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Stability of Resistive Wall Modes in the Presence of Walls with Poloidally Varying Rotation

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Abstract: Stabilisation of the Resistive Wall Mode (RWM) is an essential requirement for many confinement devices if they are to produce attractive power plant designs. This can be achieved by the introduction of a second, rotating, wall. An alternative approach arises naturally from schemes for a tokamak power plant involving a flowing lithium wall, which produces a poloidally varying rotation. The stability of RWMs in such configurations is investigated.

1. Introduction

The resistive wall mode (RWM), which grows on the timescale of the vertical field time constant of the vacuum vessel, poses a threat to steady state advanced tokamak operation. Bulk rotation of the plasma can result in the plasma seeing the wall as perfectly conducting due to the skin effect, but this can be undone if the mode 'locks' to the wall [1]. This has led to the suggestion that there should be a second, rotating wall [2] (possibly 'faked' by a suitable configuration of external sensors and coils [3]) in addition to a static one: then the plasma cannot simultaneously lock to both and the mode is predicted to be stabilised. However, the proposed use of a flowing lithium wall in a power plant [4] leads to a configuration in which the 'wall' moves poloidally in opposite directions in the upper and lower halves of the poloidal cross section. Such non-uniform poloidal 'rotation' could also tend to stabilise the RWM as it clearly cannot lock to the wall everywhere. In this paper we analyse the effect of non-uniform rotation on the RWM stability, first considering the case with poloidally non-uniform toroidal rotation, then the case of poloidal flows. As shown in Section 2, the effect of the non-uniform rotation is to couple different poloidal harmonics (labelled by poloidal mode number m) so that the matching condition at the wall leads to a recurrence relation between these harmonics involving the wall time-constant, the flow velocity of the 'wall' and the set of stability indices Δ'_m, corresponding to jumps of the eigenfunction across the wall. Introducing a model for the plasma response at a resonant surface in the bulk plasma leads to an eigenvalue problem for the RWM. Results for the critical flow for RWM stability and the corresponding poloidal mode structure for the cases of current driven and pressure driven modes are presented in Section 3. Section 4 provides conclusions.

2. Formulation of the problem

The starting point is the linearised induction equation for the magnetic perturbation $\mathbf{b}$ in the wall:

$$\frac{\partial \mathbf{b}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{b}) + \eta_W \nabla^2 \mathbf{b} \, , \tag{1}$$

where η_W is the wall resistivity (assumed uniform). Now, in the envisaged flow pattern for a power station we would have $\mathbf{V} = V(\theta)\mathbf{e}_\theta$, with $\mathbf{e}_\theta$ a unit poloidal vector. Indeed, to simulate injection and extraction points $V(\theta)$ would be discontinuous at these locations. As mentioned above, we first investigate the more tractable flow $\mathbf{V} = V(\theta)\mathbf{e}_z$, with $\mathbf{e}_z$ a unit *toroidal* vector. Decomposing the radial component of $\mathbf{b}$ as

$$b_r = \left(\sum_{m=-\infty}^{\infty} b_m \exp im\theta \right) \exp{(pt + ikz)}, \tag{2}$$

the radial component of Eqn (1) gives, assuming a cylindrical limit,

$$\sum_m p b_m \exp im\theta = \sum_m \left(-ikV(\theta) b_m + \eta_W \frac{\partial^2 b_m}{\partial r^2} - \frac{m^2}{r^2} \eta_W b_m \right) \exp im\theta. \tag{3}$$

To connect up the interior plasma with the outer vacuum region we integrate Eqn.(3) across the wall (radius a, thickness δ_W), to find

$$\sum_m \left(p + ikV(\theta)\right) \delta_W b_m \exp im\theta = \sum_m (\eta_W/a)(\Delta'_m - m^2 \delta_W/a) b_m \tag{4}$$

where Δ'_m is the well known (dimensionless) stability index at the wall of the mth poloidal harmonic.

