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# Suppression of runaway electron avalanches by radial diffusion

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## Abstract

The kinetic theory of runaway electron avalanches caused by close Coulomb collisions is extended to account for radial diffusion. This is found to slow down the growth of avalanches. An approximate analytical formula for the growth rate is derived and is verified by a three-dimensional Monte Carlo code constructed for this purpose. As the poloidal magnetic flux that is available to induce an electric field in a tokamak is limited, avalanches can be prevented altogether by sufficiently strong radial diffusion. The requisite magnetic fluctuation level is sensitive to the mode structure and the speed of the disruption. It is estimated to be  $\delta B/B \sim 10^{-3}$  for parameters typical of large tokamaks.

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## I. Introduction

The generation of runaway electrons in disruptions is a common feature in present-day tokamaks. The runaways are generated in the thermal quench of the plasma, when the plasma temperature drops to the region of 10 eV [1]. The resistivity then becomes very large and the loop voltage therefore rises rapidly – typically to a few hundred volts in the centre of the Joint European Torus (JET) [2]. A large fraction (typically around a half in many tokamaks) of the pre-disruption plasma current can then be converted to a runaway current, and the loop voltage drops to nearly zero. For future larger experiments, runaway electrons can be a severe problem since their loss to the first wall may cause severe localized surface damage. It is thus desirable to avoid runaway production in disruptions as far as possible. Magnetic fluctuations may be helpful in this respect since they increase the loss rate of runaway electrons, as has been reported from experiments in Ref [3].

The present paper investigates to what extent the presence of radial diffusion caused by magnetic fluctuations can be expected to prevent the production of runaways in large tokamaks. Our paper is organized as follows. In Sec II we briefly review the physics of runaway production, focusing on so-called “secondary” generation, which is thought to be the dominant mechanism in tokamaks with large plasma current. In the following section the influence of radial diffusion on runaway production is calculated analytically. The result is then compared with simulations done with a three-dimensional, time-dependent Monte Carlo code constructed for this purpose, as described in Sec IV. Our conclusions are summarized in Sec V, where we estimate the magnetic fluctuation level required to suppress avalanches.

## II. Secondary runaway generation

The generation of runaway electrons has been studied theoretically for over four decades. Early work focused on the formation of fast electrons through the combined effects of a constant electric field and velocity-space diffusion as described by the Fokker-Planck equation [4, 5, 6, 7]. This is sometimes referred to as primary generation of runaways. More recently it has been realized that in a tokamak with sufficiently large plasma current, close Coulomb collisions (which are not described by the usual Fokker-Planck

equation) between thermal electrons and existing runaways can lead to exponential multiplication of the latter – a so-called runaway avalanche [8, 9, 10]. The point is that a single such collision can kick the thermal electron above the critical energy for runaway acceleration by the electric field. The total number of  $e$ -foldings caused by this “secondary generation” mechanism in a disruption can be estimated as [10]

$$\int \gamma_r dt \sim \frac{I}{I_A \ln \Lambda}, \quad (1)$$

where  $I$  is the plasma current and  $I_A = 4\pi m_e c / \mu_0 e = 0.017$  MA is the Alfvén current. Consequently, in small tokamaks this mechanism is not effective, but becomes the dominant mechanism in a next-step device. It has been observed on a small scale in non-disrupting TEXTOR (Tokamak Experiment for Technology Oriented Research) plasmas [11].

Mathematically, secondary runaway production is governed by the kinetic equation

$$\frac{\partial f}{\partial t} - \frac{eE_{\parallel}\xi}{m_e c} \left( \frac{\partial f}{\partial p} - \frac{2\lambda}{p} \frac{\partial f}{\partial \lambda} \right) = C(f) + S, \quad (2)$$

for the electron distribution  $f$  [10]. The velocity-space coordinates are the relativistic momentum  $p$ , which has been normalized to  $m_e c$  (with  $m_e$  the rest mass), and  $\lambda = p_{\perp}^2 / p^2 b = (1 - \xi^2) / b$ , where  $\xi = p_{\parallel} / p$  and  $b = B / B_{\max}$  is the magnetic field normalized to its maximum on the flux surface in question. Coulomb collisions are described by the right-hand side of the equation, where  $C(f)$  denotes the Fokker-Planck collision operator. For superthermal electrons it has the form [7]

