\tau_{\|}}{\tau_{\perp}} \sim \frac{(Z_{\text{eff}} - 1)\,\Delta^2}{z^{3/2}},$$

which is small because of the orderings (2) and (3). The parallel dynamics can thus be analyzed on each flux surface separately. If this were not the case, the transport problem would be effectively two-dimensional and much more difficult.

Since $z \gg 1$, the assumption (3) implies $n_z z \ll n_i$. As a result, the electrostatic potential is approximately constant on flux surfaces, $\Phi \simeq \Phi_0(\psi)$, which can be verified *a posteriori*. The first-order drift kinetic equation for the H ions is

$$v_{\|}\nabla_{\|}\left(f_{i1} + \frac{I v_{\|}}{\Omega_i}\frac{\partial f_{i0}}{\partial \psi}\right) + \frac{e v_{\|}\nabla_{\|}\Phi_1}{T_i} f_{i0} = C_i(f_{i1}),$$

where f_{i0} is a Maxwellian at rest, $\Omega_i = eB/m_i$, and the magnetic field is $\mathbf{B} = I(\psi)\nabla\varphi + \nabla\varphi \times \nabla\psi$, so that ψ is the poloidal flux. The gradient is taken at constant magnetic moment μ and lowest-order energy $\mathcal{E}_0 \equiv m_i v^2/2 + e\Phi_0$. The first-order ion distribution function thus becomes

$$f_{i1} = -\frac{I v_\parallel}{\Omega_i}\frac{\partial f_{i0}}{\partial \psi} - \frac{e\Phi_1}{T_i} f_{i0} + h_i(\mathcal{E}_0, \mu, \psi, \sigma),$$

where $\sigma = v_\parallel/|v_\parallel|$ and h_i vanishes in the trapped domain. The electron distribution is of a similar form (but with $m_i \to m_e$ and $e \to -e$), and the impurity density can be related to Φ_1 by quasineutrality,

$$n_z = \frac{n_e - n_i}{z} = \frac{2n_0}{T_0}\frac{e\Phi_1}{z}, \tag{4}$$

where $2n_0/T_0 \equiv n_{e0}/T_e + n_{i0}/T_i$, and n_{i0}, T_{i0}, n_{e0} and T_{e0} are the densities and temperatures associated with the lowest-order, Maxwellian, distribution functions f_{i0} and f_{e0}. Since the impurities are highly charged, their perpendicular velocity is dominated by the $\mathbf{E} \times \mathbf{B}$ drift, and from the continuity equation, $\nabla \cdot (n_z \mathbf{V}_z) = 0$, it follows that there must be a parallel impurity return flow equal to

$$V_{z\parallel} = -\frac{I}{B}\frac{d\Phi_0}{d\psi} + \frac{K_z(\psi) B}{n_z},$$

where the integration constant $K_z(\psi)$ is proportional to the poloidal flow velocity.

Since the H ions generally have a different parallel flow velocity, they exert friction on the impurities. The ion-impurity collision operator is

$$C_{iz} = \frac{\nu_{iz}(v)}{2}\frac{\partial}{\partial \xi}\left(1 - \xi^2\right)\frac{\partial}{\partial \xi} + \nu_{iz}\frac{m_i v_\parallel V_{z\parallel}}{T_i} f_{i0},$$

$$\nu_{iz}(v) = \frac{3\pi^{1/2}}{4\tau_{iz}}\left(\frac{v_{Ti}}{v}\right)^3,$$

with $V_{z\parallel}$ the parallel impurity flow velocity, $v_{Ti} = (2T_i/m_i)^{1/2}$ the ion thermal speed, $\xi \equiv v_\parallel/v$, and $\tau_{iz} = 3(2\pi)^{3/2}\epsilon_0^2 m_i^{1/2} T_i^{3/2}/n_z z^2 e^4 \ln\Lambda = (n_i/n_z z^2)\tau_{ii}$ the ion-impurity collision time. The parallel friction force between H ions and impurities is