Now we must choose a suitable functional form for $V(\theta)$. A particularly tractable choice is $V(\theta) = V \sin\theta$. Inserting this expression into Eqn.(4) we see that equating each coefficient of $\exp(im\theta)$ produces a difference equation. In fact, introducing the wall time $\tau_W = a\delta_W/\eta_W$, normalising p to τ_W, and replacing V_0 by $kV_0\tau_W/2$, we have

$$p b_m + V_0(b_{m-1} - b_{m+1}) = \Delta'_m b_m - \kappa m^2 b_m \tag{5}$$

So, the redefined V_0 is the key parameter in the problem (the inclusion of the small parameter $\kappa = \delta_W/a$ merely serves to help convergence in m). The case of a poloidal flow $V_0 \sin\theta$ leads to a similar equation, but with the substitution $k \to -m/a$. More realistic poloidal flows couple more harmonics: modelling the lithium wall as counter-rotating flows in the upper and lower halves of the poloidal cross section (which we term a 'step' flow) leads to an infinite set of these harmonics, since then

$$V = \frac{4}{\pi} V_0 \sum_r \frac{\sin(2r+1)\theta}{2r+1} \tag{6}$$

Correspondingly Eqn (5) takes the form

$$pb_m + V_0 \sum_{m'} c_{mm'}(b_{m+2m'+1} - b_{m-2m'-1}) = (\Delta'_m - \kappa m^2) b_m \tag{7}$$

where $c_{mm'} = (2/\pi)m/(2m'+1)$ and $V_0 \tau_W/a \to V_0$. The Δ'_m quantities are to be calculated from the equilibrium profiles of current and pressure chosen.

It may be that one harmonic M is resonant at r_s in the plasma, where we assume the plasma response is visco-resistive: $\Delta'_M = p\tau_V$, where τ_V is the viscous diffusion time normalised to τ_W [5]. This must be matched to the jump in the radial derivative of the Mth eigenfunction at r_s. Using the matching condition at the resistive wall, $\Delta'_M(a) = p$, we obtain [6]

$$\Delta'_M(r_s) = \frac{1 + c_1 p}{c_2 + c_3 p} \tag{8}$$

which can be used to write

$$\Delta'_M(a) = \frac{-1 + c_2 p\tau_V}{c_1 - c_3 p\tau_V} \tag{9}$$

3. Results and Discussion

We consider two cases: (i) a prototype current driven RWM situation of a current profile with a non-resonant m=3 mode which is ideally unstable in the absence of a wall, having a resonant m=2 sideband; and (ii) the situation of interest for advanced tokamaks with a pressure driven resonant $m = 2$ mode which is ideally unstable without a wall. We take $\tau_V = \tau_W$ in all calculations.

For case (i) we choose current and safety factor profiles as shown in Fig 1 and calculate Δ'_m. We find $\Delta'_3 = 4.89$; the stability indices for all other harmonics with $m \neq 2$ are negative. For $m = 2$ we have c_1 = -0.017 c_2 = 1.722 and c_3 = 0.218 in Eqn (8). In Fig 2 we show the effect of increasing V_0 on the growth rate and frequency of the RWM in case (i), for a toroidal flow $V_0 \sin\theta$.

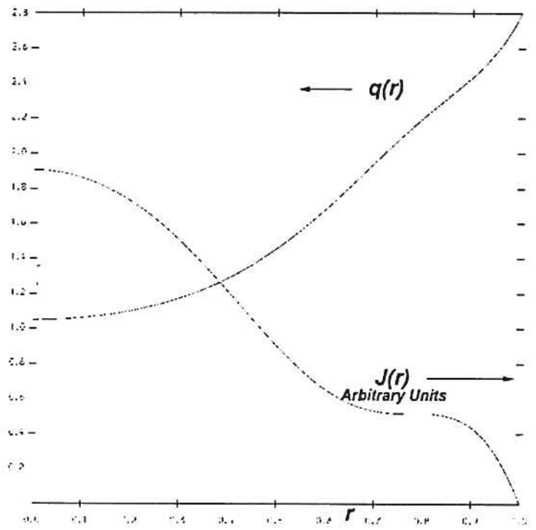

Figure 1: Current and safety factor profiles used in case (i): the $m = 3$ RWM is unstable for $V_0 = 0$