$$C(f) = \frac{1}{\tau p^2} \left[ \frac{\partial}{\partial p} (1 + p^2) f + (1 + Z_{\text{eff}}) \sqrt{1 + p^{-2}} \mathcal{L}(f) \right],$$

where  $Z_{\text{eff}}$  is the effective ion charge,  $\tau = 4\pi e_0^2 m_e^2 c^3 / n_e e^4 \ln \Lambda$  is the collision time for relativistic electrons, and

$$\mathcal{L} = \frac{1}{2} \frac{\partial}{\partial \xi} (1 - \xi^2) \frac{\partial}{\partial \xi}$$

is the Lorentz scattering operator. Relativistic effects associated with the thermal electron distribution have been neglected but are retained for the fast electrons.  $S$  is a source term of fast electrons produced by close collisions between existing runaways and slow electrons. In the limit where the former are strongly relativistic and move nearly parallel to the magnetic field while the momentum imparted to the slow electrons is



relatively small,  $S$  can be calculated by retaining only the leading term in the quantum-mechanical Møller scattering formula [12]. This approximation [10], whose accuracy was confirmed in Ref [13], gives

$$S(p, \lambda) = \frac{n_r}{2\pi\tau \ln \Lambda} \sqrt{b^{-1} - \lambda} \frac{\delta(\lambda - \lambda_2)}{p^2} \frac{\partial}{\partial p} \left( \frac{1}{1 - \sqrt{1 + p^2}} \right), \quad (3)$$

$$\lambda_2 = \frac{2b^{-1}}{\sqrt{1 + p^2} + 1}.$$

Note that this source is proportional to the density of existing runaways  $n_r$  and to the collision frequency for *close* collisions  $1/(\tau \ln \Lambda)$  rather than the usual Fokker-Planck collision frequency  $1/\tau$ . Most secondary electrons are generated with relatively low energy ( $p \ll 1$ ) and with velocity vectors nearly perpendicular to the magnetic field ( $\lambda \simeq 1$ ).

Rosenbluth and Putvinski [10] solved the orbit-average of Eq (2) analytically in several limits and constructed an interpolation formula

$$\gamma_r \tau \simeq \frac{E - 1}{\ln \Lambda} \sqrt{\frac{\pi\varphi}{3(Z_{\text{eff}} + 5)}} \left( 1 - \frac{1}{E} + \frac{4\pi(Z_{\text{eff}} + 1)^2}{3\varphi(Z_{\text{eff}} + 5)(E^2 + 4/\varphi^2 - 1)} \right)^{-1/2}, \quad (4)$$

for the runaway production rate  $\gamma_r = d \ln n_r / dt$ , whose correctness they confirmed numerically. Here  $E = |E_{\parallel}| / E_c$  is the normalized electric field,  $\varphi = 1 - 1.46\epsilon^{1/2} + 1.72\epsilon$  describes the effects of finite toroidicity,  $\epsilon = r/R$  is the inverse aspect ratio, and  $E_c = m_e c / e\tau$  is the critical electric field below which no runaway is possible [7]. Thus, we have approximately

$$\gamma_r \tau \sim \frac{E - 1}{2 \ln \Lambda}. \quad (5)$$

The estimate (1) is obtained by integrating this growth rate over time, assuming  $E_{\parallel} \gg E_c$ , using  $E_{\parallel} \simeq L(dI/dt)/(2\pi R)$ , and approximating the plasma inductance by  $L \simeq \mu_0 R$ .

### III. Radial diffusion

The analysis reviewed above assumes that there is no loss of runaway electrons. In practice, these particles undergo radial transport due to magnetic fluctuations and are thus imperfectly confined. This reduces the growth rate of runaway avalanches since the runaways leaving the plasma do not produce any new ones in close collisions with

thermal electrons. In this section, we calculate the reduction of the avalanche growth rate caused by radial diffusion described by the addition of a term

$$\frac{1}{r} \frac{\partial}{\partial r} r D \frac{\partial f}{\partial r} \quad (6)$$

on the right-hand side of Eq (2), where  $D$  is the diffusion coefficient. The effect of diffusion on primary runaway generation has previously been investigated by Catto et al [14], but this problem does not appear to have been analyzed in the context of secondary runaway production other than in the numerical study of Ref [15].

