UKAEA FUS 458

EURATOM/UKAEA Fusion

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June 2001

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On ion flow caused by the inductive electric field in a tokamak

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Abstract

It was recently pointed out that the inductive electric field in a tokamak can give rise to toroidal and poloidal plasma rotation comparable to that sometimes observed in experiments. Here it is shown that the flow velocity of heavy impurity ions, which is normally what is measured, is lower than that of the bulk plasma for rotation produced in this way. If the bulk ions and electrons are in the banana regime, the impurity rotation is at most about two thirds of the bulk plasma rotation and decreases with increasing effective ion charge ($Z_{\rm eff}$) and distance from the magnetic axis.

PACS Numbers: 52.25.Dg, 52.55.Fa, 52.25.Vy, 52.25.Fi

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Understanding the causes of tokamak plasma rotation – both toroidal and poloidal – is important since it sheds light on the transport mechanisms that are operational in the plasma. For instance, transport barriers are frequently observed to be associated with sheared rotation, and it is widely believed that sheared flow is the cause of these barriers. Conversely, plasma transport properties affect the rotation by, e.g., sustaining it against viscous damping.

The rotation velocity of each species a in an axisymmetric tokamak is given by an expression of the form

$$\mathbf{V}_a = \omega_a R\hat{\varphi} + u_{a\theta}\mathbf{B},$$

where ω_a and $u_{a\theta}$ are flux functions, R is the major radius, $\hat{\varphi}$ the toroidal unit vector, and $\mathbf{B}$ the magnetic field [1, 2, 3]. This is the most general expression for an incompressible flow tangential to flux surfaces. Note that the toroidal rotation has contributions from both ω_a and $u_{a\theta}$ while poloidal rotation is described by $u_{a\theta}$ alone. Standard neoclassical theory relates these quantities to linear combinations of radial density and temperature gradients and (for ω_a) the radial electric field. In a recent paper [4], however, it was pointed out that the inductive electric field used to drive Ohmic current also contributes significantly to $u_{a\theta}$ for bulk ions ($a = i$) in a pure hydrogen plasma, and this may be relevant to recent observations in Alcator C-Mod of spontaneous rotation in Ohmic plasmas [5]. On the other hand, it is the rotation of impurity ions that is measured in these (and most other) experiments, from Doppler broadening of line radiation, and it is well known that this velocity can be quite different from that of the bulk plasma [6]. It is the purpose of this Brief Communication to clarify this point by presenting a calculation which extends that of Ref [4] to account for the presence of a heavy impurity species in the plasma and, at the same time, simplifies the analysis by using the Hirshman-Sigmar moment formalism [2]. From this calculation, it is evident how to make the further generalization to an arbitrary number of impurities of general charges and masses.

We thus consider a plasma consisting of electrons (e), hydrogenic bulk ions (i) and heavy impurity ions (z). We treat the impurity charge as a large expansion parameter, $z \gg 1$, and assume that the impurity strength parameter $\alpha = Z_{\text{eff}} - 1 = n_z z^2/n_e$ is of order unity so that $n_z z/n_e \ll 1$ and $n_e \simeq n_i$. In the experiments reported in Ref [5] the impurity charge is $z = 19$, and as is typical in these and many other experiments, we assume that the bulk ions and electrons are in the banana regime of collisionality while the impurities are collisional. Under the influence of a toroidal electric field each species satisfies the flux-surface averaged $\langle\cdots\rangle$ force balance equations [2]

$$\langle \mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}_a\rangle = \left\langle B\left(R_{a\|} + n_a e_a E_{\|}\right)\right\rangle,$$
$$\langle \mathbf{B}\cdot\nabla\cdot\mathbf{\Theta}_a\rangle = \left\langle BH_{a\|}\right\rangle.$$