$$R_{z\parallel} = -\int m_i v_\parallel C_{iz}(f_{i1}) d^3v = -\frac{p_i I}{\Omega_i \tau_{iz}}\left(\frac{d\ln p_i}{d\psi} - \frac{3}{2}\frac{d\ln T_i}{d\psi}\right) + \frac{m_i n_i}{\tau_{iz}}\left(u - \frac{K_z}{n_z}\right) B, \tag{5}$$

where

$$u \equiv \frac{\tau_{iz}}{n_i B}\int v_\parallel \nu_{iz} h_i \, d^3v \tag{6}$$

is a flux function since $d^3v \propto B d\mathcal{E}_0 d\mu / v_\parallel$.

We now turn our attention to the parallel momentum equation for the impurities,

$$m_z n_z \mathbf{b} \cdot (\mathbf{V}_z \cdot \nabla \mathbf{V}_z) = -n_z ze \nabla_{\|} \Phi_1 - \nabla_{\|} p_z - \mathbf{b} \cdot \nabla \cdot \boldsymbol{\pi}_z + R_{z\|}.$$

Here $\mathbf{b} = \mathbf{B}/B$ is the unit vector along the field and $\boldsymbol{\pi}_z$ is the impurity viscosity tensor. If the radial electric field is of the same order as the temperature gradient, $e\Phi' \sim T'$, the flow velocities of both ion species are of the order $V_{\|} \sim \delta v_{Ti}$. The ratio between the inertial term and the friction is then

$$\frac{m_z n_z \mathbf{b} \cdot (\mathbf{V}_z \cdot \nabla \mathbf{V}_z)}{R_{z\|}} \sim \frac{\delta}{z \hat{\nu}_{ii}}.$$

For simplicity, we shall assume that this parameter is small, which is realistic in edge plasmas where the bulk ions are not far into the banana regime. This also enables us to neglect any poloidal variation in the impurity temperature, which is detemined by the impurity energy equation, where compressional heating,

$$p_z \nabla \cdot \mathbf{V}_z \sim \delta p_z v_{Ti}/L_{\|}$$

and the divergence of the diamagnetic heat flux,

$$\nabla \cdot \mathbf{q}_{z\wedge} = \nabla \cdot \left(\frac{5p_z}{2zeB} \mathbf{b} \times \nabla T_z \right) \sim \delta p_z v_{Ti}/zL_{\|},$$

tend to produce poloidal asymmetries. Both these terms are, however, overwhelmed by ion-impurity energy equilibration,

$$\frac{p_z \nabla \cdot \mathbf{V}_z}{Q_{zi}} \sim \frac{z \nabla \cdot \mathbf{q}_{z\wedge}}{Q_{zi}} \sim \frac{\delta p_z v_{Ti}/L_{\|}}{n_z (T_i - T_z)/\tau_{zi}} \sim \frac{\delta}{z \hat{\nu}_{ii}} \frac{T_i}{T_i - T_z},$$

so that $(T_i - T_z)/T_i \sim \delta / z\hat{\nu}_{ii} \ll 1$. In addition, the parallel viscosity associated with the impurities becomes smaller than the pressure gradient

$$\frac{\mathbf{b} \cdot \nabla \cdot \boldsymbol{\pi}_z}{\nabla_{\|} p_z} \sim \frac{p_z \tau_{zz} V_{\|}/L_{\|}^2}{p_z/L_{\|}} \sim \frac{\delta}{z^{3/2} \hat{\nu}_{ii}} \frac{n_i}{n_z z^2}.$$

