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Excitation of **axisymmetric** Alfvénic modes in ohmic tokamak discharges

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Abstract

Magnetohydrodynamic (MHD) mode activity in the Alfvén frequency range has been detected in the absence of energetic ions during discharges in several conventional tokamaks and spherical tokamaks, including the Tokamak Fusion Test Reactor (TFTR) and the Mega-Amp Spherical Tokamak (MAST). In TFTR the dominant toroidal mode number n was found to be zero; this is also the case in MAST discharges for which mode number information is available. The observed properties of these modes are shown to be consistent with global Alfvén eigenmodes (GAEs). Although they appear to have little or no effect on plasma performance in present-day devices, the fact that they are frequently observed in MAST ohmic discharges suggests that they could be used as a diagnostic of plasma equilibrium parameters. In principle, they could also provide the basis for a plasma heating scheme. A possible mechanism for the excitation of Alfvén eigenmodes in the absence of fast ions is suggested by two-fluid simulations of various tokamaks, in which high frequency mode activity is found to be correlated with relatively long–timescale MHD events in the plasma, such as internal reconnection events (IREs) or edge localised modes (ELMs). A simple analytical model describing the excitation of Alfvénic modes by either IREs or ELMs is proposed. The coupling of low and high frequency MHD is predicted to be strongest for radially–extended modes: this is consistent with the low mode numbers of the activity observed in TFTR and MAST.

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1 Introduction

In many tokamak experiments magnetic fluctuations in the Alfvén frequency range have been observed to be driven unstable by energetic ions, including charged fusion products [1]. Most commonly, toroidicity–induced Alfvén eigenmodes (TAEs), arising from the coupling of shear Alfvén waves with the same toroidal mode number n and poloidal mode numbers m and $m+1$, have been observed in the presence of energetic ions at frequencies $\omega \simeq c_A/2qR$, where c_A is the Alfvén speed, q is tokamak safety factor, and R is major radius. However, modes with Alfvénic frequency scaling have also been detected during ohmic discharges in several devices, including the Axial Symmetric Divertor Experiment (ASDEX) Upgrade [2], the Tokamak Fusion Test Reactor (TFTR) [3], and, most recently, the Mega–Amp Spherical Tokamak (MAST) [4, 5]. These observations were unexpected, since Alfvénic modes are generally thought to be driven by energetic particles, with speeds comparable to or greater than the Alfvén speed c_A. Nonthermal ion tails have been observed using a neutral particle analyser in ohmic MAST discharges following internal reconnection events (IREs) [6], but modes are often excited in the Alfvén frequency range without such tails simultaneously being detected. In a strictly ohmic plasma, TAEs can be weakly–damped, thereby making it possible for them to be driven unstable by the introduction of super–Alfvénic ions. If, however, no such ions are launched externally into the plasma, or produced internally via fusion reactions (the latter can be ruled out as a significant source of energetic particles in the case of MAST plasmas produced so far), it is necessary to find other mechanisms to account for the detection of modes in the Alfvén frequency range.

In Section 2 we present observations of Alfvénic mode activity during ohmic discharges on MAST, and in Section 3 we show that both these observations and the earlier observations of ohmic mode activity in TFTR are consistent with the excitation of global Alfvén eigenmodes. Using fluid simulations (Section 4) and a simple analytical model (Section 5), we propose a mechanism for the excitation of such modes that does not require the presence of energetic particles.

2 Alfvénic activity in ohmic MAST plasmas

The MAST spherical tokamak currently produces deuterium plasmas with major radius $R \simeq 0.8\,\mathrm{m}$, minor radius $a \simeq 0.6\,\mathrm{m}$, plasma currents I_p up to about 1 MA, axial toroidal magnetic fields B_0 in the range 0.2–0.5 T, and electron densities n_e typically in the range $1\text{–}2\times 10^{19}\,\mathrm{m}^{-3}$. High frequency (up to 500 kHz) magnetohydrodynamic (MHD) activity in the plasma is measured using Mirnov coils both inboard and outboard of the confined plasma. MHD modes with frequencies of typically 200–300 kHz have been detected in many ohmic discharges, with a wide range of plasma parameters. The measured amplitudes are lower than those of modes observed in a similar frequency range in neutral beam–heated plasmas. The signals measured on outboard coils are generally stronger than those measured on inboard coils, suggesting that the modes are ballooning in character.

