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Abstract

A physically motivated, theoretical basis for defining scrape-off layer (SOL) widths, is proposed. Applying it to theoretical models proposed for transport in the SOL shows how the width is affected by the form of the cross-field transport and the parallel transport to target plates. For a cross-field diffusivity $\chi_\perp$ and a parallel loss time $\tau_{||}$, the estimate $\Delta = C(\chi_\perp \tau_{||})^{1/2}$, with C typically in the range 0.6 - 1.0 and $\chi_\perp$ and $\tau_{||}$ evaluated at the SOL-core interface, is found to provide a physically meaningful and accurate expression for the power scrape-off layer width Δ.

1 Introduction

There is some ambiguity in defining widths for temperature, density, power flux etc in the scrape-off layer (SOL). For instance, one could use half-widths or fit simple or more complex experimental forms to the profiles. Theoretically one often uses the simple estimate

$$\Delta \sim \bar{\Delta} \equiv (\chi_\perp \tau_{||})^{1/2} \qquad (1)$$

where $\chi_\perp$ is a cross-field diffusivity in the SOL and $\tau_{||}$ represents the time for transport along open magnetic field lines to the limiter or divertor plates. In this note a prescription based on a physically motivated theoretical basis (a variational principle) is proposed, using a simple model for energy transport in the SOL for illustration. This procedure could be extended to cover coupled energy and particle transport in more realistic divertor configurations.

2 Physical SOL Model

We consider the transport of energy in the SOL, taking it to be described by a combination of thermal diffusion with coefficient $\chi_\perp$, possibly anomalous, across the magnetic field and transport along the magnetic field, either by thermal conduction or free streaming, to divertor plates (this transport along the field is characterised by a 'loss-time' $\tau_{||}$).

Thus

$$\frac{d}{dx}\chi_\perp \frac{dT}{dx} - \frac{T}{\tau_{||}} = 0 \qquad (2)$$

where x is the distance into the SOL from the edge of the core plasma and T is the temperature in the SOL; we assume n is a constant for simplicity. In general $\chi_\perp$ can be a non-linear function of T and dT/dx; $\tau_{||}$ is either proportional to $T^{-1/2}$ for free streaming or $T^{-5/2}$ for thermal conduction.

3 A Non-Linear Variational Principle

Equation (2) potentially has a strongly non-linear character. It is nevertheless possible to generate a variational formulation suitable for its solution [1]. One does this by considering $\chi_\perp$ and $\tau_{||}$ as functions of an auxiliary variable T^*, constructing a variational principle with respect to variations in T at constant T^*, and setting $T = T^*$ as a subsidiary condition at the end. It is readily verified that eqn (2) follows from the variation according to the above prescription, of

$$J = \int_0^\infty dx L(T^*, T) \tag{3}$$

where

$$L = \frac{1}{2}\left[\chi_\perp^* \left(\frac{dT}{dx}\right)^2 + \frac{T^2}{\tau_{||}^*}\right] \tag{4}$$

with the asterisk implying the quantity is a function of T^*.

The physical basis for this procedure has been discussed by Prigogine [2]. For dissipative systems one can replace the Lagrangian principle of classical mechanics by a minimum entropy prediction principle which can readily be formulated for linear versions of eqn (2). The non-linear case is treated by introducing two temperatures: T^*, the average temperature profile satisfying the transport equation, and T the temperature distribution treated as a fluctuating quantity. The extremum of eqn (3) determines the most probable distribution $T(T^*)$ of the fluctuating quantity T for a given average distribution T^*. Finally one has to impose the consistency condition $T = T^*$.

4 Boundary Conditions

The solution of eqn (2) must be subject to a boundary condition on the heat flux Q per unit density incident from the plasma core, ie

$$\chi_\perp \left.\frac{dT}{dx}\right|_0 = -Q \tag{5}$$

where subscript zero implies the quantity is evaluated at $x = 0$. This can be accomplished by considering the variational quantity

$$K = J/N^2 \tag{6}$$

where

$$N = \left. \frac{T}{\sqrt{2\tau_{\parallel}^*}} \right|_0 \tag{7}$$

and allowing for variations δT_0 at $x = 0$. Variation of expression (6) with respect to T, ie $\delta K = 0$, leads to both eqns (2) and (5).