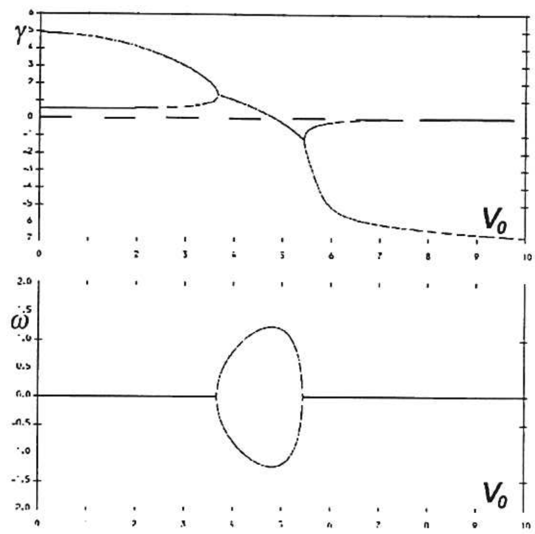

Figure 2: Growth rate and frequency against V_0 for case (i) with toroidal flow $V_0 \sin\theta$

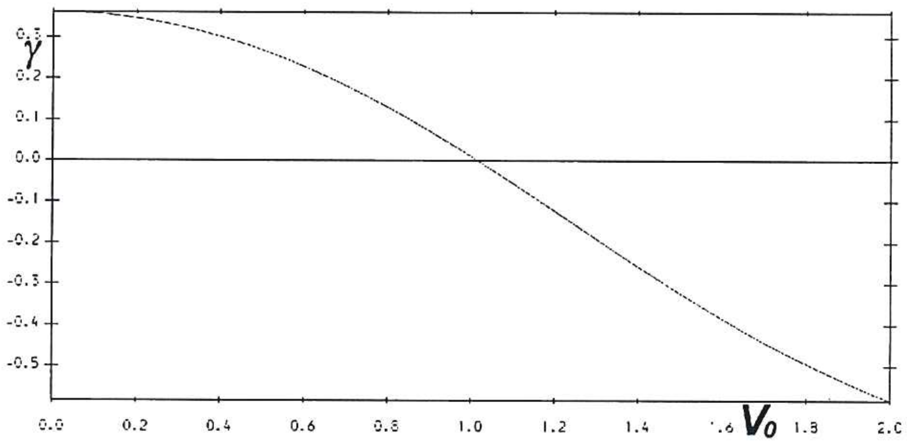

Figure 3: Growth rate against V for case (ii) with poloidal flow $V_0 \sin\theta$

In case (ii) we simulate the effects of an unstable, pressure driven, $m = 2$ mode in toroidal geometry by generic stability parameters ε and δ: ε represents the degree of instability of the ideal mode in the absence of a wall, δ the degree of stability of the tearing mode in the presence of a perfectly conducting wall. Then [1]

$$\Delta_2'(r_s) = \frac{1-\delta p}{-\varepsilon + p} \tag{10}$$

We choose $\delta = 1$ and $\varepsilon = 0.1$ in Eqn (10). For the other harmonics we calculate $\Delta_0' = -2$, $\Delta_{\pm 1}' = -2q_a(q_a \mp 1)^{-1}$, with $q_a = 2.8$, and take $\Delta_m' = -2|m|$ for all other

m. In Fig 3 we show the variation of growth rate with flow for a poloidal flow $V_0 \sin\theta$ (the case of a toroidal flow is similar).