The activity is generally bursting in character, on a timescale of typically 0.1 ms or less, although in a given discharge the bursting can persist at a low amplitude for periods of up to 200 ms. The mode frequency is approximately constant during each burst and between adjacent bursts. Figure 1 shows a narrow–band mode persisting for approximately 50 ms, drifting slowly in frequency. Such behaviour is often observed in the case of TAEs, the slow frequency drift reflecting the evolution of the plasma current, density profile and/or toroidal field B_0: the TAE frequency scales approximately as $B_0/qn_e^{1/2}$. In general it has not been possible to determine whether high frequency modes in ohmic MAST discharges have a similar frequency scaling, but it seems likely that slow variations in frequency such as that shown in Fig. 1 arise from changes in plasma equilibrium parameters. Toroidal mode number information has been determined for high frequency activity in several ohmic MAST discharges, on the assumption that the intrinsic burst lifetimes are longer than the time windows used to Fourier transform the data (because of noise limitations, the latter cannot be reduced below 32 μs). In every case the $n = 0$ component is found to be dominant, as in the case of high frequency activity in ohmic TFTR discharges [3]. It has not been possible to measure the poloidal mode numbers of these modes, due to low signal–to–noise ratios in coils other than those in the outer midplane.

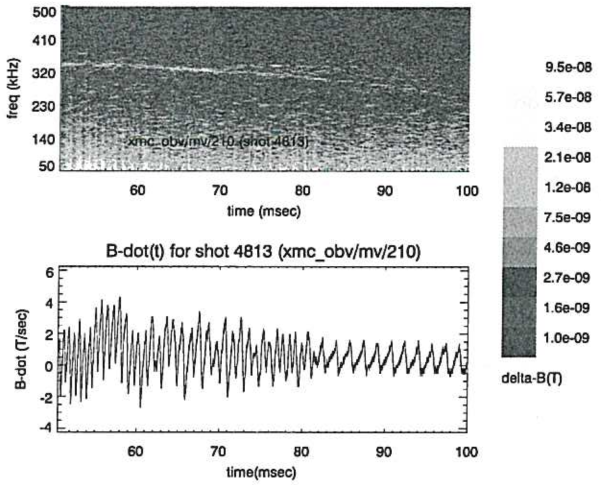

Figure 1: Mirnov coil spectrogram (top) and time trace (bottom) from MAST ohmic discharge #4813. The spectrogram shows a discrete mode in the Alfvén frequency range persisting for about 50 ms.

There is often evidence of a correlation between mode excitation and MHD activity at much lower frequencies. An example of this, from the first experimental campaign on MAST, is shown in Fig. 2. The time trace (bottom) shows a sequence of MHD events, each of which has a broad Fourier spectrum (top). For 20 ms or so following the last of these MHD events, a measurable signal persists at a frequency of about 320 kHz. In this case it is not known whether the $n = 0$ component was dominant (high frequency mode number identification only became possible in the second MAST campaign).

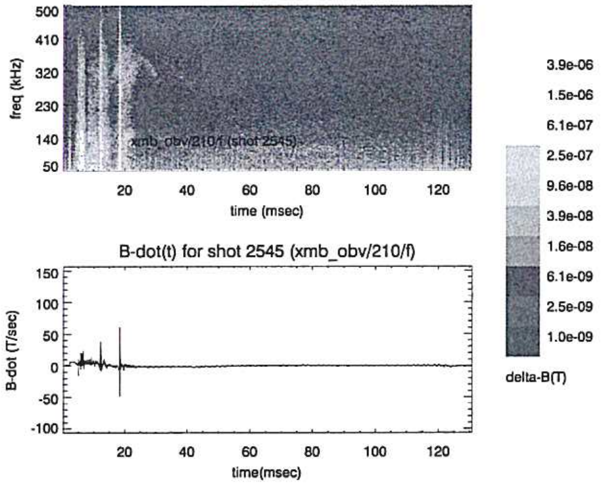

Figure 2: Mirnov coil spectrogram (top) and time trace (bottom) from MAST ohmic discharge #2545. The spectrogram shows a discrete mode in the Alfvén frequency range, apparently excited by a sequence of broadband MHD events in the plasma and persisting for about 20 ms.

3 Alfvén continuum and eigenmodes in ohmic MAST plasmas

TAEs are discrete Alfvén eigenmodes arising from the coupling of Fourier harmonics with poloidal mode numbers m and $m + 1$. In the limit of a cylindrical plasma, singularities occur in the eigenmode equations for $\omega = \pm k_{\|m} c_A$, where $k_{\|m} = (n - m/q)/R$ and $c_A \equiv B_0/\sqrt{\mu_0 \rho}$, μ_0 being free space permeability and ρ the plasma mass density [7]. **Note that the quantity c_A is evaluated using the magnetic field on–axis B_0, whereas ρ is a flux surface quantity, i.e. it varies with distance from the magnetic axis. Thus, c_A is only strictly equal to the Alfvén speed at the magnetic axis. It is important to make this distinction in the case of MAST, since the total magnetic field in a spherical tokamak varies strongly on flux surfaces that are far from the axis. In the rest of the paper, c_A will be used to denote the flux surface quantity $B_0/\sqrt{\mu_0 \rho}$ rather than the true local Alfvén speed.** For each singularity in the eigenmode equations there exists a (strongly damped) eigenmode. Since ρ and q vary with distance from the magnetic axis, the singularities define a continuous eigenvalue spectrum. For $n \neq 0$, the continuous spectra can cross over: setting $k_{\|m} = -k_{\|m+1}$, one finds that the crossing point occurs where the safety factor $q = (m + 1/2)/n$ and hence the frequency is $\omega = k_{\|m} c_A = c_A/2qR$. In a toroidal plasma a gap occurs in the continuum close to the cylindrical crossing point: TAEs, with frequencies lying inside such gaps, are nonsingular and hence susceptible to excitation by energetic particles. If, however, $n = 0$, the equation $k_{\|m} = -k_{\|m+1}$ cannot be satisfied and $n = 0$ TAEs cannot therefore exist. We thus conclude that the modes identified as having $n = 0$ in ohmic MAST and TFTR discharges cannot be TAEs. Ellipticity–induced and triangularity–induced Alfvén eigenmodes also arise from the crossing of cylindrical continua, and thus must also have finite n.