5 The Scrape-off Layer Width

Using the solution of eqn (2) in eqn (3) for J (with $T^* = T$) after integration by parts, one obtains

$$J = \left(\frac{QT}{2}\right)_0 \tag{8}$$

One can also write Q, using eqn (5), and eqn (2), as

$$\begin{aligned} Q = -\chi_\perp \left. \frac{dT}{dx} \right|_0 &= \int_0^\infty \frac{d}{dx}\left(\chi_\perp \frac{dT}{dx}\right) dx \\ &= \int_0^\infty \frac{T dx}{\tau_{\parallel}} \end{aligned} \tag{9}$$

It follows from eqn (6) that

$$K = \int_0^\infty \frac{T dx}{\tau_{\parallel}} \Big/ \left(\frac{T}{\tau_{\parallel}}\right)_0 \tag{10}$$

Since this represents the width of the heat flux onto the divertor target this provides a natural definition of the SOL width Δ, ie

$$K = \Delta \tag{11}$$

6 An Example

As an illustration of this approach we consider the model

$$\chi_\perp = \frac{k_\perp T^\alpha}{L_T^\beta} \quad , \quad \frac{1}{L_T} = -\frac{1}{T}\frac{dT}{dx} \tag{12}$$

where the exponents α and β are constants and $k_\perp$ will in general depend on plasma parameters; examples can be found in Ref 3. The parallel loss processes can be represented in the form

$$\frac{1}{\tau_{\parallel}} = k_{\parallel} T^\gamma \tag{13}$$

where for free streaming $k_{\parallel} \propto 1/L_{\parallel}$ and $\gamma = 1/2$ and for thermal conduction $k_{\parallel} \sim n/L_{\parallel}^2$ and $\gamma = 5/2$, with $L_{\parallel}$ the distance along the field lines to the divertor plate.

Taking trial functions $T = T_0 e^{-\lambda x}, T^* = T_0 e^{-\lambda^* x}$ and following the prescription in section 3 (ie, evaluating K in terms of T_0, λ and λ^*, varying it with respect to λ to determine a minimum $\lambda(\lambda^*)$ and then setting $\lambda = \lambda^*$ to finally obtain λ), we find

$$\Delta = \left[\frac{(\alpha+1)(\gamma+2)^2}{(\alpha+2)^2} \frac{k_\perp}{k_\parallel} T_0^{\alpha-\gamma} \right]^{1/(\beta+2)} \frac{4+3\alpha+(\alpha+1)\gamma}{(\alpha+1)(\gamma+2)^2} \tag{14}$$

(It is necessary to assume $\alpha > -1$ to find a solution, but this is usually the case [3].) Invoking the definitions (12) and (13) to eliminate T_0 in favour of $\chi_{\perp 0}, \tau_{\parallel 0}$ and L_{T0}, and expressing L_{T0} in terms of $\chi_{\perp 0}, \tau_{\parallel 0}$ and Δ using eqns (5 - 8) and (11), one finally obtains

$$\Delta = f \bar{\Delta}_0 \tag{15}$$

where $\bar{\Delta}_0$ is as defined in eqn (1), but evaluated specifically in terms of separatrix parameters, and the form factor f is given by

$$f = \left[\frac{(4+3\alpha+\alpha\gamma+\gamma)^{(\beta+2)/2}}{(\alpha+2)} \right]^{1/(\beta+1)} \frac{1}{(\alpha+1)^{1/2}(\gamma+2)} \tag{16}$$

The factor f tends to be in the range 0.6 - 1.0, the departure from unity being greatest for the collisional case $\gamma = 5/2$; the dependence on β is rather weak (assuming $\beta \geq 0$, which will ususally be the case for models based on turbulence driven by temperature or pressure gradients). The range of values for f is exemplified by the choices: (i) gyro-Bohm cross-field transport and collisional parallel transport ($\alpha = 3/2$, $\gamma = 5/2$, $\beta = 0$), and (ii) classical collisional transport across and collisionless streaming along the field ($\alpha = -1/2$, $\gamma = 1/2$, $\beta = 0$). These lead to $f = 0.59$ and 1.03, respectively. Other published models for α and β are to be found in Ref 3.

7 Discussion

It should be stressed that the procedure we employ only minimises the normalised entropy production functional K, eqn (6), with respect to T for a given choice of T^*. This is different from minimising K with respect to *both* T and T^*, and does not necessarily lead to the same minimum. In practice we apply this to a particular class of functions for T and T^* containing free parameters in order to determine these parameters. However, the actual solution to eqn (2) may lead to a higher value of K; ie we do not have a true minimising principle. It is possible to construct an example where an exact solution is possible ($\chi_\perp = 1$, $\tau_\parallel = (6T)^{-1}$ with solution $T = (a+x)^{-2}$, where $a = (2/Q)^{1/3}$) that leads to a value $K = 0.42/Q^{1/3}$, whereas the exponential trial function used in obtaining eqn (16) leads to $K = 0.40/Q^{1/3}$.

Conclusions

For a simple model of energy transport in the SOL, a procedure based on a variational formulation, related physically to a non-linear generalisation of minimum entropy production, leads to a natural definition of a SOL width. Such an approach could be generalised to more complex and realistic descriptions of a tokamak divertor.

The procedure provides a means to define and determine a SOL width for a given transport model, using appropriate trial functions for the temperature profile T, which can then be compared with direct measurements of the distribution of heat flux on a divertor target plate. The specific transport model considered serves to illustrates how the SOL width depends on the detailed structure of the anomalous thermal diffusivity. It is found to be well represented by the simple estimate (1), multiplied by a factor which is typically in the range $0.6 - 1.0$ and evaluated at the separatrix, no matter what the exponents α, β and γ are.

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References

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