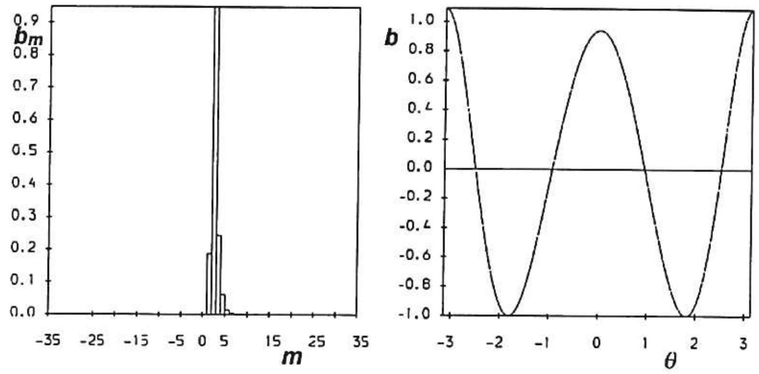

Figure 4: Case (ii) with $V = V_0 \sin\theta$ and $V_0 = 1.0$: (a) Fourier spectrum of b_m; (b) the poloidal eigenmode structure at $z = 0$.

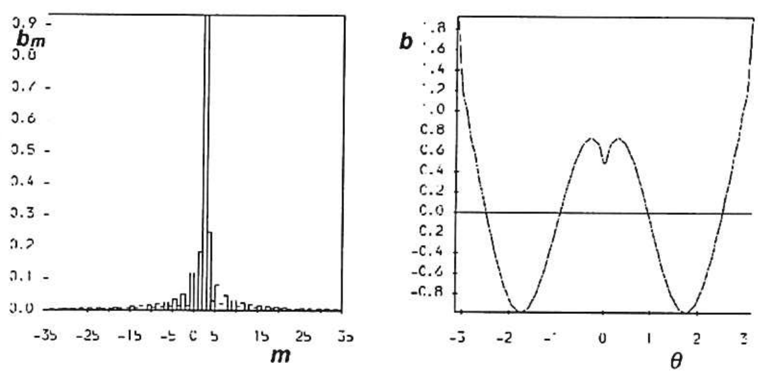

Figure 5: As in Fig 4, but with a step function flow

Figures 2 and 3 show a stabilising influence of a poloidally varying flow on the RWM. In case (i), Fig 2, the situation is more complex: with increasing flow two unstable modes eventually coalesce to produce a mode with a real frequency which, with further increase in flow, stabilises before separating to produce two modes, one of which becomes marginally unstable. Thus there is at least a window of stable flows: $4.85 < V_0 < 7.81$. The situation in case (ii), Fig 3, is much simpler: a steady decrease in growth rate with flow until one arrives at a stable mode. Calculations have also been carried out with more complex forms for $V(\theta)$, retaining a greater number, N, of harmonics in the Fourier representation of the step flow in Eqn (6). The results are quantitatively similar, growth rates decreasing smoothly until the mode stabilises at a critical V_0, although the eigenfunctions become progressively more peaked around $\theta = \pi$ (where the poloidal flow vanishes), with a corresponding broader spectrum of Fourier harmonics. Figures 4 and 5 show the eigenfunctions (for $z = 0$) and Fourier decompositions for cases with $N = 1$ and $N \to \infty$, respectively. The critical flow for stability changes relatively little: $N = 1, V_0 = 1.016; N = 3, V_0 = 1.025$; and $N = 35, V_0 = 1.025$.

4. Conclusions

Our investigations of the introduction of a poloidally varying flow, stimulated by the lithium blanket concept, show that it is possible to stabilise the RWM in this way. This encourages a more comprehensive study of different equilibria and corresponding Δ'_m to see how robust this conclusion is and what are the requirements on V_0. For the cases with poloidal flow studied we find the critical value of the dimensionless parameter V_0 for stability is near unity, lower than for the toroidal case (the actual flows differ by a further a/R factor where R is the major radius). Physically this corresponds to a flow $V \sim a/\tau_W \sim \eta_W/\delta_W$. For lithium with $\delta_W \sim 1$cm, this yields $V \sim 25\text{m.s}^{-1}$. Similar results have been found in Ref 4, although a truncation at three poloidal harmonics used there can produce misleading results, particularly at larger V. The parameter V_0 can be recognised as the magnetic Reynolds number for the 'wall'. However, it is always possible to simulate the stabilising influence of a poloidally rotating wall by a system of programmed coils, which allows one to separate the idea from the lithium wall concept; this raises the question of what is the optimal configuration for this scheme.

Acknowledgements

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