There is, however, a class of Alfvénic mode that *can* have $n = 0$: the global Alfvén eigenmode (GAE) [8]. Unlike TAEs, these have a single dominant poloidal mode number m and thus do not require toroidicity in order to exist. They arise from a coupling of shear and compressional Alfvén waves that can occur in a plasma with finite density gradient and magnetic shear. **Villard and Vaclavik [9] have proposed that the $n = 0$**

modes observed in TFTR can be identified as GAEs. For the particular case of a cylindrical plasma in which $(k_{\|m}^2 c_A^2)^2$ has a single minimum at minor radial distance $r = r_0 \neq 0$, Mahajan and co-workers [8] computed the GAE frequency eigenvalues. These lie below $k_{\|m} c_A$ ($k_{\|m}$ being evaluated at $r = r_0$), and tend towards this value as the magnetic shear $s \equiv (r/q)dq/dr \rightarrow 0$ or the density scale length $L \rightarrow \infty$. As in the case of the shear wave, a singularity occurs in the eigenmode equation for ω precisely equal to $k_{\|m} c_A$, and so $s \neq 0$ and finite L are required in order to avoid continuum damping. **It should be noted, however, that Villard and Vaclavik [9] obtained GAE continuum damping rates that are low enough (typically of the order of a few percent of the real frequency) to suggest that GAEs with frequencies in the continuum could still be excited, given a sufficiently strong drive.**

We have used the CSCAS code [10] to compute the continuous $n = 0$ Alfvén spectrum for one time slice ($t = 220\,\mathrm{ms}$) in MAST discharge #4284: this is shown in Fig. 3. Continuous spectrum eigenvalues are represented by the letter X, the three different branches of the continuum corresponding to $m = 1$, 2 and 3 (in order of increasing frequency). The two other curves in Fig. 3 represent models of the q-profile and the normalised density profile that are broadly consistent with data from this shot.

All three continuum branches shown in Fig. 3 satisfy the condition that $k_{\|m} c_A$ has a minimum at finite r. Despite the tight aspect ratio of MAST plasmas, the $n = 0$ continua are far enough apart that toroidal coupling is weak and the continua are given essentially by the cylindrical limit, namely $|\omega| = mc_A(r)/q(r)R$. A local minimum often occurs towards the edge of conventional and spherical tokamak plasmas, particularly when the density profile is relatively flat compared to the q-profile. Using the ideal MHD code MISHKA-1 [11] we have searched for discrete eigenmodes with frequencies lying below the local minima in Fig. 3. Three such modes were found to exist, all with normalised frequencies $\omega R/c_A(0) < 1$, i.e. below the $m = 1$ continuum. Figure 4 shows one of

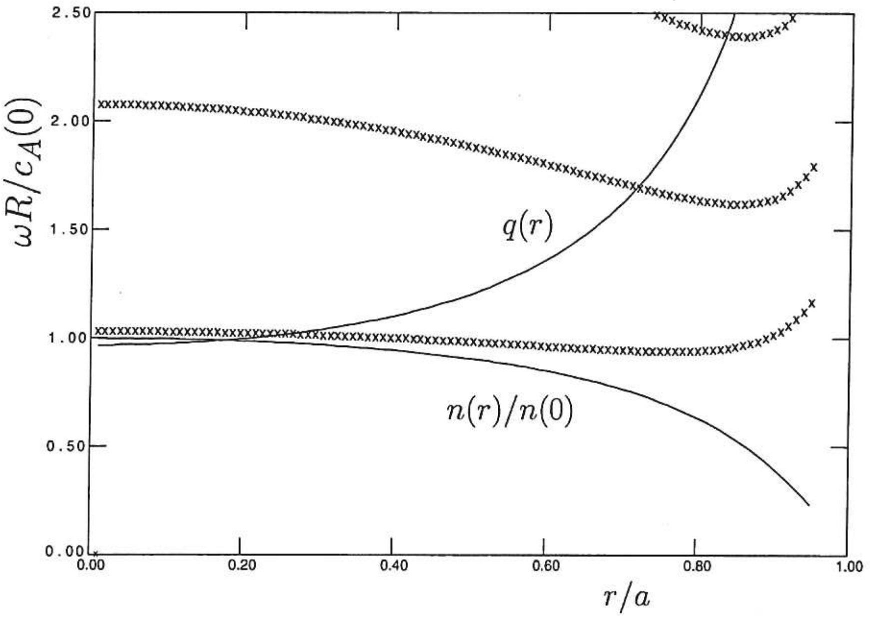

Figure 3: Continuous Alfvén spectrum for $n = 0$ in MAST discharge #4284 at $t = 220\,\mathrm{ms}$, computed using CSCAS. The quantity r/a is the square root of poloidal magnetic flux normalised to its value at the plasma edge.

