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The injection of neutral particle beams counter to the plasma current direction in the Mega-Ampère Spherical Tokamak (MAST) [A. Sykes, R. J. Akers, L. C. Appel *et al.*, Nucl. Fusion **41**, 1423 (2001)] leads to substantial losses of energetic beam ions and also rapid toroidal rotation. The electrodynamic consequences of energetic ion loss on tokamak plasmas are explored in the light of results from the MAST counter injection experiments and test particle calculations of the current density due to escaping ions. Previous authors have noted that there are two possible consequences of such a current: either a compensating bulk plasma return current is set up, or the plasma behaves as an insulator, with the energetic ion current balanced by a displacement current rather than a conduction current. Radial electric fields and hence toroidal flows occur in both cases, but higher fields are predicted in the insulating case. Such fields are important because they can confine both fast ions and bulk plasma (via the suppression of turbulent transport). The return current scenario, which appears to be operative during counter injection in MAST, is shown to be applicable if there is a sufficiently high level of momentum transport in the bulk ions; electrons cannot carry the return current, although they contribute to an ambipolar particle flux on the plasma confinement timescale. The insulating scenario may be applicable to high confinement regimes in burning tokamak plasmas.

I. Introduction

Neutral beam injection (NBI) is a well-established and very efficient means of heating tokamak plasmas [1, 2, 3]. Recent results from the Mega-Ampère Spherical Tokamak (MAST) indicate that beam injection counter to the direction of the plasma current results in substantial losses of beam ions and can also cause the plasma to spin toroidally at supersonic speeds [4, 5]. The purpose of the present paper is to elucidate the electrodynamics of this process and, in so doing, to make predictions regarding energetic ions losses and the effects thereof in tokamaks generally.

The concepts to be discussed are exemplified by a typical neutral beam injection scenario in MAST [4]. The line-averaged plasma density is typically about 2×10^{19} m^{-3}, and the plasma current is of the order of 1 MA. A beam consisting of deuterium atoms with energies mostly around 40 keV is injected in the midplane at a tangency major radius R of about 0.7 m. The plasma magnetic axis is close to $R = 0.9$ m while the outboard plasma edge is at $R \simeq 1.4$ m. We are concerned with the differences that are observed when the direction of beam injection relative to that of the plasma current is reversed. The beam energy $\varepsilon_{\rm NBI}$ is such that fast ions are produced in the plasma primarily by charge exchange with bulk ions; direct ionisation cross sections are four to five times smaller (see Fig. 5.3.1 in Ref [2]). In Fig. 1 we show calculated beam ion production contours in the midplane for counter-injected MAST discharge #8321. It can be seen that most of the fast ions appear at around $R = 1.25 - 1.3$m. The beam geometry is such that, in the absence of electric fields, almost all of the fast ions born here will be in trapped orbits.

We use a cylindrical (R, φ, Z) coordinate system; the toroidal magnetic field is in the negative φ direction, as is the current. The poloidal field is thus positively oriented along the flux surfaces in the (R, Z) plane (so that $B_Z > 0$ in the outboard midplane). The injection velocity $v_\varphi > 0$ for counter-injected ions. It follows from the invariance of canonical angular momentum p_φ that the trapped ions move outwards in radius from their birth position during the course of their bounce orbit. In the majority of cases the outboard leg of this orbit lies outside the plasma boundary. Counter-injected fast ions are thus susceptible to charge-exchange loss due to collisions with neutrals outside the last closed flux surface. In the case of co-injection, p_φ invariance causes trapped ions to move inwards in radius after injection. Thus, very few of them are expected to be in loss orbits.

When the beam is switched on for the first time, the 40 keV neutral beam atoms reach a point where the bulk plasma density is sufficiently high for them to undergo charge-exchange. The bulk ion turns into a neutral, with energy around 1 keV. In its place we have a 40 keV fast ion which is in a trapped orbit (the electric fields in the initially ohmic bulk plasma can be no more than a $\mathrm{kVm^{-1}}$ or so). It has a rather large Larmor radius, of order 0.1 m. In MAST, the toroidal and poloidal fields are both around 0.2 T close to the birth position ($R \simeq 1.3$ m) of the fast ions in loss orbits. Thus the bounce orbit width is of the same order as the Larmor radius. As soon as it is born, the fast ion travels along the inboard leg of its bounce orbit, reaches the tip and then during its reverse leg exits the plasma volume. Such an ion is not necessarily lost immediately: there is a finite probability of the ion re-entering the plasma that depends on the neutral density outside the last closed flux surface. The mean free path of the fast ions for charge-exchange in this region can be longer than the distance traversed on the outboard leg of their bounce orbits.