The addition of the diffusion term (6) adds another dimension to the kinetic equation, which then becomes very difficult to solve analytically. [Already without diffusion the kinetic equation (2) poses a formidable mathematical problem, which was, however, successfully solved in Ref [10].] However, an approximate analytical solution can nevertheless be found by noting that there is a separation of time scales in the problem. The acceleration to relativistic speed of an electron that has recently been knocked above the runaway threshold by a close collision occurs on a time scale  $\tau_{acc} = \tau/E$ , but the time scale for growth of the runaway population is much longer: as we have already noted, it is  $\gamma_r^{-1} \sim (2 \ln \Lambda) \tau_{acc}$ . Therefore we expect that finite radial diffusion can interrupt the avalanche even if the diffusion is not strong enough to alter the velocity-space kinetics of the runaway generation process.

The momentum space can thus be divided into a low-energy region ( $p < p_*$ ) where the avalanche mechanism operates undisturbed, and a high-energy region ( $p > p_*$ ) where the accelerating runaways undergo radial diffusion [16]. The flux of runaways from the former to the latter region is given by the Rosenbluth-Putvinski formula (4). In the simple, but unrealistic, case where the diffusion coefficient is independent of energy, integrating the kinetic equation (2) over the high-energy region then gives the following simple equation for the runaway density

$$\frac{\partial n_r}{\partial t} = \gamma_r n_r + \frac{1}{r} \frac{\partial}{\partial r} r D \frac{\partial n_r}{\partial r},$$

where the growth rate  $\gamma_r$  is given by Eq (4). (The source term  $S$  can be ignored for  $p > p_*$ ). If, for simplicity, the plasma has circular cross section, this equation is easily solved by separation of variables. Keeping only the (Bessel) eigenfunction corresponding to the lowest eigenvalue,

$$n_r(r, t) \propto J_0(kr), \quad k = 2.4/a,$$

where  $a$  is the minor radius, one obtains the confinement time  $\tau_e = 1/k^2 D$ . If this is shorter than the inverse avalanche growth rate  $\gamma_r^{-1}$  no avalanche occurs. Thus, in this simple case the avalanche is expected to be interrupted if

$$D > D_{\text{crit}} = \gamma_r \left( \frac{a}{2.4} \right)^2. \quad (7)$$

The diffusion coefficient  $D$  of fast electrons is thought to be governed by magnetic turbulence in the plasma [17], but its precise value is not known. In the simplest picture, the electrons follow stochastic magnetic field lines and thus diffuse radially out of the plasma with a diffusion coefficient of the order

$$D_{RR} = \pi q v_{\parallel} R \left( \frac{\delta B}{B} \right)^2, \quad (8)$$

which was first derived by Rechester and Rosenbluth [18]. Here  $\delta B$  is the magnetic fluctuation level, and the magnetic field is assumed to be fully stochastic. However, runaway confinement is known to be much better than that predicted by Rechester-Rosenbluth diffusion [17]. It was first pointed out by Mynick and Krommes [19] that the finite Larmor radius and magnetic drift velocity of the fast electrons can considerably reduce the diffusion coefficient. Some of their results were later disputed by Myra and Catto [20] (and subsequently corrected in a joint paper [21]), who found that the poloidal component of the drift does not, in fact, reduce the diffusion coefficient in broad-band magnetic turbulence. Their analysis shows that the effect of magnetic drift on diffusion is generally very complicated and depends on details of the turbulence such as its poloidal localisation and radial correlation length. In general, drifts tend to be important when the orbit width  $\Delta_e$  exceeds the mode width  $\Delta_m$ . The details can vary substantially, however, and the diffusion coefficient may decrease slowly (algebraically) or rapidly (exponentially) with  $\Delta_e/\Delta_m$  depending on the poloidal structure of the turbulence. In any case, since the orbit width for a relativistic electron increases linearly with  $p$ ,

$$\Delta_e \simeq \frac{q m_e c}{e B} = \frac{q p}{B_T} 1.7 \text{ mm}, \quad (9)$$

the confinement improves progressively for fast electrons as they are accelerated by the electric field. Here  $q$  is the tokamak safety factor and  $B_T$  the magnetic field strength in Teslas.