With these simplifications, the parallel momentum equation reduces to

$$n_z ze \nabla_{\|} \Phi_1 + T_i \nabla_{\|} n_z = R_{z\|}, \tag{7}$$

from which we can now calculate the poloidal impurity rotation K_z by noting the solubility constraint $\left\langle BR_{z\|} \right\rangle = 0$, where

$$\langle \cdots \rangle = \oint \frac{(\cdots) d\theta}{\mathbf{B} \cdot \nabla \theta} / \oint \frac{d\theta}{\mathbf{B} \cdot \nabla \theta}$$

is the flux surface average, with θ the poloidal angle. Thus solving for the poloidal flow K_z appearing in (5), and inserting the resulting friction force in the parallel momentum equation (7), we obtain

$$\left(T_i + \frac{n_z z^2}{2n_0} T_0\right) \nabla_{\|} n_z$$
$$= -\frac{p_i I}{\Omega_i \tau_{iz}} \left(\frac{d \ln p_i}{d\psi} - \frac{3}{2} \frac{d \ln T_i}{d\psi}\right) \left(1 - \frac{\langle n_z \rangle}{n_z} \frac{B^2}{\langle B^2 \rangle}\right) + \frac{m_i n_i u}{\tau_{iz} n_z} \left(n_z - \frac{\langle n_z B^2 \rangle}{B^2}\right) B,$$

where we have used (4) to eliminate the electric field. Finally, we write this equation in dimensionless form by introducing $n \equiv n_z / \langle n_z \rangle$, $b \equiv B / \langle B^2 \rangle^{1/2}$, $\alpha \equiv \langle n_z \rangle z^2 T_0 / 2n_0 T_i$,

$$\gamma \equiv -\frac{eu}{IT_i} \left\langle B^2 \right\rangle \left(\frac{d \ln p_i}{d\psi} - \frac{3}{2} \frac{d \ln T_i}{d\psi}\right)^{-1}, \tag{8}$$

and a modified poloidal angle coordinate ϑ defined by

$$d\vartheta \equiv \frac{\langle \mathbf{B} \cdot \nabla \theta \rangle}{\mathbf{B} \cdot \nabla \theta} d\theta,$$

so that the flux-surface average is equivalent to an average over ϑ. The parallel momentum equation then becomes

$$(1 + \alpha n) \frac{\partial n}{\partial \vartheta} = g \left[n - b^2 + \gamma \left(n - \left\langle n b^2 \right\rangle\right) b^2\right], \tag{9}$$

$$g \equiv -z^2 \frac{IB}{\Omega_i \tau_{ii} \langle \mathbf{B} \cdot \nabla \theta \rangle} \left(\frac{d \ln p_i}{d\psi} - \frac{3}{2} \frac{d \ln T_i}{d\psi}\right).$$

It is straighforward to verify that $g = O(\Delta)$. This parameter measures the steepness of the pressure and temperature profiles, and is assumed to be small in conventional neoclassical theory [12]. The pressure gradient is then larger than the friction force, and it is immediately clear from (9) that the impurity density is nearly constant on each flux surface, $n \simeq 1$, with a small *up-down asymmetry* [10],

$$(1 + \alpha) \frac{\partial n}{\partial \vartheta} = g(1 - b^2) + O(g^2) \qquad (g \ll 1),$$

which changes sign if the toroidal field is reversed. It is not affected by a reversal of the plasma current.

In the opposite limit of very steep gradients, $g \gg 1$, the friction force exceeds the pressure gradient and causes a substantial rearrangement of impurities within each flux surface. If we expand n in powers of g^{-1},

$$n = n_0 + n_1 + O(g^{-2}) \qquad (g \gg 1), \tag{10}$$

the lowest-order solution is *in-out asymmetric*,

$$n_0 = \frac{\gamma}{1 - \langle (1+\gamma b^2)^{-1} \rangle} \frac{b^2}{1+\gamma b^2}. \tag{11}$$

The first-order term n_1 contains up-down asymmetry and is determined by

$$(1+\alpha n_0)\frac{\partial n_0}{\partial \vartheta} = g\left[n_1 + \gamma\left(n_1 - \left\langle n_1 b^2 \right\rangle\right) b^2\right]. \tag{12}$$

In a torus with small inverse aspect ratio, $\epsilon = r/R \ll 1$, the variation of the impurity density is small, $n - 1 = O(\epsilon)$, [10]. In the opposite limit of tight aspect ratio, there are very few circulating particles, so that $u \to 0$ and thus $\gamma \to 0$. Then $n_0 \to b^2$, implying a much larger impurity density on the inside of the flux surface than on the outside – by an order of magnitude in the edge of a typical spherical tokamak. This should be an experimentally verifiable prediction of the theory. It is, however, worth noting that rapid toroidal rotation (which is ruled out by our orderings) has the opposite effect since the centrifugal force pushes impurities to the outside of each flux surface [13].

