UKAEA FUS 406

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March 1999

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To be published in Computer Physics Communications as part of the proceedings of the First International Conference on Modern Trends in Computational Physics, Dubna, 1998

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The problem of evolution of toroidal plasma equilibria

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Abstract

This paper is devoted to an advanced mathematical model for a self-consistent description of the evolution of free boundary toroidal plasmas, with a description of numerical algorithms for the solution of the appropriate non-linear system of integro-differential equations, and discussion of some results from the model.

Key words: toroidal plasma; equilibria evolution; coupled non-linear partial differential equations; controlled fusion.

1 Introduction

The problem of mathematical modelling of plasma evolution in axisymmetric toroidal systems with magnetic confinement, such as tokamaks, is considered. A large number of different processes, distinguished by characteristic time scales, develop in toroidal plasmas. Here we study relatively slow processes in the gradual transition of the toroidal plasma from one quasi-equilibrium state to another, caused by self-consistent interactions between the plasma and the magnetic field.

It is very important to develop accurate models of plasma evolution, since, for example, these allow predictions of (a) how the plasma shape changes with time, (b) how to access so-called "advanced regimes" of improved plasma

Preprint submitted to Elsevier Preprint *25 March 1999*

confinement, and (c) the influence of non-ohmic currents on the evolution of plasma equilibria.

An advanced equilibrium evolution model was developed by the authors and implemented in the code SCoPE. The main features of the model and the code are discussed in the paper.

Calculations for cases close to conditions observed in the START spherical tokamak at Culham Science Centre, UK are presented. Parameters were found for which the plasma eventually evolved to an "advanced" regime. Results of calculations are in qualitative agreement with measurements on START. Another example of the theory and code application is discussed, i.e. the calculation of self-consistent time evolution of the electron distribution function and equilibrium in the presence of current drive by radio-frequency (RF) waves.

2 Formulation of the problem

The evolution of free boundary toroidal plasma equilibria can be reduced to the solution of the system of two coupled strongly non-linear equations [1–3] (for the purpose of this paper, equations for the spatial transport of energy and particles [1] have been neglected from the system and replaced with prescribed time-dependent density and temperature profiles)

$$R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi}{\partial R}\right)+\frac{\partial^2\psi}{\partial Z^2}=-\mu_0 R j_\varphi, \tag{1}$$

$$\frac{\partial}{\partial t}\left(\int \tilde{F}dZ\right)\frac{\partial\psi}{\partial Z}-\frac{\partial\psi}{\partial t}\tilde{F}=\frac{1}{\mu_0\sigma_\parallel}\left(\frac{\partial\tilde{F}}{\partial R}\frac{\partial\psi}{\partial R}+\frac{\partial\tilde{F}}{\partial Z}\frac{\partial\psi}{\partial Z}\right)+\frac{j_\varphi R}{\sigma_\parallel}\tilde{F}-$$
$$-\frac{R^2}{\sigma_\parallel}\vec{j}_{\mathrm{add}}\cdot\vec{B},\quad \text{in } \Gamma_p \tag{2}$$

$$j_\varphi=\begin{cases} R\dfrac{\partial p(t,\psi)}{\partial\psi}+\dfrac{1}{2\mu_0 R}\dfrac{\partial F(t,\psi)^2}{\partial\psi} & \text{in } \Gamma_p \\ \sum\limits_{i=1}^{L} J_i(t)\delta(R-R_i)\delta(Z-Z_i) & \text{outside } \Gamma_p \end{cases},$$

$$F(t,\psi)=\int\limits_{\psi=\mathrm{const}}\tilde{F}(t,R,Z)B_{\mathrm{pol}}^{-1}\,dl\Big/\int\limits_{\psi=\mathrm{const}}B_{\mathrm{pol}}^{-1}\,dl.$$

Here R and Z are major radius and vertical co-ordinates, $B_{\mathrm{tor}}=F/R$ and B_{pol} are the toroidal and poloidal magnetic fields (TF, PF), $p(t,\psi)$ is plasma pressure, $J_i(t)$ are currents in the vessel wall, solenoid and poloidal field coils, $\vec{j}_{\mathrm{add}}$ are non-Ohmic plasma currents (driven by plasma pressure or non-inductive heating). Expressions for pressure-driven currents (bootstrap, Pfirsch-Schluter and diamagnetic currents) and for neo-classical conductivity $\sigma_{\|}$ are taken from Refs. [4–6]. The unknown functions are the poloidal flux ψ and current F, which gives B_{tor}. Coupling of equations (1), (2) assumes that the plasma evolves through equilibrium states. The system (1), (2) is completed by the following initial and boundary conditions