The situation is thus considerably more complicated than indicated by the simple criterion (7), and we need to calculate how a velocity-dependent diffusion process affects a runaway avalanche. To this end, we again integrate the kinetic equation (2) in the high-energy region,  $p > p_*$ , but this time only take the integral over perpendicular momentum. Since most runaways have nearly parallel momenta,  $p \simeq p_{\parallel}$ , the pitch-angle scattering operator is approximately

$$\mathcal{L} \simeq \frac{1}{2p_{\perp}} \frac{\partial}{\partial p_{\perp}} p_{\perp} \frac{\partial}{\partial p_{\perp}}$$

and is therefore annihilated by the integration. Thus we obtain the following equation for the fast electrons

$$\tau \frac{\partial F}{\partial t} + (E - 1) \frac{\partial F}{\partial p} = \frac{\tau}{r} \frac{\partial}{\partial r} r D(p) \frac{\partial F}{\partial r}, \quad (10)$$

where  $F$  is the distribution function in parallel momentum,

$$F = \int f d^2 p_{\perp}.$$

The second term in Eq (10) contains the difference between acceleration by the electric field and collisional drag. As before, the population of high-energy electrons governed by Eq (10) is fed from below (in  $p$ ) by the avalanche mechanism, which operates in the region ( $p < p_*$ ) according to Eq (4) and thus provides a boundary condition on  $F(p_*)$ ,

$$(E - 1)F(p_*, r, t) = \gamma_r \tau n_r = \gamma_r \tau \int_{p_*}^{\infty} F(p) dp. \quad (11)$$

Note the unconventional form of this condition: the boundary data depends on the integral of the solution.

It is not difficult to find separable solutions to Eq (10),

$$F(p, r, t) = J_0(kr) \exp \left[ \gamma t - \frac{\tau}{E - 1} \int_{p_*}^p (\gamma + k^2 D(p')) dp' \right]. \quad (12)$$

In general, the full solution must be sought as a superposition of several of these eigensolutions, but in the present case the boundary condition (11) is satisfied by a single eigensolution if the growth rate  $\gamma$  is chosen so that

$$E - 1 = \gamma_r \tau \int_{p_*}^{\infty} dp \exp \left[ -\frac{\tau}{E - 1} \int_{p_*}^p (\gamma + k^2 D(p')) dp' \right]. \quad (13)$$

More generally, Eqs (10) and (11) are satisfied by any sum of such solutions corresponding to different Bessel eigenvalues  $k$ . Again, for our purposes it is sufficient to

keep only the term corresponding to the the lowest  $k$  since this term has the highest growth rate. Therefore, the relevant solution is given by Eq (12) with  $k = 2.4/a$ .

In a disruption, the electric field is normally large in the sense that  $E \gg 1$ , so that the runaway threshold is very low,  $p_* \ll 1$ . We can then take  $p_* = 0$  in Eq (13), which determines the avalanche growth rate  $\gamma$  if the runaway diffusion coefficient  $D(p)$  is known. If the latter is independent of  $p$ , we simply obtain

$$\gamma = \gamma_r - k^2 D, \quad (14)$$

which reproduces the criterion (7). As already remarked, in practice  $D(p)$  decreases with increasing  $p$ ; in fact, the integral

$$\int_0^\infty D(p) dp \quad (15)$$

usually converges fairly rapidly. In this case, Eq (13) can be simplified further by noting that, because of Eq (5),

$$\frac{\gamma\tau}{E-1} \lesssim \frac{1}{2 \ln \Lambda} \ll 1,$$

so that the first term of the integral within the exponent in Eq (13) is small over the convergence region of the term involving  $D(p')$ . The integration of the latter can thus be extended to infinity, which gives us a completely explicit formula for the growth rate,

$$\gamma = \gamma_r \exp \left( -\frac{k^2 \tau}{E-1} \int_0^\infty D(p) dp \right). \quad (16)$$