these eigenmodes, the upper and lower frames representing velocity components in the minor radial (v_r) and poloidal (v_θ) directions respectively. In the case of v_r the eigenmode components with poloidal mode number m and $-m$ are equal in both magnitude and sign, while for v_θ the $\pm m$ components have opposite sign. In both cases it is apparent that the dominant poloidal mode numbers are $m = \pm 1$, although there are also contributions from $|m| \geq 2$. The normalised frequency in this case is $\omega R/c_A(0) \simeq 0.975$ **(close to the minimum of the $m = 1$ curve in Fig. 3)**: taking $B_0 = 0.4\,\mathrm{T}$, $n_e(0) = 1.5 \times 10^{19}\,\mathrm{m}^{-3}$, $R = 0.8\,\mathrm{m}$ this gives a mode frequency of about 310 kHz, which is typical of high frequency modes observed in ohmic MAST discharges (cf. Figs. 1 and 2). The other two eigenmodes have $\omega R/c_A(0) \simeq 0.80$ and 0.72.

Although the highest frequency eigenvalue is approximately equal to a value of $k_{\|}c_A$

close to the plasma edge, it should be noted from Fig. 4 that the eigenmode is global, in the sense that it extends over the entire plasma volume. In Ref. [3] it is stated that the $n = 0$ modes observed in TFTR were localised at the outer plasma edge. However, this conclusion is based on an observed correlation between the values of plasma edge parameters and the mode frequency and amplitude: there was no direct information on mode structure. The available data from TFTR are thus not inconsistent with radially–extended GAEs, **and computations of such modes using TFTR–like parameters yield profiles that are indeed global in character [9].**

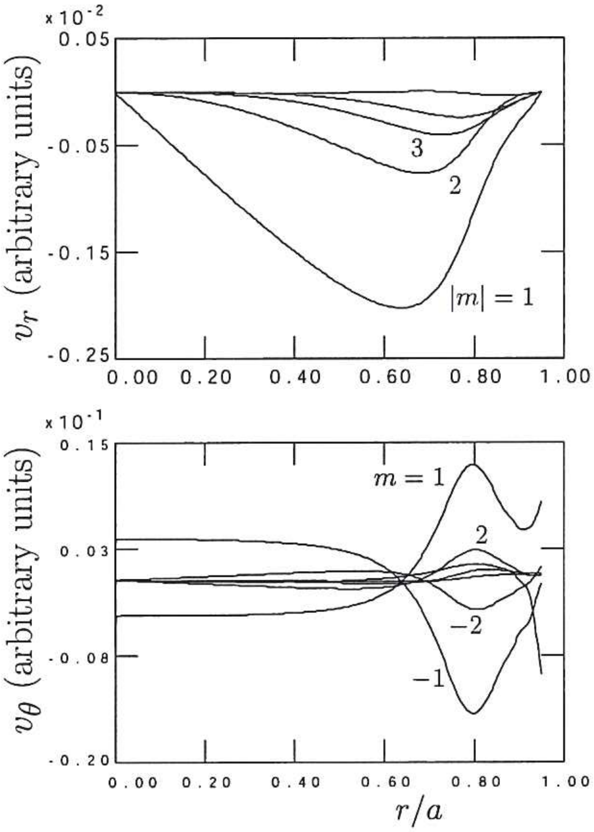

Figure 4: GAE eigenfunction with $n = 0$ computed for the MAST equilibrium used to generate Fig. 3. The quantities v_r and v_θ are the radial and poloidal components of velocity. The dimensionless frequency eigenvalue is $\omega R/c_A(0) \simeq 0.975$.

4 Two–fluid simulations

Modes in the Alfvén frequency range have been observed in simulations of conventional tokamaks carried out using the **electromagnetic** two–fluid CUTIE code [12]. **This code uses a periodic cylinder model of a tokamak plasma, and includes such effects as visco–resistive tearing, ballooning, drift and shear Alfvén modes. The equilibrium magnetic field in the code is spatially–varying and has finite curvature: these are the essential elements required for the existence of both GAEs [13] and TAEs [7]. However, the flux surfaces are assumed to be circular and their Shafranov shift is neglected. The code cannot therefore be used to describe quantitatively processes occuring in MAST plasmas, which are characterised by tight aspect ratio and flux surfaces which are highly noncircular and have a significant Shafranov shift.** It does, nevertheless, provide simulated data from large aspect ratio devices that may shed light on the excitation of high frequency modes during ohmic shots in MAST.