III Radial transport

The rearrangement of impurities on each flux surface we have just calculated has surprising implications for the neoclassical transport. The radial neoclassical flux of H ions is driven by the parallel ion-impurity friction force and is equal to

$$\langle \Gamma_i^{\rm neo} \cdot \nabla\psi \rangle = \left\langle \frac{I R_{z\|}}{eB} \right\rangle = \langle \mathbf{B} \cdot \nabla\theta \rangle \frac{\langle p_z \rangle I}{e\langle B^2 \rangle}\left[\left\langle \frac{n}{b^2} \right\rangle - 1 + \gamma\left(1 - \left\langle n b^2 \right\rangle\right)\right] g. \tag{13}$$

It increases linearly with the gradients when g is small, and is then proportional to $\langle b^{-2} - 1 \rangle g$ as is characteristic of Pfirsch-Schlüter transport. (Note that the particle flux scales like the Pfirsch-Schlüter value although the H ions are in the banana regime [14].) However, when the profiles become so steep that $g \gg 1$ then Eqs (10) - (13) show that

$$\langle \Gamma_i^{\rm neo} \cdot \nabla\psi \rangle \propto g^{-1}, \tag{14}$$

The contributions to the flux from both n_0 and n_1 vanish, and as a result the neoclassical particle flux *decreases* with increasing gradients when the latter become sufficiently steep!

We have tacitly assumed that γ does not vary too much when g increases. This assumption can be justified in the limits of large and small aspect ratio. The function

h_i, which determines u and hence γ by Eqs (6) and (8), is obtained from the solution of $\left\langle \left(B/v_{\|}\right) C_i\left(f_{i1}\right)\right\rangle = 0$ [1, 2]. The redistribution of impurities which occurs when $g = O(1)$ affects this equation by making the ion-impurity collision frequency vary over the flux surface and by changing $V_{z\|}$. If the inverse aspect ratio is small, these effects are no larger than $O(\epsilon)$. The function h_i is therefore not much affected by the impurity redistribution and is approximately equal to that found in the conventional theory. The quantity u is thus proportional to the gradients, which makes γ independent of g. In the opposite limit of tight aspect ratio, γ is small since the function h_i is non-zero only in the small circulating domain of velocity space.

As the neoclassical channel is suppressed, classical transport becomes relatively more important. The classical particle flux is

$$\left\langle \Gamma_i^{\mathrm{cl}} \cdot \nabla\psi \right\rangle = \left\langle \frac{R^2 \nabla\varphi \cdot \mathbf{R}_{z\perp}}{e} \right\rangle,$$

where $\mathbf{R}_{z\perp}$ is the perpendicular friction force,

$$\mathbf{R}_{z\perp} = \frac{m_i n_i}{\tau_{iz}} \left(\mathbf{V}_{i\perp} - \mathbf{V}_{z\perp} - \frac{3}{2 m_i \Omega_i} \mathbf{b} \times \nabla T_i \right).$$

Since the difference in diamagnetic velocities is $\mathbf{V}_{i\perp} - \mathbf{V}_{z\perp} \simeq \mathbf{b} \times \nabla p_i / n_i e B$, and $\nabla\varphi \cdot (\mathbf{b} \times \nabla\psi) = -B_\theta^2/B$, the classical flux becomes

$$\left\langle \Gamma_i^{\mathrm{cl}} \cdot \nabla\psi \right\rangle = -\frac{p_i B^2}{m_i \Omega_i^2 \tau_{iz} n_z} \left(\frac{d \ln p_i}{d\psi} - \frac{3}{2} \frac{d \ln T_i}{d\psi} \right) \left\langle \frac{R^2 B_\theta^2}{B^2} n_z \right\rangle. \tag{15}$$

It is immediately apparent that this flux can also be affected by the redistribution of impurities. To understand the behavior of the total (classical + neoclassical) transport, it is instructive to take the limits of large and tight aspect ratio, now considered separately.