The NBI power P_{NBI} is around 2 MW in MAST and hence $P_{\mathrm{NBI}}/\varepsilon_{\mathrm{NBI}} \sim 3 \times 10^{20}$ energetic ions are born per second. The majority of these ions are in trapped orbits that cross the plasma boundary, if unrestrained by electric fields, and are eventually lost from the plasma (although the loss does not necessarily occur in the first bounce orbit, for the reason discussed above). There is, of course, a current associated with this net radial transport of energetic ions out of the plasma: if all of the ions were lost, the current would be $eP_{\mathrm{NBI}}/\varepsilon_{\mathrm{NBI}} \simeq 50$ A. This is negligible compared to the plasma current in MAST (~ 1 MA), and therefore the loss of fast ions can have no significant effect on the tokamak magnetic field. However, the total current $\mathbf{j}$ must satisfy the equation of charge conservation

$$\frac{\partial \rho}{\partial t} + \nabla \cdot \mathbf{j} = 0, \tag{1}$$

where ρ is charge density. A net ρ will be produced in the plasma unless the escaping fast ion current is balanced by a return current carried by one or more other plasma species, thereby ensuring that $\mathbf{j}$ remains divergence-free. The total current can be written as

$$\mathbf{j} \;=\; \mathbf{j}_i + \mathbf{j}_e + \mathbf{j}_{\mathrm{f}}. \tag{2}$$

Throughout this paper the subscripts i, e and f refer to bulk (deuterium) ions, electrons and fast ions; we neglect for the time being the effects of plasma impurities. We can then write $\rho = e(n_i + n_{\mathrm{f}} - n_e)$ and the electric field $\mathbf{E}$ associated with any net charge in the plasma is determined by Gauss's law

$$\nabla \cdot \mathbf{E} \;=\; \frac{\rho}{\epsilon_0}, \tag{3}$$

where ϵ_0 is vacuum permittivity. Clearly the charge-exchange process leaves both electron density n_e and total ion density $n_i + n_{\mathrm{f}}$ unaffected. Before neutral beam injection begins electron and ion currents are ambipolar and thus there is no loss of charge from the system. The electric field is static (apart from low amplitude turbulent fluctuations which, again, only ever cause ambipolar particle transport) and is determined by momentum balance in the ion and electron fluids.

The rapid loss of energetic beam ions thus causes one of two things to happen. The first possibility is that bulk electrons or ions move across closed flux surfaces enclosing the fast ion production region, producing a return current [5] that cancels $\mathbf{j}_{\mathrm{f}}$, so that $\mathbf{j}$ remains divergence-free. The second possibility, that $\mathbf{j}_{\mathrm{f}}$ is balanced by a displacement current rather than a bulk plasma conduction current, was mentioned briefly by Helander and co-workers [5] but not discussed by them in detail. This scenario can be illustrated by the following thought experiment. Consider a vacuum stellarator field into which a neutral beam is injected and ionised with lasers. The fast ions, if sufficiently energetic, are not confined by the stellarator magnetic field and are lost. The electrons, on the other hand, with their much smaller Larmor radii, are magnetically confined and hence their current cannot cancel that of the fast ions; there are no bulk ions to provide return currents in this case.

The supersonic toroidal rotation velocities observed in counter-injected MAST plasmas indicate the presence of strong radial electric fields. It is important to understand the physical origin of such fields, for two reasons. First, it is possible that counter-injected fast ions could be confined electrostatically, if the potential drop between the birth position and the plasma edge is large enough. Second, electric fields that are sufficiently strongly-sheared in the radial direction can cause turbulent transport to be suppressed, and thereby induce a transition to high confinement (H-) mode (see e.g. Ref [6]). A key aim of the present paper is to estimate the magnitude of the radial electric field in the two scenarios described above. In Secs. II and III we investigate these scenarios in detail. We demonstrate that which of the two cases is applicable to a real experiment depends upon transport in the bulk ion channel. If the bulk ions are sufficiently well-confined, the scenario without return currents is the only one consistent with the basic principles of charged particle electrodynamics. To clarify some of the issues raised in our discussion of the two scenarios, we present in Sec. IV test particle calculations of the current density due to escaping beam ions under various conditions in a model counter-injected MAST discharge. Finally in Sec. V we present conclusions and discuss the possible significance of our results for devices other than MAST.