Figure 5 shows time series (left plots) and Fourier power spectra (right plots) of magnetic fluctuations in a CUTIE simulation of an H–mode discharge in the larger aspect ratio COMPASS–D tokamak [14]. The upper plots correspond to $r \simeq 0.8a$, while the lower plots correspond to $r \simeq a$. At both minor radii, there is a sharply–defined peak in the spectrum centred at a frequency $\nu \simeq 250\,\mathrm{kHz}$. This is much higher than typical drift mode frequencies. The mode extends from the plasma edge to about half the minor radius. Similar phenomena have been observed in many other runs of CUTIE. These include simulations of the Dutch RTP tokamak [15] as well as COMPASS–D. In some cases the mode activity peaks at a radial location closer to the plasma core than the edge. The modes invariably appear immediately after either a reconnection event or an edge localised mode (ELM): the high frequency $n = 0$ modes observed in TFTR were also found to be strongly correlated with ELMs [3]. An important point to note here is that there are no fast particle effects in CUTIE, since it is a strictly fluid code, nor does it include charge exchange effects, which were invoked in Ref. [3] to account for the high frequency modes in TFTR.

The narrow–band features in the spectra of Fig. 5 have peak frequencies that are somewhat lower than typical values of $k_{\|m} c_A$ for $n = 0$, $m = 1$ close to the edge of the COMPASS–D plasma being simulated. **However, it is possible that the high frequency modes observed in the CUTIE simulations are not $n = 0$, $m = 1$ GAEs. The frequency spectra in Fig. 5 pertain to a single toroidal location, and therefore do not give any direct indication of toroidal mode structure. In any case, CUTIE is a fully nonlinear code, and snapshots of mode activity are generally from the nonlinear phase of the underlying instability, in which strong coupling between different toroidal mode numbers is likely to occur, thereby complicating the unique identification of mode numbers.** It should also be noted that GAEs with finite n exist [8], with frequencies that can be lower than those of modes with $n = 0$, $m = 1$. There is moreover a possibility that the high frequency modes in CUTIE simulations are in fact TAEs rather than GAEs: although not fully toroidal, CUTIE takes into account the poloidal variation in the equilibrium magnetic field, and hence the coupling of poloidal mode numbers characteristic of TAEs.

Loop voltages resulting from reconnection events in CUTIE, $V_{\rm loop}$, are typically in the range 10–50 V and last for a time $\Delta t \sim 10\,\mu$s. The corresponding electric field $E = V_{\rm loop}/2\pi R$ would accelerate ions of charge Ze, mass m_i, initial speed v_i and energy $\mathcal{E}_i$ to final energies of at most

$$\mathcal{E}_f \simeq \mathcal{E}_i + \frac{ZeV_{\rm loop} v_i \Delta t}{2\pi R} + \frac{Z^2 e^2 V_{\rm loop}^2 \Delta t^2}{8\pi^2 m_i R^2}. \qquad (1)$$