A Large aspect ratio

In a torus with large aspect ratio and circular cross section, $b^2(\vartheta) = 1 - 2\epsilon \cos\vartheta + O(\epsilon^2)$, and we can expand the impurity density similarly,

$$n(\vartheta) = 1 + n_c \cos\vartheta + n_s \sin\vartheta + O(\epsilon^2).$$

The solution then found from (9) is

$$n_s = \frac{2\epsilon(1+\alpha)g}{(1+\alpha)^2 + (1+\gamma)^2 g^2},$$

$$n_c = \frac{2\epsilon(1+\gamma)g^2}{(1+\alpha)^2 + (1+\gamma)^2 g^2}.$$

Thus, the in-out asymmetry increases monotonically with increasing gradient, while the up-down asymmetry has a maximum at $g = (1+\alpha)/(1+\gamma)$. It is now straightforward to evaluate the fluxes (13) and (15) to obtain

$$\left\langle \Gamma_i^{\rm cl} \cdot \nabla\psi \right\rangle + \langle \Gamma_i^{\rm neo} \cdot \nabla\psi \rangle = \frac{\epsilon p_z}{q^2 e} \left(1 + \frac{2q^2}{1 + \left(\frac{1+\gamma}{1+\alpha}\right)^2 g^2} \right) g.$$

where $q = rB/RB_\theta$ is the safety factor. The second term represents the neoclassical contribution and exceeds the first, classical term by the Pfirsch-Schlüter factor $2q^2$ when the gradients are weak, $g \ll 1$. On the other hand, if the profiles are steep ($g \gg 1$) the neoclassical flux is suppressed and classical transport dominates. As the latter is not much affected by the weak $[O(\epsilon)]$ impurity redistribution, the flux then increases linearly with g. The total flux is non-monotonic if $q > 2$.

Figure 1 shows the fluxes as functions of the normalized gradients g in a torus with safety factor $q = 3$. The total flux (solid line) depends on the gradients in a way characteristic of bifurcating systems [16]. If the flux is raised above the local maximum, a sudden transisition occurs to a state with much steeper gradients. Conversely, if the flux is decreased below the local minimum of the curve, the gradients suddenly become much smaller. Sudden transitions between states of high and low confinement are thus possible.

B Tight aspect ratio

At tight aspect ratio the situation is slightly different. Not only is the neoclassical transport suppressed when the gradients are large, but the classical transport is also affected. Since $n = b^2$ when $g \gg 1$ most ion-impurity collisions then occur on the inside of each flux surface, and the step size in the ion-impurity collisional walk is reduced, resulting in weaker classical transport, cf Ref [13].

To illustrate the transport when g varies from small to large values, we have solved Eq (9) numerically, with a periodic boundary condition, for the equilibrium shown in Fig 2. This has been obtained by magnetic reconstruction of experimental data from a discharge (no 35096) in the Small Tight Aspect Ratio Tokamak (START) at Culham [15]. The transport fluxes calculated from (13) and (15) are shown in Fig 3. The neoclassical flux (dotted line) is completely suppressed when $g \gg 1$, and the classical diffusion coefficient is reduced by a factor

$$\frac{\left\langle \Gamma_i^{\mathrm{cl}} \cdot \nabla\psi \right\rangle^{(g\gg 1)} / g}{\left\langle \Gamma_i^{\mathrm{cl}} \cdot \nabla\psi \right\rangle^{(g\ll 1)} / g} = \frac{\langle R^2 B_\theta^2 \rangle}{\langle R^2 B_\theta^2 / B^2 \rangle \langle B^2 \rangle} \simeq 0.27.$$

At tight aspect ratio it is also possible for the classical diffusion coefficient to exceed the neoclassical one when $g \ll 1$. An example is shown in Fig 4, where we have calculated the transport in a different START discharge (no 36544), in which the toroidal magnetic field was much lower than in the previous example. The classical transport is about twice as large as the neoclassical transport when $g \ll 1$, and dominates completely when $g > 1$. The total flux is a monotonic function of the gradients, and no bifurcation can occur. However, the plasma confinement is still significantly enhanced when $g \gg 1$, with a particle diffusivity less than one third of that for $g \ll 1$.