Following Hirshman and Sigmar, we relate the viscosities $\mathbf{\Pi}_a$ and $\mathbf{\Theta}_a$ to the poloidal flux of particles and heat by

$$\frac{1}{\langle B^2\rangle}\left\langle \begin{array}{c} \mathbf{B}\cdot\nabla\cdot\mathbf{\Pi}_a \\ \mathbf{B}\cdot\nabla\cdot\mathbf{\Theta}_a \end{array}\right\rangle = \begin{pmatrix} \hat{\mu}_{a1} & \hat{\mu}_{a2} \\ \hat{\mu}_{a2} & \hat{\mu}_{a3} \end{pmatrix}\begin{pmatrix} u_{a\theta} \\ 2q_{a\theta}/5p_a \end{pmatrix} = \mathbf{M}_a\cdot\mathbf{u}_a$$

and

$$\frac{1}{\langle B^2\rangle}\left\langle \begin{array}{c} BR_{a\|} \\ BH_{a\|} \end{array}\right\rangle = \sum_b \begin{pmatrix} l_{11}^{ab} & -l_{12}^{ab} \\ -l_{21}^{ab} & l_{22}^{ab} \end{pmatrix}\begin{pmatrix} u_{b\theta} \\ 2q_{b\theta}/5p_b \end{pmatrix} = \sum_b \mathbf{L}_{ab}\cdot\mathbf{u}_b, \qquad (1)$$

where $q_{a\theta} = \mathbf{q}_a\cdot\nabla\theta/\mathbf{B}\cdot\nabla\theta$ is the contravariant component of the poloidal heat flux and $p_a = n_a T_a$ the pressure of species a. These expressions are obtained by expanding the distribution functions in Sonine polynomials and truncating after two terms; as usual this approximation turns out to be accurate within a few percent, i.e., as accurate as the Coulomb logarithm in the collision operator. In order to isolate the inductive electric field as the single driving term, we have assumed in Eq (1) that all radial gradients vanish, so that $\omega_a = 0$ for each species. (This assumption is satisfied if the density and temperature gradients are sufficiently weak and, for the gradient of the electrostatic potential, i.e., the radial electric field, if the plasma is viewed from a rigidly toroidally rotating frame of reference.) The coefficients

$$\hat{\mu}_{aj} = \frac{3\langle(\nabla_{\|}B)^2\rangle}{\langle B^2\rangle}\mu_{aj}$$

and

$$l_{jk}^{ab} = m_a n_a\left(\delta_{ab}\sum_c \frac{M_{ac}^{j-1,k-1}}{\tau_{ac}} + \frac{N_{ab}^{j-1,k-1}}{\tau_{ab}}\right).$$

which can be looked up in Refs [2] and [3], summarize all kinetic information needed to evaluate the flows. Neoclassical effects are described by the viscosity coefficients $\hat{\mu}_{aj}$, which thus depend on collisionality and flux-surface geometry. In the banana regime

$$\begin{pmatrix} \hat{\mu}_{e1} \\ \hat{\mu}_{e2} \\ \hat{\mu}_{e3} \end{pmatrix} = \frac{m_e n_e}{\tau_{ee}}\frac{f_t}{f_c}\begin{pmatrix} 0.533 + (1+\alpha) \\ -0.625 - 1.5(1+\alpha) \\ 1.386 + 3.25(1+\alpha) \end{pmatrix},$$

$$\begin{pmatrix} \hat{\mu}_{i1} \\ \hat{\mu}_{i2} \\ \hat{\mu}_{i3} \end{pmatrix} = \frac{m_i n_i}{\tau_{ii}} \frac{f_t}{f_c} \begin{pmatrix} 0.533 + \alpha \\ -0.625 - 1.5\alpha \\ 1.386 + 3.25\alpha \end{pmatrix},$$

with $\tau_{ab} = 3(2\pi)^{3/2}\epsilon_0^2 m_a^{1/2} T_a^{3/2}/n_b e_a^2 e_b^2 \ln\Lambda$, f_t the effective fraction of trapped particles [2], and $f_c = 1 - f_t$. For a tokamak with circular cross section and small inverse aspect ratio, $\epsilon \ll 1$, the trapped particle fraction is $f_t \simeq 1.46\epsilon^{1/2}$. The friction coefficients are

$$\mathbf{L}_{ii} = -\frac{m_i n_i}{\tau_{iz}} \begin{pmatrix} 1 & \frac{3}{2} \\ \frac{3}{2} & \frac{13}{4} + \frac{\sqrt{2}}{\alpha} \end{pmatrix}, \qquad \mathbf{L}_{iz} = \frac{m_i n_i}{\tau_{iz}} \begin{pmatrix} 1 & 0 \\ \frac{3}{2} & 0 \end{pmatrix},$$