Evaluating the right hand side using CUTIE simulation parameters for COMPASS–D,

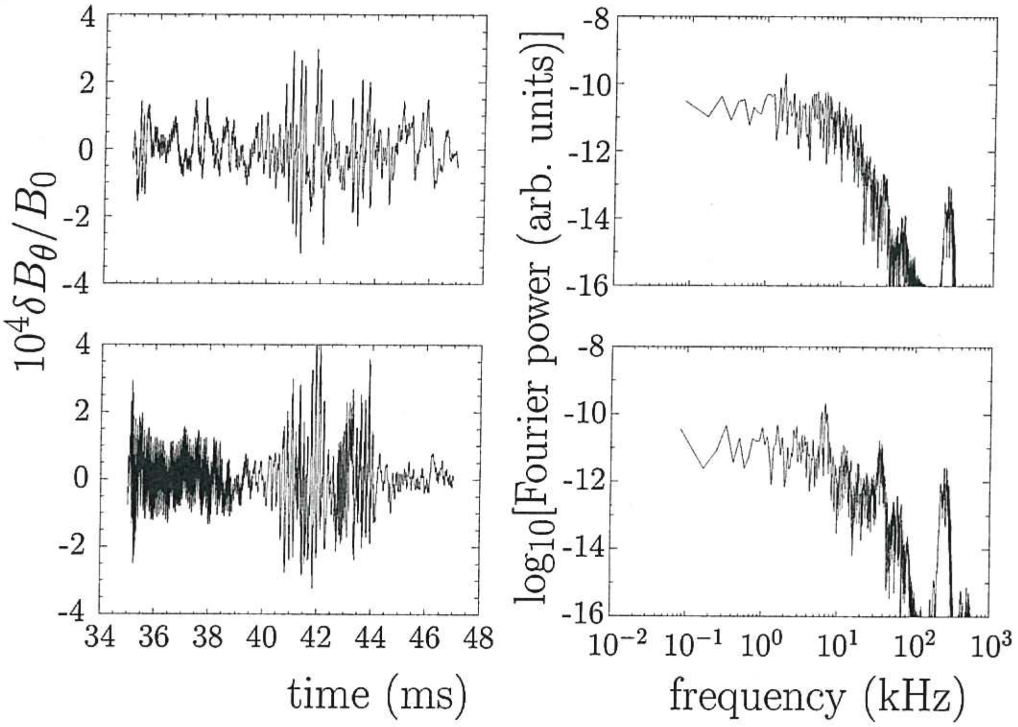

Figure 5: Time series (left plots) and Fourier power spectra (right plots) of magnetic fluctuations in a CUTIE simulation of an H–mode discharge in COMPASS–D. The upper plots correspond to a minor radial distance $r \simeq 0.8a$, where a is the total minor radius, while the lower plots correspond to $r \simeq a$.

with $V_{\text{loop}} = 50\,\text{V}$ and $\mathcal{E}_i = 250\,\text{eV}$ (a typical ion temperature), we obtain $\mathcal{E}_f - \mathcal{E}_i \simeq 22\,\text{eV}$. Thus, the reconnection events in the simulations would accelerate ions by only a modest amount. No super–Alfvénic ions would be generated by such events, and fast ion excitation of Alfvén eigenmodes would not therefore occur if reconnection events similar to those observed in the CUTIE simulations of COMPASS–D took place in a real tokamak plasma. Somewhat higher values of V_{loop} ($\sim 100\,\text{V}$) in the plasma core, persisting for $\Delta t \sim 1\,\text{ms}$, have been inferred by Helander and co–workers [6] in the case of IREs in MAST. According to Eq. (1), deuterons can then be accelerated to energies of up to around 10 keV. Particle spectra measured immediately after IREs on MAST extend up to energies of this order [6]. The spectra fall off rapidly with energy, however, and the number of super–Alfvénic ions generated by IREs in MAST appears to be very small. They are thus unlikely to drive Alfvén eigenmodes unstable.

5 Analytical model

The correlation observed in CUTIE simulations between high frequency mode excitation and low frequency MHD activity, combined with the evidence of a similar experimental correlation in ohmic MAST discharges (Fig. 2), suggests that the time evolution of an Alfvén eigenmode (AE) displacement in ohmic plasmas can be represented by a simple driven oscillator equation of the form

$$\dot{a} + i\omega_0 a = \alpha \dot{f}(t), \tag{2}$$

where $a(t)$ is the mode magnetic field, ω_0 is the frequency eigenvalue, $f(t)$ is a magnetic field perturbation arising from an impulsive MHD event in the plasma [e.g. an IRE or an edge localised mode (ELM)], and α is a measure of the coupling between the impulsive event and the AE (reflecting the fact that the MHD event is localised in a region of the plasma which in general does not coincide with the peak amplitude of the AE). The AE here could be either a GAE or a TAE. An appropriate representation of $f(t)$ is suggested by loop voltage time traces during IREs in MAST: these typically exhibit an asymmetric negative spike in the voltage, with the rise more rapid than the decay. This suggests that a suitable choice for $f(t)$ might be

$$f(t) = e^{t/\tau_1}, \quad t < 0 \tag{3}$$

$$f(t) = e^{-t/\tau_2}, \quad t > 0 \tag{4}$$

where $\tau_1 < \tau_2$. The amplitude of f is normalised to unity, with $t = 0$ corresponding to the peak field perturbation. Solving Eq. (2) for this $f(t)$, and taking the real part, we obtain

$$\mathrm{Re}[a(t)] = \frac{\alpha e^{t/\tau_1}}{1 + (\omega_0 \tau_1)^2}, \qquad t < 0 \tag{5}$$

$$\mathrm{Re}[a(t)] = \frac{\alpha \left[\cos\omega_0 t - \omega_0\tau_1 \sin\omega_0 t\right]}{1 + (\omega_0\tau_1)^2} + \frac{\alpha \left[e^{-t/\tau_2} - \cos\omega_0 t\right]}{1 + (\omega_0\tau_2)^2} - \frac{\alpha\omega_0\tau_2 \sin\omega_0 t}{1 + (\omega_0\tau_2)^2}, \quad t > 0 \tag{6}$$

If $/\omega_0$ is much **smaller** than the rise and decay time constants of the driving term (τ_1, τ_2), Eq. (6) reduces to

$$\mathrm{Re}[a(t)] \simeq -\frac{\alpha}{\omega_0}\left(\frac{1}{\tau_1} + \frac{1}{\tau_2}\right) \sin\omega_0 t. \tag{7}$$

For $t > 0$ the ratio of the AE amplitude to the driver amplitude is thus equal to $(\alpha/\omega_0)(1/\tau_1 + 1/\tau_2)$. The full solution is plotted in Fig. 6 for $\omega_0\tau_1 = 2\pi$, $\tau_2 = 3\tau_1$: the **bold** line indicates the profile of $f(t)$. The key point here is that the solution for $t > 0$ is a sinusoidal oscillation whose amplitude is essentially the Fourier transform of the driving term. Since, for this particular choice of $f(t)$, the Fourier transform falls off only slowly with frequency (as $1/\omega_0$), the time–asymptotic AE mode amplitude is substantial, despite the fact that $\omega_0 \gg 1/\tau_2$. It is important to note that the high frequency mode remains in the plasma after the driving term has decayed to a negligible level: such behaviour is reminiscent of the spectrogram shown in Fig. 2. The above analytical solution can be easily generalised to include damping. Figure 2 suggests, however, that the damping time of the high frequency modes can be long compared to that of impulsive MHD events that may be driving them.