C General remarks

It should be pointed out that the neoclassical *heat flux* is generally not expected to be a non-monotonic function of the gradients. Not only ion-impurity collisions, but also ion-ion collisions, drive the heat flux. The redistribution of impurities reduces the ion-impurity friction but does not affect the ion-ion collisions.

As we have seen, the transition to improved particle confinement need not be sudden but can also be gradual, depending on whether the flux decreases for large g or merely increases at a slower rate. This is sensitive to the magnitude of the classical or anomalous transport and also to processes we have neglected. In order to estimate the maximum gradient allowed in the present ordering, we note that, when inertia and compressional heating are included, the solution (10) is expected to acquire additional terms of order $O(\delta / z\hat{\nu}_{ii})$, which compete with the $O(g^{-2})$ term when the gradients become so steep that

$$\delta \geq \frac{1}{z\hat{\nu}_{ii}^{1/3}} \qquad \Rightarrow \qquad \Delta \geq \hat{\nu}_{ii}^{2/3} z.$$

This suggests that our banana-regime analysis is limited to $g < 6$ for realistic impurities. However, as follows from the work by Hsu and Sigmar [10], the phenomenology can be similar in the Pfirsch-Schlüter regime ($\hat{\nu}_{ii} > 1$). Indeed, it is apparent from Sec II above that although the ion-impurity friction force may depend on the collisionality, most other aspects of the impurity dynamics do not. Crucially, in the equation that controls the behavior of the impurities (9), only the definitions of g and γ are sensitive to the collisionality, not the form of the equation itself.

IV Summary

In conclusion, in an impure toroidal plasma with steep gradients, heavy impurity ions undergo a spontaneous rearrangement on each flux surface, reducing their parallel friction with the bulk ions. This is the driving force for the neoclassical flux, which therefore decreases if the gradients become sufficiently steep. This gives rise to the possibility of a transport bifurcation. Indeed, the relations between particle flux and the gradients shown in Figs 1 and 3 are remarkably similar to that postulated by Hinton and Staebler [16] for the anomalous heat flux.

The neoclassical heat flux is less influenced than the particle flux by the impurity redistribution, since heat is transported by both ion-ion collisions and ion-impurity collisions but only the latter are affected by the redistribution. However, if $Z_{\text{eff}} - 1 = O(1)$ so that the contributions from these different classes of collisions are comparable, the heat flux is significantly reduced as the contribution from ion-impurity collisions becomes less important.

The circumstance that neoclassical transport is suppressed when the gradients are steep may be of importance for interpreting tokamak experiments where the transport appears to be lower than the conventional neoclassical prediction [17]. More speculatively, the presence of a neoclassical transport bifurcation could be related to the H-mode (which involves a reduced anomalous heat flux). For instance, the latter could be triggered by a suddenly improved neoclassical particle confinement, leading to a steepened pressure gradient, increased radial electric field, and shear stabilization of the plasma turbulence.

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Fig 1. The neoclassical ion particle flux (dotted curve), the classical flux (dashed line), and the total flux (solid curve) vs gradient in a toroidal plasma with circular cross section, large aspect ratio, and safety factor $q = 3$.