$$
\begin{gathered}
\psi(0,R,Z)=\psi_0(R,Z), \quad F(0,\psi)=F_0(\psi), \\
\lim_{R\to 0}\psi = \lim_{\substack{R\to\infty \\ Z\to\infty}}\psi = 0, \quad \tilde{F}(t,R,Z)\Big|_{\Gamma_p} = \frac{\mu_0}{2\pi}I_{\mathrm{rod}}(t)
\end{gathered}
\tag{3}
$$

where Γ_p is the free plasma boundary, defined as the closed flux surface of maximum width, and $I_{\mathrm{rod}}(t)$ is the current down the central rod which produces the TF. The function $\psi_0(R,Z)$ can be obtained from the solution of Eq. (1) with $p(t=0,\psi)$ and $F_0(\psi)$ prescribed. However, instead of setting $F_0(\psi)$, it is usually more convenient to set the total plasma current and the current density profile $j_\varphi(t=0,\psi)$ and define $F_0(\psi)$ from the expression for j_φ. Also, in order to tie the initial pressure and current profiles to the initial plasma geometry rather than to the initial flux, we set them as functions of r, the half-width of the flux surface: $p(t=0,r(\psi))$ and $j_\varphi(t=0,r(\psi))$.

Considering the external parameters in Eqs. (1)-(3) as controls, one can formulate different control problems. In particular, one can adjust the coil currents to keep the total plasma current a given function of time and/or to maintain a specified plasma shape.

Equilibrium evolution has been considered in many papers, e.g. Refs. [1–3,7,8]. One of the new features of the approach presented here is the use of the parallel Ohm's law (Eq. (2)) in cylindrical (R,Z) co-ordinates. Unlike other approaches, which use an analytically-averaged 1D Ohm's law equation, we first solve the 2D equation for Ohm's law in (R,Z) numerically, and then average the solution over flux surfaces. Use of (R,Z) co-ordinates for both Eqs. (1) and (2) results in a less complicated algorithm for the solution of the free boundary problem than the inverse variables technique or mixed inverse variables/(R,Z) technique, but at some expense in CPU time.

3 Account of current driven by RF-waves

Current driven by RF-waves can be calculated from the electron distribution function and then added to $\vec{j}_{\mathrm{add}}$ in Eq. (2). Here we consider lower hybrid current drive (LHCD). The simplest description of the evolution of the electron distribution function in the presence of LHCD is a 1D kinetic equation [9]

$$\frac{\partial f_e}{\partial t} = \frac{n_e e^4 \ln\Lambda\,(2+Z_i)}{4\pi\varepsilon_o^2 m_e^2}\frac{\partial}{\partial v_\|}\left\{\frac{T_e}{m_e v_\|^3}\frac{\partial f_e}{\partial v_\|} + \frac{f_e}{v_\|^2}\right\} +$$
$$+\frac{\partial}{\partial v_\|}\left(D(v_\|)\frac{\partial f_e}{\partial v_\|}\right) + \frac{e}{m_e}E_\|\frac{\partial f_e}{\partial v_\|}, \qquad (4)$$

where D is the quasilinear LH diffusion coefficient, and $v_\|$ and $E_\|$ are the components of the velocity and electric field parallel to the magnetic field. Standard notation from Ref. [2] is used. The Maxwellian distribution can be taken as the initial condition. The boundary conditions at infinity are $f_e(t, -\infty) = f_e(t, +\infty) = 0$.

Solution of Eq. (4) gives the total Ohmic and LH current. However the contribution of the Ohmic current is already treated neo-classically in Eq. (2). In order to pick out the contribution of the LH current we solve two independent kinetic equations: the first one is the original Eq. (4) and the second one is Eq. (4) without the quasilinear diffusion operator (i.e. with $D = 0$). Then the difference between the currents found from these two equations

$$j_{LH} = j^{(1)} - j^{(2)}, \quad j^{(i)} = e\int v_\| f_e^{(i)} d^3v$$

is used in the evolution equation (2). Thus Eqs. (1), (2) and (4) become coupled through $E_\|$, calculated in Eqs (1), (2), and through j_{LH}, calculated in Eq. (4).

For the quasilinear coefficient the following simple model can be used to specify interaction of LH waves with electrons in a particular region of velocity space

$$D = D_0 \exp\left[-\frac{(u-u_0)^2}{(\Delta u)^2} - \frac{(r-r_0)^2}{(\Delta r)^2}\right].$$

Here $u \equiv v_\|/v_e$, $v_e \equiv \sqrt{2T_e/m_e}$ and D_0 is chosen so that the LH current is equal to a specified fraction of the total plasma current.