This simple formula is the main result of the present paper. Note that although the avalanche growth rate is reduced by the presence of radial diffusion, it cannot become negative if the integral (15) converges. The physical reason for this is that the confinement time of the fast electrons approaches infinity sufficiently quickly as they are accelerated by the electric field. To understand this in a simple way, consider the simple time-dependent diffusion equation

$$\frac{\partial n_r}{\partial t} = \frac{1}{r} \frac{\partial}{\partial r} D(t) \frac{\partial n_r}{\partial r},$$

which models the transport of electrons whose diffusion coefficient changes with time (because of acceleration in our problem). By introducing the new time variable

$$t' = \int_0^t D(t) dt,$$

this equation is transformed into an ordinary diffusion equation. It is immediately clear that if the integral defining  $t'$  converges as  $t \rightarrow \infty$ , a fraction

$$\exp\left(-k^2 \int_0^\infty D(t) dt\right)$$

of the electrons will always stay in the system, regardless of the absolute value of the diffusion coefficient. The avalanche growth rate (16) contains exactly this reduction factor.

#### IV. Monte Carlo simulation

In order to verify the ideas described above and to fully simulate the avalanche process, we have constructed a Monte Carlo code, ARENA (Avalanche of Runaway Electrons, Numerical Analysis code). This code numerically solves the orbit-average of Eq (2) along the lines described in Ref [10], but with the additional diffusion term (6).

The Monte Carlo solution of this equation involves following a large number of Monte Carlo particles in phase space  $(r, p, \lambda)$ ; the phase space positions of these particles are changed periodically, at time intervals  $\Delta t$ , by applying Monte Carlo operators representing collisions, the accelerating electric field, and the diffusion term (6). The Monte Carlo operator for the latter is given by

$$\Delta r = \frac{\partial D}{\partial r} \Delta t + \zeta \sqrt{2D\Delta t},$$

where  $\zeta$  is a random number with zero mean and unit variance. Monte Carlo operators for the orbit-averaged effects of Coulomb collisions and the electric field are given in Ref [10]. It should be noted here that a Monte Carlo particle given by  $(r, p, \lambda)$  represents a number of real particles uniformly distributed in time along the unperturbed guiding-center orbit given by the three invariants  $(r, p, \lambda)$ .

Thus, the procedure for obtaining the numerical solution is as follows: at the beginning of the calculation a large number of Monte Carlo particles are initialized with a Maxwellian velocity distribution and with a radial density profile which can be freely specified; after each time step,  $\Delta t$ , the particle positions in phase space are updated with the aid of the Monte Carlo operators; in addition, new runaway particles are added in accordance with the source term (3); diagnostic output (runaway growth rate, aver-



aged energy etc) are produced after suitably spaced time intervals (i.e. some fraction of a collision time).

The ARENA code has been benchmarked carefully (without diffusion) against the Rosenbluth-Putvinski formula (4), which has been found to be accurate in all physically relevant cases. Adding radial diffusion, we have used the code to verify the approximate analysis given in Sec III. If the diffusion coefficient is taken to be independent of momentum, the growth rate is found to drop linearly with increasing diffusion, in practically exact agreement with Eq (14), see Fig 1. In the more realistic case where the diffusion coefficient does depend on momentum, we have verified Eq (16) by running the code with a radial diffusion coefficient given by

$$D(p) = D_0 e^{-(p/\Delta p)^2},$$

for various values of the width  $\Delta p$ . The runaway electrons were initialized with the (almost flat) radial density profile  $n_r(r) = n_{r0}[1 - 0.9(r/a)^2]^{0.1}$ , and the ratio between the accelerating field and the critical one was  $E = |E_{||}/E_c| = 100$ . The result of the simulations is shown in Fig 2, where the growth rate obtained from the simulations and the one from Eq (16) are plotted as functions of  $\Delta p$ . As can be seen, the agreement is good, which indicates that Eq (16) is a convenient and accurate estimate of the reduction of the runaway growth rate caused by momentum-dependent diffusion. If  $\Delta p$  becomes very large the agreement is worse since the integral (15) then converges slowly. In such cases the more accurate expression (13) should be used to calculate the growth rate.