As noted earlier, fixed–frequency modes in ohmic MAST discharges typically have frequencies of around 250 kHz. IREs in MAST appear to occur on timescales of not less than about $100\,\mu$s. Taking $\tau_1 = 100\,\mu$s, $\tau_2 = 300\,\mu$s and, for example, $\alpha = 0.1$, we obtain from Eq. (7) a predicted AE amplitude that is smaller than the amplitude of the driving perturbation by a factor of 8×10^{-4} **(in this case $\omega_0\tau_1$, $\omega_0\tau_2$ are much**

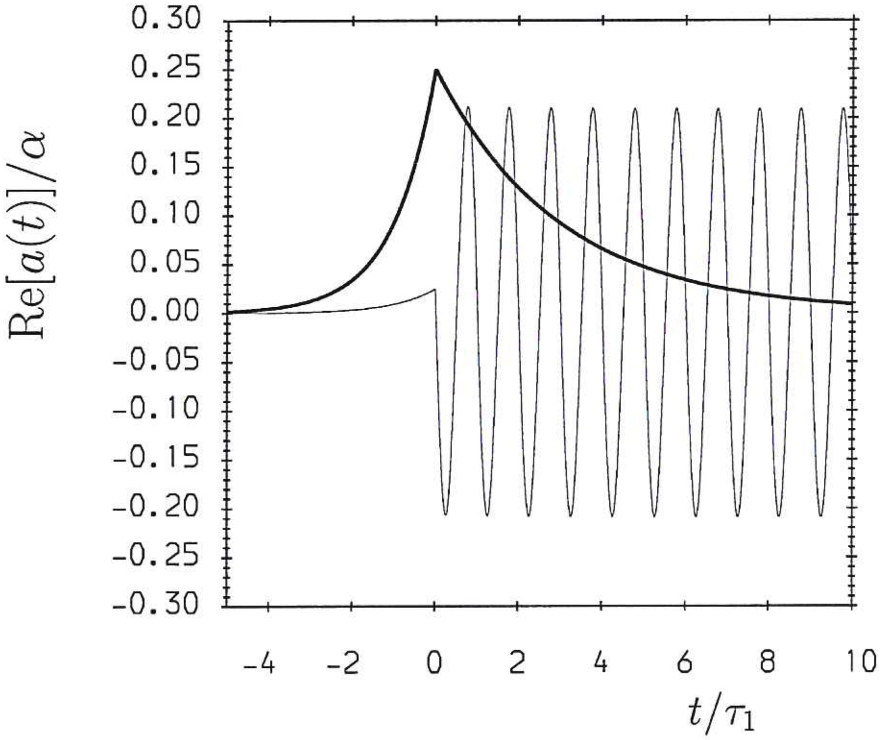

Figure 6: Real part of the solution of Eq. (2), normalised to α, for $\omega\tau_1 = 2\pi$, $\tau_2 = 3\tau_1$. The bold line shows the time profile of the driving term (the magnitude of this term has been reduced by a factor of 4).

larger than the values used to illustrate schematically solutions of Eq. (2) for $\omega_0\tau_1$, $\omega_0\tau_2 \gg 1$ **in Fig. 6)**. In CUTIE simulations the typical amplitude of magnetic field perturbations associated with tearing modes is found to be around $10^{-2}B_\theta$, where B_θ is the poloidal field. Setting the latter equal to 0.2 T for MAST, Eq. (7) gives a high frequency mode amplitude of around 2×10^{-6} T. There are calibration uncertainties in the measured amplitudes of high frequency modes in ohmic MAST shots, but they are likely to be of the order of 10^{-8} T. Since the modes are detected using Mirnov coils outside the plasma, the peak amplitudes are always substantially higher than the measured values. Nevertheless, the fact that credible estimates of the relevant parameters yield high frequency mode amplitudes that are comparable to or greater than estimates of the measured amplitudes suggest that the modes may indeed be excited via such a mechanism.