$$\mathbf{L}_{zi} = \frac{m_i n_i}{\tau_{iz}} \begin{pmatrix} 1 & \frac{3}{2} \\ 0 & 0 \end{pmatrix}, \qquad \mathbf{L}_{zz} = -\frac{m_i n_i}{\tau_{iz}} \begin{pmatrix} 1 & 0 \\ 0 & \alpha\sqrt{2} + \frac{15}{2} \end{pmatrix},$$

$$\mathbf{L}_{ie} = \frac{m_e n_e}{\tau_{ee}} \begin{pmatrix} 1 & \frac{3}{2} \\ 0 & 0 \end{pmatrix} = \alpha^{-1}\mathbf{L}_{ze}, \qquad \mathbf{L}_{ee} = -(1+\alpha)\frac{m_e n_e}{\tau_{ee}} \begin{pmatrix} 1 & \frac{3}{2} \\ \frac{3}{2} & \frac{13}{4} + \frac{\sqrt{2}}{1+\alpha} \end{pmatrix},$$

where zero entries denote quantities of order $(m_i/m_z)^{1/2} \ll 1$ or $(m_e/m_i)^{1/2} \ll 1$. We do not need $\mathbf{L}_{ei}$ and $\mathbf{L}_{ez}$ as the electron flow velocity is much larger than those of the ion species. As far as the electrons are concerned, the ions are practically stationary and the electron-ion friction force is independent of the ion flow velocity.

We thus obtain the following system of equations for each species

$$\mathbf{M}_a \cdot \mathbf{u}_a = \sum_b \mathbf{L}_{ab} \cdot \mathbf{u}_b + \mathbf{E}_a,$$

where

$$\mathbf{E}_a = \begin{pmatrix} n_a e_a \left\langle E_\parallel B \right\rangle / \langle B^2 \rangle \\ 0 \end{pmatrix}.$$

For the electrons, this reduces to

$$(\mathbf{M}_e - \mathbf{L}_{ee}) \cdot \mathbf{u}_e = \mathbf{E}_e,$$

from which the electron flow velocity can be evaluated by matrix inversion, yielding the usual neoclassical reduction of electric conductivity due to trapping. The ion flows are then obtained by solving the 4×4 system of equations

$$\begin{pmatrix} \mathbf{M}_i - \mathbf{L}_{ii} & -\mathbf{L}_{iz} \\ -\mathbf{L}_{zi} & -\mathbf{L}_{zz} \end{pmatrix} \begin{pmatrix} \mathbf{u}_i \\ \mathbf{u}_z \end{pmatrix} = \begin{pmatrix} \mathbf{E}_i - \mathbf{L}_{ie} \cdot \mathbf{u}_e \\ -\mathbf{L}_{ze} \cdot \mathbf{u}_e \end{pmatrix}, \qquad (2)$$

where we have noted that the viscosity is negligible for collisional impurities and that the electric force acting on the impurities is smaller than their friction against electrons,

$$\frac{R_{ze\|}}{n_z ze E_\|} \sim \frac{m_e j_\| / \alpha \tau_{ei}}{n_z z e^2 E_\|} \sim z \gg 1.$$

The motion of the impurities is thus simply determined by the balance of the impurity-electron and impurity-hydrogen friction forces.

The first element of the right-hand side of Eq (2) is equal to the sum of the electric force on the bulk ions and their friction against the electron population, and is sometimes written as

$$\frac{\left\langle \left(n_i e E_\| + R_{ie\|}\right) B \right\rangle}{\langle B^2 \rangle} = \frac{n_i e \langle E_* B \rangle}{\langle B^2 \rangle}, \tag{3}$$

where $E_* = E_\| + R_{ie\|}/n_i e$ is the "effective electric field" felt by a bulk ion [1]. In a pure plasma (no impurities) embedded in a straight magnetic field this vanishes, $E_* = 0$, and the ions do not "feel" the electric field; it is exactly cancelled by the friction force from the electrons. The reason for this cancellation is that the electric force acting on the electrons must necessarily equal their friction against ions. As pointed out in Ref [7], the situation is different in an impure tokamak plasma since the electric force on the electrons is balanced not only by friction against bulk ions, but also by the mirror force (causing trapping) and friction against impurity ions. As a result of these effects, the first row in Eq (2) equals Eq (3) with