## V. Conclusions

Our analytical calculation and numerical simulations demonstrate that radial diffusion slows down runaway avalanches, so that the growth rate  $\gamma_r$  calculated in Ref [10] is reduced according to Eq (13). If the integral (15) converges quickly enough, this equation is reduced to the more explicit Eq (16). At first sight, this expression suggests that introducing a large diffusive loss of fast electrons does not prevent an avalanche from occurring, since the growth rate (16) cannot become negative. However, in practice there are at least two reasons why sufficiently rapid diffusion should still be able to effectively prevent avalanches. Firstly, the runaway energy is not unlimited, as assumed in



Sec III. Several physical processes (like synchrotron radiation or cyclotron instability) limit the energy and hence the confinement time of runaways, so that avalanches may be completely stabilized. This may, however, require a very large diffusion coefficient. Secondly, and perhaps more importantly, for practical purposes it is sufficient to reduce the growth rate to a level where there are not enough volt-seconds of induced electric field to achieve significant avalanche growth. Thus, although complete avalanche stabilisation by radial diffusion may be difficult, it is nevertheless possible to reduce the avalanche size to a manageable, or even negligible, level.

In order to estimate the magnetic fluctuation level required for this, we use the approximate expression (1) for the total number of avalanche  $e$ -foldings, modified by the reduction factor from Eq (16),

$$\int \gamma dt \sim \frac{I}{I_A \ln \Lambda} \exp \left( -\frac{k^2 \tau}{E-1} \int_0^\infty D(p) dp \right). \quad (17)$$

We assume that the diffusion coefficient  $D(p)$  is equal to the Rechester-Rosenbluth value (8) for small  $p$  and falls off at higher energies when the orbit width (9) becomes comparable to the mode width of the magnetic turbulence [20]. The mode width is, of course, quite uncertain; it has recently been estimated to be 0.5 - 4 cm in TEXTOR-94 [23]. This determines the value of the  $D(p)$  integral, which we write as

$$\int_0^\infty D(p) dp = p_{\text{crit}} \pi q c R \left( \frac{\delta B}{B} \right)^2,$$

where  $p_{\text{crit}}$  characterizes the speed of convergence. Requiring that the number of  $e$ -foldings (17) should not be larger than some number  $N$  defining the maximum tolerable avalanche size implies the following lower limit on the magnetic fluctuation level,

$$\frac{\delta B}{B} \gtrsim \frac{a}{2.4} \sqrt{\frac{E-1}{\pi q c R \tau p_{\text{crit}}} \ln \left( \frac{I}{N I_A \ln \Lambda} \right)}.$$

This limit is practically independent of the argument within the logarithm, and becomes for typical large-tokamak parameters about  $10^{-3}$ . (For JET,  $E_{\parallel} \lesssim 10^4$  V/m,  $R = 3$  m,  $a = 1$  m,  $n_e = 2 \cdot 10^{19}$  m<sup>3</sup>, and  $p_{\text{crit}} > 1$ ). This greatly exceeds the fluctuation level in quiescent plasmas but is not unrealistic in a disruption, where  $\delta B/B \sim 10^{-2}$  has been reported [24]. Thus, it appears possible that the naturally occurring, or any externally induced, magnetic fluctuations could significantly reduce the size of secondary runaway avalanches.

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but much smaller than  $E/\tau k^2 D$ , so that diffusion does not affect electrons in the low-energy region  $p < p_*$ .