The least well–determined parameter in the above calculation is α. This is likely to depend critically on the radial extent of the high frequency mode. If the mode were highly localised and peaked at a radial location distant from that of the MHD event represented by $f(t)$, one would expect α and hence the mode amplitude to be extremely small [cf. Eq. (7)]. The mode width of a GAE with $n = 0$ is approximately equal to r_0/m [16]: the dominant contribution to the mode shown in Fig. 4 comes from $|m| = 1$, and so the mode extends over essentially the entire plasma volume. One would then expect a relatively strong coupling between high and low frequency MHD, i.e. a relatively high value of α. Moreover, the $n = 0$ GAE frequency increases linearly with m while the predicted mode amplitude scales as $1/\omega$ [Eq. (7)]. TAEs have a frequency which is approximately a factor of two lower than that of $n = 0$, $m = 1$ GAEs, and TAEs with low m have a similarly global radial structure [7]. The mechanism proposed here thus favours the excitation of GAEs and TAEs with low mode numbers. Although the poloidal mode numbers of the high frequency $n = 0$ modes observed in ohmic MAST discharges have not been determined experimentally, their frequencies and tentative identification as GAEs appear to indicate that the dominant poloidal mode number is likely to be $m = 1$.

The growth rate γ of fast particle–driven TAEs increases linearly with m (if the fast ion orbit width is less than the mode width) [17]. If the TAE saturates as a result of particle trapping, the saturation amplitude scales as γ^2, i.e. as m^2 [18]. Thus, in contrast to GAEs or TAEs driven by MHD events, one would expect fast particle–driven TAEs to have high mode numbers. In the JET tokamak, for example, fast particle–driven TAEs typically have $n \sim m \sim 6 - 11$ [19].

The driving term in Eq. (2) could in principle arise from any MHD instability in the plasma. As noted previously, a correlation has been observed between IREs and high energy ion tails in ohmic MAST discharges. Calculations by Helander and co–workers [6] indicate that electric fields induced by the reconnection process are sufficient to account for the formation of these tails. If the tail extends up to speeds comparable to c_A, it is possible that the ions could excite Alfvénic modes, or affect their evolution if they are already present. It should emphasised, however, that there are many examples of high frequency modes being excited in ohmic MAST discharges without nonthermal ion tails simultaneously being detected. In any case, the CUTIE simulations described in the previous section show that an energetic particle population is not a prerequisite for high frequency mode excitation.

6 Summary and discussion

High frequency MHD activity observed in ohmic MAST and TFTR plasmas with toroidal mode number $n = 0$ has been shown to be consistent with the excitation of global Alfvén eigenmodes (GAEs). In the case of MAST, this is the first time that modes identified as GAEs have been observed in spherical tokamak geometry. Using two–fluid simulations and an analytical model we have demonstrated that it is possible to generate GAEs or toroidicity–induced Alfvén eigenmodes (TAEs) in tokamaks via impulsive MHD events in the absence of energetic particles (whose presence is normally assumed to be required for the excitation of Alfvén eigenmodes). This mechanism favours the excitation of radially–extended GAEs or TAEs with low mode numbers.

Although the modes observed in ohmic MAST shots have fixed frequencies, they are generally bursting rather than continuous [5]. Indeed, in some cases individual bursts persist only for as long as the MHD events that appear to be driving them. Observations of high frequency magnetic fluctuations during IREs in the Small Tight Aspect Ratio Tokamak (START) [20] may provide an earlier example of this type of behaviour. It is possible that such modes lie inside the Alfvén continuum, and as such are strongly damped. One could easily represent this case in the model proposed in Section 5 by adding a damping term to Eq. (2). It should be stressed, however, that there are instances in ohmic MAST plasmas of high frequency modes persisting for much longer times (see e.g. Fig. 2).

In contrast to TFTR and MAST, there is evidence from soft X–ray data that the high frequency modes in ASDEX Upgrade ohmic plasmas had $m = 0$ [2]. If this interpretation is correct, it would appear to rule out $n = 0$ GAEs (either m or n must be nonzero in order for a GAE to have finite frequency). However, it would also contradict the TAE interpretation given in Ref. [2], since TAEs must have $m \neq 0$. Moreover, some of the other characteristics of the activity seen in ASDEX Upgrade are similar to those of the modes observed in the other two machines. For example, the high frequency activity in both ASDEX Upgrade and TFTR is strongly correlated with ELMs [2, 3].

Although the modes in TFTR and MAST identified here as GAEs can be driven unstable in ohmic plasmas, one would expect them to interact with energetic particles whose speeds are close to c_A. Li and co–workers [21] have shown that GAEs with $n = 0$, $m = 1$ can be destabilised by an isotropic Maxwellian distribution of fusion α–particles, while Gorelenkov [22] has computed GAE drive in the presence of anisotropic α–particles. Early theoretical work on GAEs was prompted largely by the possibility that they could provide an alternative radio–frequency heating scheme [8, 13]. Appert and co–workers [13] concluded that the most efficient such scheme would be one involving antenna excitation of GAEs with $n = 0$: the results of the present paper indicate that these are also the GAEs most likely to be generated spontaneously in ohmic plasmas. The apparent ease with which such modes can be excited in both conventional and spherical tokamaks suggests that in practice they are very weakly damped, and for this reason may be unsuitable for efficient plasma heating.

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