II. Return current scenario

Helander and co-workers [5] did not consider in detail the dynamics of the return current. We present in this section a description of the return current scenario specifically during the early phase of counter-current NBI, on timescales that are long enough for charge neutrality to have been established, so that $\nabla \cdot \mathbf{j} = 0$, but shorter than the plasma confinement time or fast particle slowing-down time. We shall assume that return currents in this phase are entirely associated with bulk ions; it will be demonstrated later that electrons cannot carry such currents, under typical conditions. This result does not preclude the ambipolar loss of electrons and bulk ions on longer, plasma confinement timescales.

When neutral beam injection begins and the fast ions are born, almost all are in trapped orbits that intersect the last closed flux surface. We suppose, with Helander and co-workers [5], that a return current $\mathbf{j}_i$ is produced in response to the loss of fast ions. This return current gives rise to a Lorentz force term in the bulk ion momentum balance equation

$$m_i n_i \frac{d\mathbf{v}_i}{dt} = \left[-\nabla p_i - e n_i \nabla \Phi + \mathbf{j}_i \times \mathbf{B} + \mathbf{F}_{visc}\right]. \tag{4}$$

Here, m_i, e, $\mathbf{v}_i$, p_i denote bulk ion mass, charge, fluid velocity and pressure, Φ is electrostatic potential and $\mathbf{F}_{visc}$ is the viscous force on the bulk ion fluid. It follows from this equation that

$$\mathbf{j}_i \cdot \nabla \Psi = \frac{\mathbf{F}^* \cdot (\mathbf{B} \times \nabla \Psi)}{B^2},$$

where Ψ is the poloidal flux function and

$$\mathbf{F}^* = \mathbf{F}_{visc} + e n_i \mathbf{E} - \nabla p_i - m_i n_i \frac{d\mathbf{v}_i}{dt},$$

where we have used $\mathbf{E} = -\nabla \Phi$. We assume, for simplicity, that inertia and viscosity are negligible in the radial momentum balance. In the absence of significant bulk ion flows we must then have

$$\mathbf{E} \cdot \nabla \Psi = \frac{dp_i}{d\Psi} \frac{|\nabla \Psi|^2}{e n_i}.$$

If, additionally, a flow $\mathbf{v}_i$ is present, the appropriate expression for the electric field is

$$\mathbf{E} \cdot \nabla \Psi = -\mathbf{v}_i . (\mathbf{B} \times \nabla \Psi) + \frac{dp_i}{d\Psi} \frac{|\nabla \Psi|^2}{e n_i},$$

i.e.

$$E_\Psi = -\mathbf{e}_\Psi.(\mathbf{v}_i \times \mathbf{B}) + \frac{dp_i}{d\Psi}\frac{|\nabla\Psi|}{en_i}, \qquad (5)$$

where $\mathbf{e}_\Psi = \nabla\Psi/|\nabla\Psi|$. The second term in this expression is of order T_i/ea ($\simeq$ 1 kVm^{-1}), where T_i is bulk ion temperature and a is plasma minor radius. In the case of counter-injected MAST discharges, this can be neglected in comparison with the flow-induced contribution ($\simeq$ 40 kVm^{-1}) described by the first term.

A simple model that helps us to understand what happens in a qualitative fashion can be constructed by using constitutive relations for the ion viscous forces. Since we are neglecting such forces in the $\nabla\Psi$ direction we can write

$$\mathbf{F}_{visc} = -m_i n_i \left[\nu_\theta^* v_i^\theta \mathbf{e}_\theta + \nu_\varphi^* v_i^\varphi \mathbf{e}_\varphi\right]. \qquad (6)$$