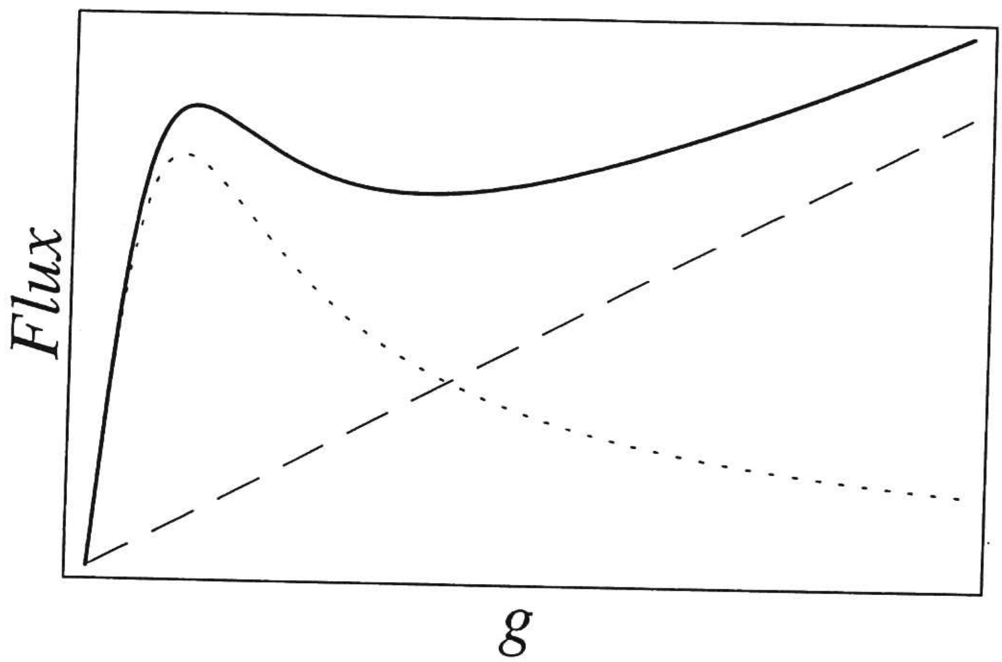

Fig 2 A magnetically reconstructed flux surface close to the edge of START dischage no 35096.