$$\frac{\langle E_* B \rangle}{\left\langle E_\| B \right\rangle} = \frac{\left(\mu_{e2} - l_{12}^{ee}\right)\mu_{e2} - \left(\mu_{e3} - l_{22}^{ee}\right)\mu_{e1}}{\left(\mu_{e1} - l_{11}^{ee}\right)\left(\mu_{e3} - l_{22}^{ee}\right) - \left(\mu_{e2} - l_{12}^{ee}\right)^2}$$

$$\simeq \frac{\alpha}{1+\alpha} + \frac{3.96 + 2.59x + \alpha(4.21 + 3.24x) + \alpha^2(1+x)}{2.59(0.65+x)(1.44+x) + \alpha(3.24+\alpha)(1+x)^2}\,\frac{x}{1+\alpha} \tag{4}$$

where $x = f_t/f_c$. The first term in this expression reflects the fact that the total friction on electrons from both ion species is proportional to $Z_{\rm eff} = 1 + \alpha$, where the term α represents the contribution from impurities so that, in a straight magnetic field, they take up a fraction $\alpha/(1+\alpha)$ of the momentum imparted to the electrons by the electric field. The second term describes the effect of electron trapping. It is clear that for realistic parameters in a tokamak, $\alpha \sim x \sim 0.5$, the bulk ions "feel" a significant fraction (more than half) of the parallel electric field.

We now turn to the ion rotation, which is obtained by solving the system of equations (2). The resulting expressions are very complicated in general; we give only limiting forms. In the limit of very few impurities, the rotation velocities of bulk and impurity ions, respectively, are

$$\hat{u}_{i\theta} = \frac{m_i n_i \langle B^2 \rangle}{e\tau_{ii} \left\langle E_\| B \right\rangle} u_{i\theta} \simeq \frac{3.98(1.02+x)(1.53+x)}{(0.65+x)(1.44+x)(2.17+x)}, \qquad \alpha \to 0,$$

$$\hat{u}_{z\theta} = \frac{m_i n_i \langle B^2 \rangle}{e\tau_{ii} \left\langle E_\| B \right\rangle} u_{z\theta} \simeq \frac{0.73(1.5+x)(3.84+x)}{(0.65+x)(1.44+x)(2.17+x)}, \qquad \alpha \to 0.$$

Thus the rotation speed of trace impurities is thus about 2/3 of the plasma rotation near the magnetic axis (where $x = 0$) and falls off toward the edge. If the impurities are more numerous, their speed is even smaller when compared with that of the bulk ions, as shown in Fig 1. In the Lorentz limit

$$\hat{u}_{i\theta} \simeq \frac{13/4}{(1+x)\alpha}, \qquad \alpha \to \infty,$$

$$\hat{u}_{z\theta} \simeq \frac{1}{(1+x)\alpha^2}, \qquad \alpha \to \infty,$$

and close to the magnetic axis,

$$\hat{u}_{i\theta} \simeq \frac{3.25(0.71+1.24\alpha+\alpha^2)}{(1+\alpha)(0.75+1.95\alpha+\alpha^2)}, \qquad f_t \to 0,$$

$$\hat{u}_{z\theta} \simeq \frac{3.78+3.83\alpha+\alpha^2}{(1+\alpha)(2.41+\alpha)(0.75+1.95\alpha+\alpha^2)}, \qquad f_t \to 0.$$

The impurity rotation is always in the same direction as the bulk ion rotation but is considerably smaller if α (or, to a lesser extent, f_t) gets large.