Here, the subscripts/superscripts θ, φ denote poloidal and toroidal directions; $\mathbf{e}_\theta$, $\mathbf{e}_\varphi$ are unit vectors; and ν_θ^*, ν_φ^* are phenomenological momentum relaxation rates. Typically, ν_θ^* is related to the ion-ion collision time, while ν_φ^* is likely to be governed by both neoclassical and turbulent radial momentum transport processes, including charge-exchange. As a further simplification, we assume that Φ is a flux function and neglect the inertia and pressure terms in the poloidal and toroidal components of Eq. (4). We thus obtain the following expressions for the bulk ion flows in terms of the radial ion current j_i^Ψ:

$$v_i^\theta = \frac{j_i^\Psi B_\varphi}{m_i n_i \nu_\theta^*}, \qquad (7)$$

$$v_i^\varphi = -\frac{j_i^\Psi B_\theta}{m_i n_i \nu_\varphi^*}. \qquad (8)$$

Substituting these expressions into the radial momentum balance relation [Eq. (5)], and neglecting the pressure gradient term, we obtain

$$\begin{aligned} E_\Psi &= B_\varphi v_i^\theta - B_\theta v_i^\varphi \\ &= j_i^\Psi \left[\frac{B_\varphi^2}{m_i n_i \nu_\theta^*} + \frac{B_\theta^2}{m_i n_i \nu_\varphi^*}\right]. \end{aligned} \qquad (9)$$

The radial electric field and the radial bulk ion current are thus related by a simple Ohm's law, the effective radial resistivity being given by

$$\begin{aligned} \eta_i^\Psi &= \left[\frac{B_\varphi^2}{m_i n_i \nu_\theta^*} + \frac{B_\theta^2}{m_i n_i \nu_\varphi^*}\right] \\ &= \mu_0 \left[\frac{(V_A^\varphi)^2}{\nu_\theta^*} + \frac{(V_A^\theta)^2}{\nu_\varphi^*}\right], \end{aligned} \qquad (10)$$

where $V_A^\varphi = B_\varphi/(\mu_0 m_i n_i)^{1/2}$, $V_A^\theta = B_\theta/(\mu_0 m_i n_i)^{1/2}$ are Alfvén speeds defined in terms of the toroidal and poloidal magnetic field components respectively, μ_0 being vacuum permeability. The resistivity is determined in general by a combination of ion-ion collisions and turbulent transport processes. Note, however, that in this case the resistivity falls as the relaxation rates increase; this dependence is precisely opposite to that found in the case of parallel electric fields (see e.g. Sec. 2.16 in [2]). In the limit $\nu_\varphi^*, \nu_\theta^* \to 0$ bulk ions are perfectly tied to magnetic field lines and the effective radial resistivity must then be infinite. Since electrons, by virtue of their smaller mass, are generally more effectively tied to field lines than ions, it follows that the electric field required to provide momentum balance would be much higher if the return current were carried by electrons rather than bulk ions. This is reflected by the scaling of resistivity with particle mass m in Eq. (10), if ν_φ^* and ν_θ^* are determined by either like-particle collisions ($\propto m^{-1/2}$) or $\mathbf{E} \times \mathbf{B}$ turbulence (independent of m). For typical MAST parameters, the electric field associated with a return current carried by electrons would be easily sufficient to confine electrostatically the fast ions, and so the need for a return current would not in fact arise.

The reciprocal of η_i^Ψ is the effective radial electrical conductivity σ^Ψ, which is a measure of the bulk ion radial mobility. In the absence of dissipation and turbulence $\nu_\theta^* = \nu_\varphi^* = 0$ and σ^Ψ vanishes. In such cases, there can be no return currents and the plasma behaves like an insulating dielectric. The fast ion losses cannot then be balanced by return currents in the bulk ions (nor, *a fortiori*, in the electrons) but only by the vacuum displacement current. This implies that the electric potential of the plasma relative to the surrounding region must fall as the plasma gets progressively more negatively charged. The process will halt when the drop in potential is enough to confine all fast ions: at that point there can be no further fast ion losses, and the entire plasma drifts in the toroidal and poloidal directions due to the radial electric field generated by the lost fast ions. This scenario will be discussed in more detail in Sec. III.

Returning to Eq. (9), we see that if $j_{\rm f}^\Psi = -j_i^\Psi$ in quasi-steady conditions, with a constant source rate for the fast ions through the neutral beam, the electric field will be negative and its size will depend upon ion momentum relaxation rates. Using Eqs. (7) and (8) we can obtain a simple relation between the relaxation rates and the bulk ion flow components:

$$v_i^\theta = -v_i^\varphi \frac{B_\varphi \nu_\varphi^*}{B_\theta \nu_\theta^*}. \qquad (11)$$

Careful experimental measurements of the profiles of the ion flows v_i^θ, v_i^φ combined with knowledge of the toroidal and poloidal magnetic field components could thus be used to estimate the ratio $\nu_\theta^*/\nu_\varphi^*$.