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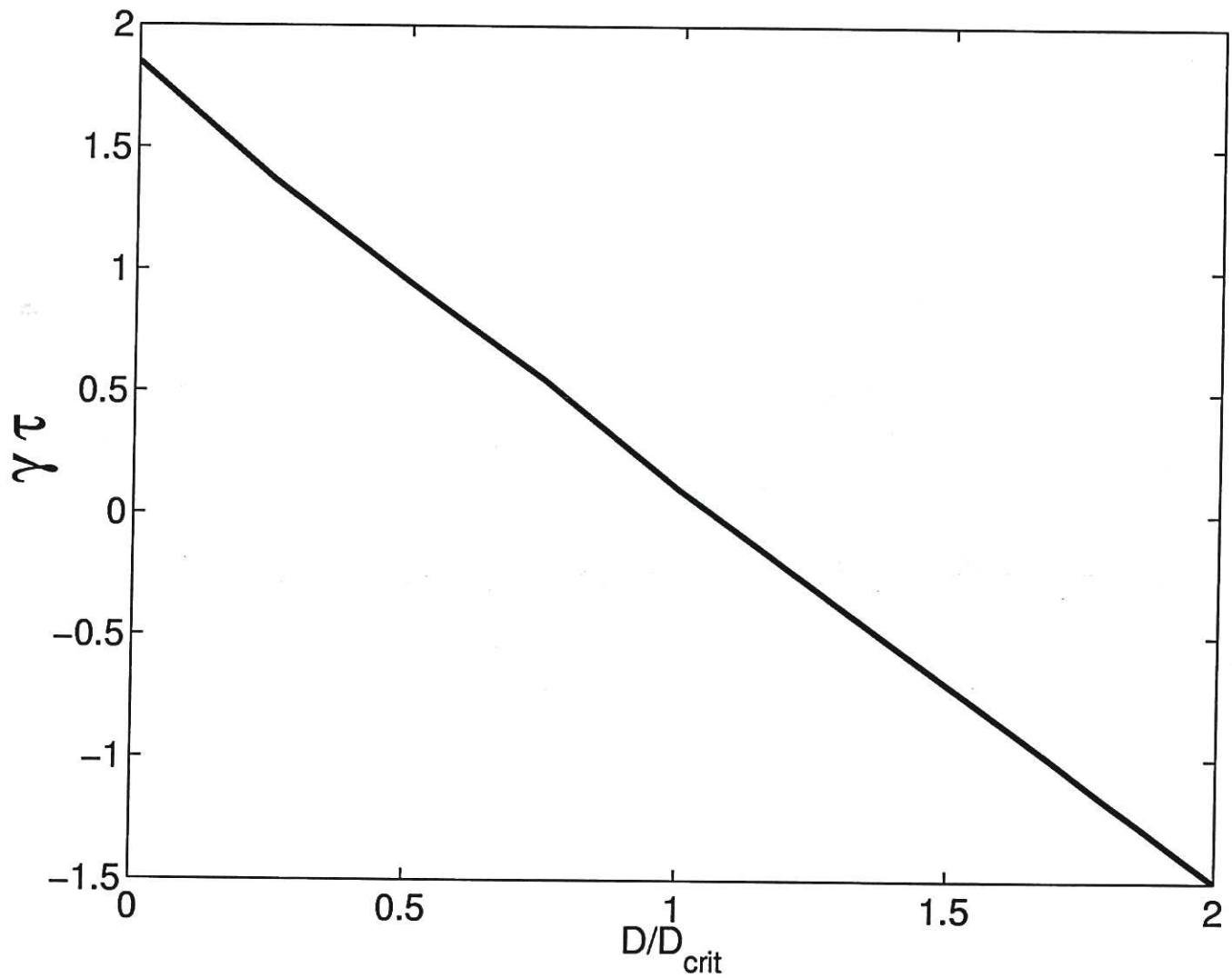


Fig 1. The normalized avalanche growth rate  $\gamma\tau$  vs the diffusion coefficient in the case when the latter is assumed to be independent of momentum. The growth rate decreases linearly with  $D$  and vanishes when  $D = D_{crit}$ , as predicted by Eq (14).