In contrast, the poloidal rotation caused by radial gradients usually calculated in neoclassical theory does not depend strongly on the impurity content and stays finite in the limit $\alpha \to \infty$. These flows are [6]

$$u_{i\theta} = \frac{IT_i}{e\langle B^2 \rangle} \frac{1.17+3.64\alpha+1.99\alpha^2}{1+0.46x+(2.58+1.65x)\alpha+1.33(1+x)\alpha^2} \frac{d\ln T_i}{d\psi},$$

$$u_{z\theta} = \frac{IT_i}{e\langle B^2 \rangle} \left[-\frac{d\ln p_i}{d\psi} + \frac{1.17+0.69x+(3.64+2.47x)\alpha+1.99(1+x)\alpha^2}{1+0.46x+(2.58+1.65x)\alpha+1.33(1+x)\alpha^2} \frac{d\ln T_i}{d\psi} \right],$$

where $I = RB_\varphi$ and ψ is the poloidal flux function, so that $\mathbf{B} = I(\psi)\nabla\varphi + \nabla\varphi \times \nabla\psi$. Hence it was noticed in Ref [6] that in the edge pedestal, where the pressure gradient is very steep, the poloidal rotation of impurities is larger than and in the opposite direction to the bulk ion rotation.

In conclusion, our results may be summarized as follows. Considering a tokamak plasma consisting of collisionless electrons and ions and collisional impurities, we have calculated the "effective electric field" (4) acting on the ions and its effect on ion flow. Although this effective electric field vanishes for a pure plasma in a straight magnetic field, this not at all true in a realistic tokamak discharge. In fact, for typical tokamak parameters $E_*/E_{||} \gtrsim 0.5$. As pointed out in Ref [4], this electric field drives a poloidal and toroidal ion flow, but as we have seen here the corresponding flow of heavy impurities (which is what is measured) is significantly smaller than that of the bulk plasma. This behavior is the opposite to that of rotation caused in the more orthodox neoclassical way by radial gradients, which tends to be larger for impurities than for bulk ions, making the detection of rotation driven by the Ohmic field correspondingly more difficult.

The fact that $E_*/E_{||}$ is so large implies that that it is not unreasonable to expect that ion runaway may sometimes occur in tokamaks, as suggested by Furth and Rutherford [7]. During normal tokamak operation the toroidal electric field is too low for this, but just like runaway electrons tend to be produced in tokamak disruptions, one could expect that ion runaway could occur during reconnecting instabilities where the toroidal electric field becomes very large. Work is in progress to assess whether this can explain recent observations of bursts of high-energy ions following internal reconnection events in the Mega-Ampere Spherical Tokamak (MAST) [8].

Helpful discussions with P J Catto and R J Hastie are gratefully acknowledged. This work was funded jointly by the UK Department of Trade and Industry and Euratom.

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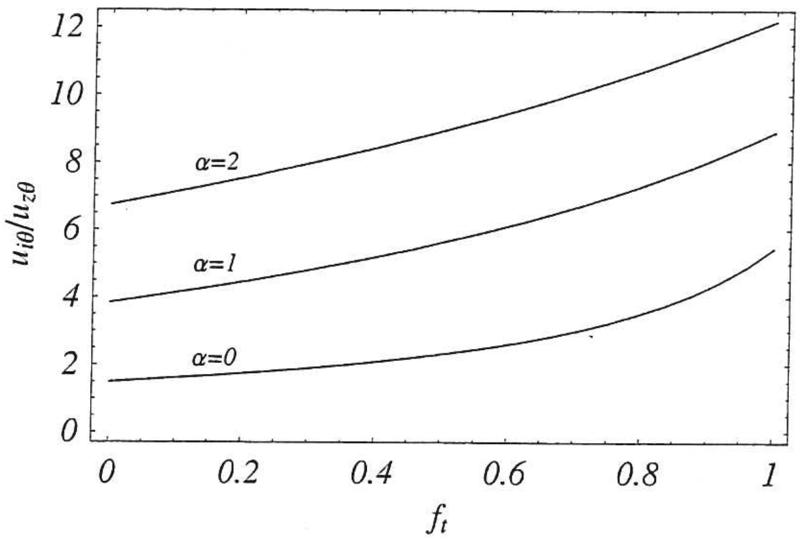

Fig.1. Ratio of bulk ion and impurity ion poloidal flows as a function of the effective fraction of trapped particles, f_t, for different values of the impurity strength parameter $\alpha=n_z z^2/n_i$. When f_t or α is large, the bulk ion flow is much larger than that of the impurities.