4 Numerical algorithm

In this section we concentrate only on one of the important features of the numerical approximation of the non-linear integro-differential system of equations (1)-(4). This is the time discretisation.

System (1), (2) is an unusual system, since time derivatives enter only the evolution equation (2). A convenient way to split it over time is

$$\frac{\chi^n - \chi^{n-1}}{\tau}\frac{\partial\psi^{n-1}}{\partial Z} - F^{n-1}\frac{\partial\psi^{n-1}}{\partial F}\frac{1}{\tau}\frac{\partial(\chi^n - \chi^{n-1})}{\partial Z} -$$

$$- \frac{\partial\chi^n}{\partial Z}\left(\frac{\partial\psi^{n-1}}{\partial P}\frac{\partial P^{n-1}}{\partial t} + \frac{\partial\psi^{n-1}}{\partial\psi_{st}}\frac{\partial\psi_{st}^{n-1}}{\partial t}\right) =$$

$$= \frac{1}{\mu_0\sigma_{\parallel}^{n-1}}\left(\frac{\partial^2\chi^n}{\partial R\partial Z}\frac{\partial\psi^{n-1}}{\partial R} + \frac{\partial^2\chi^n}{\partial Z^2}\frac{\partial\psi^{n-1}}{\partial Z}\right) +$$

$$+ \frac{j_{\varphi}^{n-1}R}{\sigma_{\parallel}^{n-1}}\frac{\partial\chi^n}{\partial Z} - \frac{R^2}{\sigma_{\parallel}^{n-1}}(\vec{j}_{\text{ext}}\vec{B})^{n-1}, \tag{5}$$

$$R\frac{\partial}{\partial R}\left(\frac{1}{R}\frac{\partial\psi^n}{\partial R}\right) + \frac{\partial^2\psi^n}{\partial Z^2} = -\mu_0 R j_{\varphi}^n, \tag{6}$$

$$\chi = \int \tilde{F} dZ, \quad \psi_{st} = \mu_0\sum_{i=1}^{L} G(R, Z, R_i', Z_i')J_i(t),$$

G is the Green function [2], n is the index of the time layer, τ is the time step.

Eq. (5) can be solved with standard difference methods for non-rectangular meshes. Solution of the equilibrium eq. (6) can be found with the Lackner method (see, e.g., [2]). control problems can be solved with, for example, the co-ordinate descent method. The plasma boundary in the free boundary problem is defined as the closed flux surface of maximum width after the solution of eq. (6).

The kinetic equation (4) can be included in two ways depending on the time scales under consideration. If we consider a time scale of the order of the characteristic current drive time then Eq. (4) should be evolved one time step for each n. If a larger time scale is considered then it is reasonable to solve Eq. (4) until steady-state for each n.

5 Description of the software

The code SCoPE ("Self-COnsistent Plasma Evolution") has been developed to solve Eqs. (1)-(4) and applied to study equilibrium evolution. SCoPE was developed with use of object oriented and package technologies. It includes a program generator, which can generate a particular program for computer modelling. The latest version of SCoPE includes 120 subroutines in Fortran-77. The code is more than 20000 lines long and contains about 30 variants of physical models and several different numerical algorithms.

The software was used with various compilers and operating systems on SUN, IBM RISC and IBM PC computers. For a relatively simple calculation the characteristic time for the calculation of 100 time steps on a grid $(N_R, N_Z) =$

(50, 50) is about 1 hour on IBM RISC workstation. However, for the solution of control problems, the CPU time can substantially increase to ~10 hours. The requirement for RAM is determined by the storage of 35 two dimensional arrays and is therefore relatively modest on a grid (50,50). The Hard Disk space required for the output files for one run is typically ~10 Mb.

6 Example: Access to "advanced regimes"

There is currently great interest in "optimised magnetic shear" regimes in tokamaks as they may allow steady-state operation with only a modest amount of non-inductive current drive - the remainder of the current would be driven by the plasma pressure. In these regimes, the magnetic shear dq/dr (where q is the safety factor of the flux surface of radius r [2]) is close to zero, or even negative, at or near the plasma centre. Access to these regimes has been demonstrated in conventional aspect ratio tokamaks, and it is important to determine whether these regimes show similar promise in tight aspect ratio "spherical tokamaks" (STs), in which the hole through the centre of the torus is very small and which may offer a more compact option for a fusion device. Experimental demonstration of regimes with steady-state potential is an important objective of the STs presently being constructed (e.g. MAST, NSTX) [10].