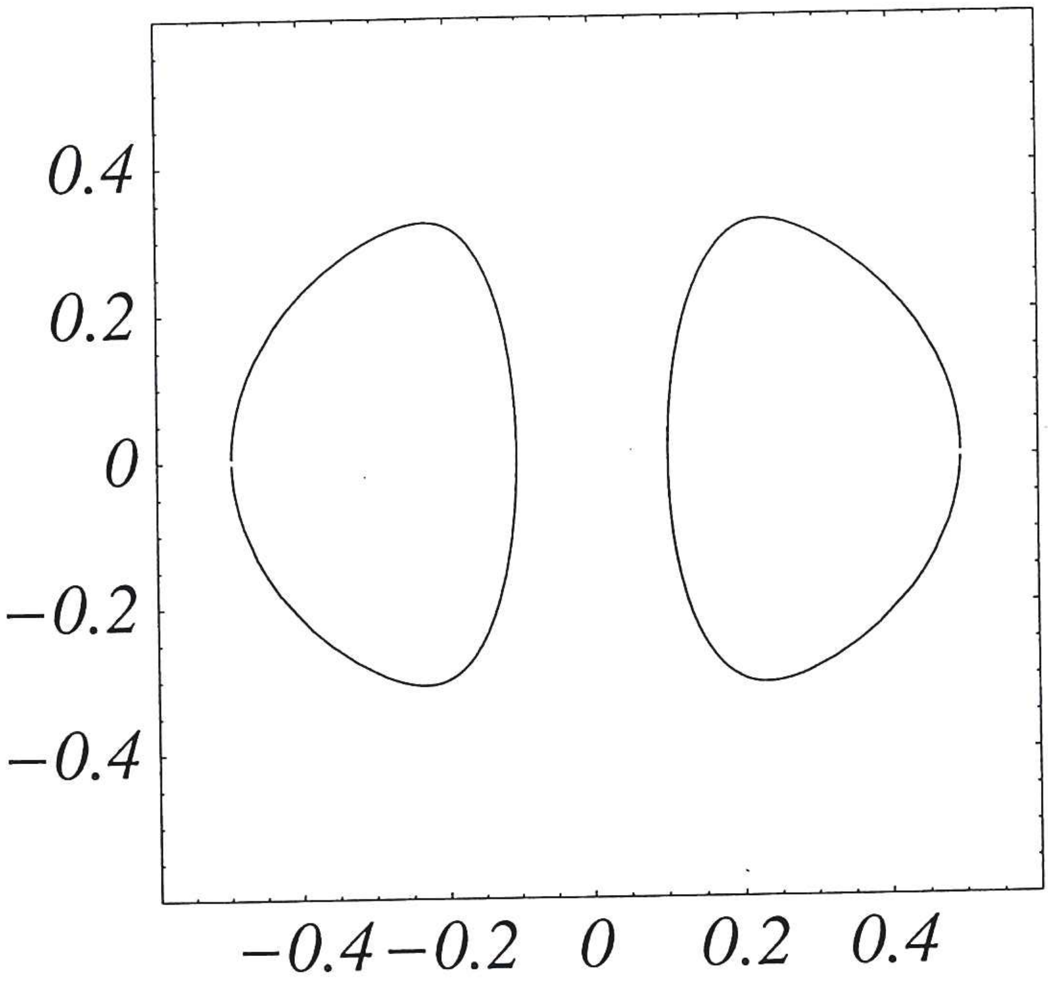

Fig 3. Same as Fig 1, but for START discharge no 35096, assuming $\gamma = 0$, $\alpha \ll 1$.

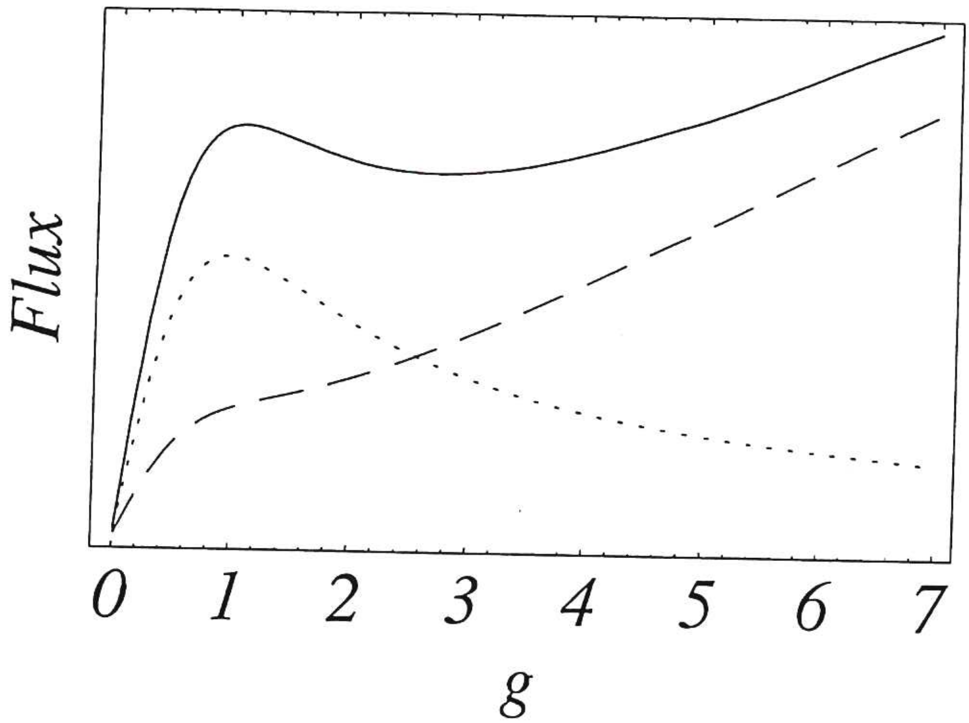

Fig 4. Same as Fig 3, but for START discharge no 36544.