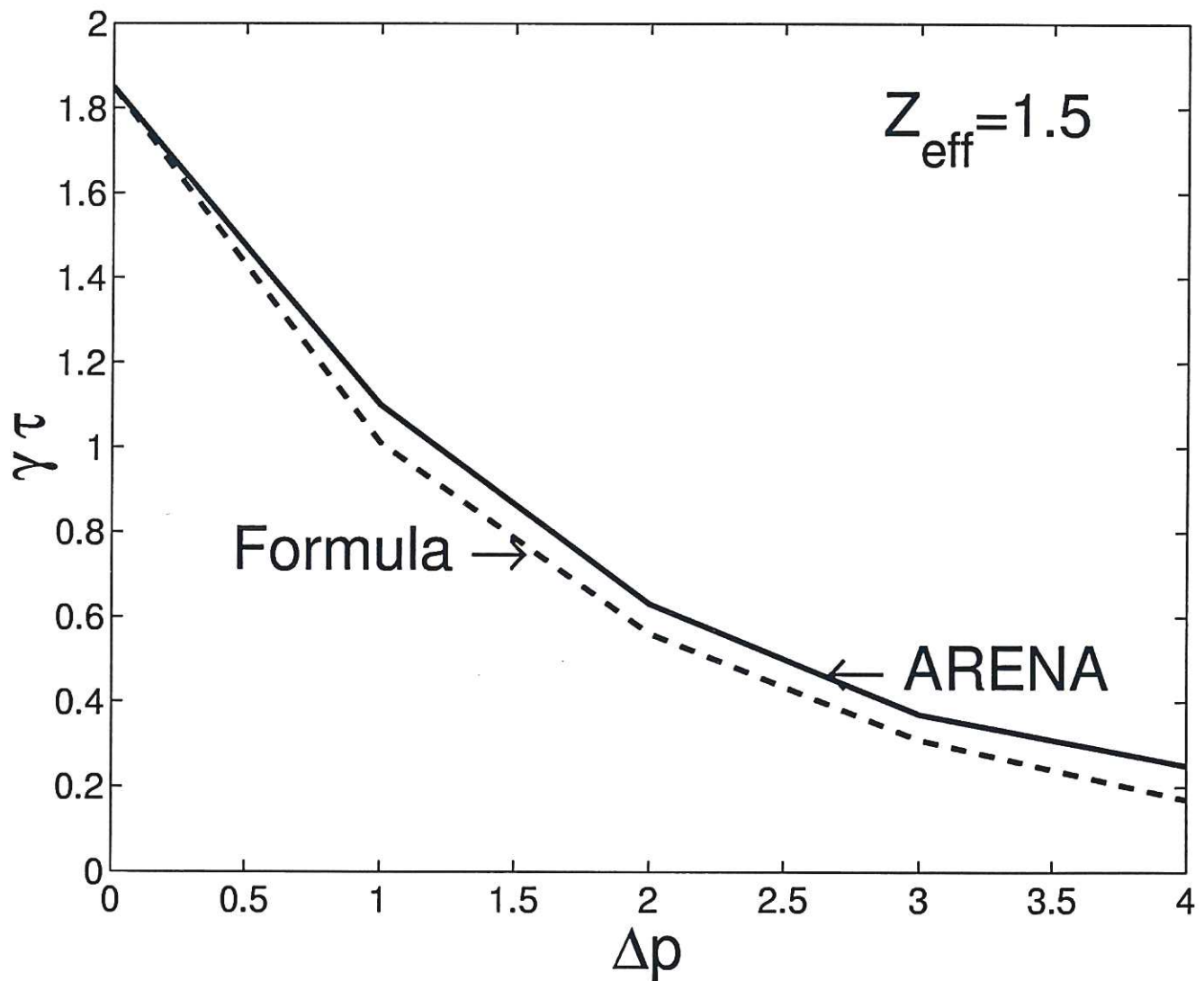


Fig 2. The normalized avalanche growth rate  $\gamma\tau$  vs  $\Delta p$  for the diffusion coefficient  $D(p) = D_0 e^{-(p/\Delta p)^2}$ , with  $D_0 = R^2/\tau$ ,  $a/R = 0.31$ , and  $Z_{\text{eff}} = 1.5$ . The solid line shows the result from the numerical simulation, and the dotted line the analytical approximation (16).