The equilibrium evolution model (1)-(3) can be used to study advance regimes. Use of an evolution model, instead of a fixed time equilibrium, is required since it permits study of the access to and duration of the advanced regime.

A START-like free boundary plasma has been considered. The time dependence of currents in the PF coils was adjusted to maintain the plasma position in the equatorial plane to within 5 % accuracy during the simulation. The following initial parameters were used: major and minor radii $R_{mag\ axis} = 0.34$ m and $a = 0.23$ m, vertical elongation $\kappa = 1.6$, $I_{\rm rod} = 500$ kA ($B(R_{mag\ axis}) =$ 0.3T), density $n_e = n_i = 0.4 \times 10^{20}$ $(0.5(1 - (r/a)^n) + 0.5)$ m^{-3}, and temperature $T_e = T_i = 0.3$ $(0.7(1-(r/a)^n)+0.3)$ keV, with $n = 3$, and current density profile $j_0(1 - (r/a)^2)$ with j_0 adjusted to give 200 kA total current (r is the flux surface half-width in the poloidal plane).

Several groups of runs were done for various pressure profiles and initial current profiles with increasing pressure (more details and plots can be found in Ref. [11]). The simulations continued after the pressure rise phase. In all cases, we observed the evolution of the plasma to an "advanced" regime with a very flat q profile, which was sustained until approximate steady-state was reached. It was found that the flatter the pressure profile at the magnetic axis, the deeper is the shear reversal in the final q-profile. This is because flatter

$p(r)$ gives larger off-axis contribution to the toroidal current density j_φ.

The first experiments to access the "advanced" tokamak regime at low aspect ratio have been performed on the START tokamak [12]. Although there were no direct measurements of $q(r)$, there were some indications of a possible negative central shear regime formation, which is in good qualitative agreement with the modelling. Access to this regime depends only weakly on the initial conditions, but depends more on the details of the plasma pressure.

7 Example: Equilibria evolution in the presence of LH waves

Here we present an example of calculations of self-consistent time evolution of the electron distribution function and equilibrium in the presence of current drive by LH waves. A model ITER-like fixed boundary plasma has been considered $a = 2.8$ m, $R_0 = 8$ m with, for simplicity, circular cross-section, $B_0 = 5.7$ T, density and temperature $n_e(0) = n_i(0) = 10^{20}$ m^{-3} and $T_e(0) = T_i(0) = 10$ keV with parabolic profiles. The total plasma current was 8 MA and kept constant over time. For the quasilinear coefficient, the following parameters were used: $u_0 = 2.5$, $\Delta u = 1$, $r_0 = 2a/3$, $\Delta r = a/5$. D_0 was adjusted to make the LH current $\sim 20\%$ of the total plasma current.

At each evolution step τ the kinetic equation was solved with a much smaller time step $\Delta\tau$ until steady-state was reached. The equlibrium evolution step τ was chosen to cover 30 seconds in 300 steps. The LHCD operator was switched on at 2.5 seconds, allowing initial plasma equilibria and $E_{||}$ evolve to some self-consistent state without RF current drive.

Different regimes were considered with various ratios of the local current diffusion time $\tau_d = 10^{-7}(\Delta r)^2\sigma_{||}(r_0)$ to the collision time of the heated electrons $\tau_c = (u_0)^3\tau_0$, $\tau_0 = 4\pi\varepsilon_o^2 m_e^2 v_e^3/(e^4 n_e \Lambda)$. Here $\sigma_{||}(r_0)$ is the plasma conductivity, which enters the evolution equation, τ_0 and u_0 are in terms of the v_e and n_e at r_0. Different ratios were obtained by varying the density and the LHCD radial width Δr. It was shown that quasi-steady-state for the electric field and the current density can be obtained.

8 Conclusions

The equilibrium evolution code SCoPE has been developed to study plasma evolution. The code allows solution of free and fixed plasma boundary problems taking account of non-ohmic currents, neo-classical plasma resistivity, the effects of the wall of the vessel (i.e. image currents), and currents in the solenoid and the poloidal field coils. It has options to solve kinetic and/or transport equations for the evolution of RF-driven currents, plasma density and temperature. Examples of the use of the code for the study of different phenomena in toroidal plasmas have been presented.

9 Acknowledgement

The MSU work was partly funded by the Russian Foundation for Basic Research, grant Nos. 96-01-01171a, 96-15-96205. The UKAEA work was jointly funded by the UK Department of Trade and Industry and by Euratom.

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