In the case of counter injection in MAST $B_\varphi < 0$, $B_\theta > 0$ and $j_i^\Psi < 0$. It follows from Eq. (8) that $v_i^\varphi > 0$, i.e. the bulk ions drift toroidally in the direction of beam injection. We infer similarly from Eq. (7) that $v_i^\theta > 0$; thus the ion poloidal flow is in the same direction as the poloidal field. Typically in MAST $B_\varphi \simeq B_\theta$ whereas $v_i^\theta \ll v_i^\varphi$, so that $|v_i^\theta B_\varphi| \ll v_i^\varphi B_\theta$, and it then follows from Eq. (9) that

$$\begin{aligned} E_\Psi &= -B_\theta v_i^\varphi \\ &= -j_\mathrm{f}^\Psi \frac{B_\theta^2}{m_i n_i \nu_\varphi^*}. \end{aligned} \tag{12}$$

Thus,

$$\nu_\varphi^* = j_\mathrm{f}^\Psi \frac{B_\theta}{m_i n_i v_i^\varphi}. \tag{13}$$

All the quantities on the right hand side of Eq. (13) can be measured directly or estimated. This equation can also be written in the form

$$\left|\frac{v_i^\Psi}{v_i^\varphi}\right| = \frac{\nu_\varphi^*}{\Omega_\theta},$$

where $\Omega_\theta = eB_\theta/m_i$ is the poloidal cyclotron frequency of the bulk ions. The work done per unit volume per unit time on these ions by the electric field is

$$\begin{aligned} E_\Psi j_i^\Psi &= \eta_i^\Psi (j_\mathrm{f}^\Psi)^2 \\ &= \eta_i^\Psi \left[\frac{m_i n_i \nu_\varphi^* v_i^\varphi}{B_\theta}\right]^2 \\ &= m_i n_i \nu_\varphi^* (v_i^\varphi)^2. \end{aligned} \tag{14}$$

As noted by Helander and co-workers [5], the energy loss from the plasma due to escaping fast ions is mitigated by the fact that these ions climb an electric potential hill and thus lose kinetic energy as they escape; this energy is returned to the bulk plasma at a rate given by Eq. (14).

It is useful at this point to make some simple estimates for MAST conditions. Let us assume that $\nu_\varphi^* \simeq 1/\tau_E$ where the energy confinement time $\tau_E \simeq 100$ ms [4]. Thus $\nu_\varphi^* \simeq 10\ \mathrm{s}^{-1}$. Taking $v_i^\varphi = 4 \times 10^5\ \mathrm{ms}^{-1}$ (a typical toroidal rotation velocity in counter-injected discharges), $B_\theta = 0.1$ T, $n_i = 2 \times 10^{19}\mathrm{m}^{-3}$, we obtain from Eq. (13)

$$\begin{aligned} j_\mathrm{f}^\Psi &= m_i n_i v_i^\varphi \nu_\varphi^* / B_\theta \\ &\simeq 2.5\ \mathrm{Am}^{-2}. \end{aligned}$$

From Eq. (12) we find that the associated radial electric field $E_\Psi \simeq 40$ kVm^{-1} and the power density delivered to the bulk ions due to this is $P_{\rm return} = E_\Psi j_i^\Psi \simeq 100$ kWm^{-3}. Taking the effective minor radius $a = 0.75$ m and the major radius $R_0 = 1$ m, the surface area of the MAST plasma is estimated to be $S \simeq 4\pi^2 \times 0.75 \simeq 30$ m^2. The number of fast ions lost per second is then $\dot{N}_{\rm loss} = S j_{\rm f}^\Psi / e \simeq 4.7 \times 10^{20}$ s^{-1}. As noted in Sec. I, the injection rate of fast ions is 3×10^{20} s^{-1}: this is of the same order of magnitude as our estimate of the loss rate, indicating that the return current scenario is consistent with most of the fast ions being lost. Poloidal rotation measurements in MAST suggest that $v_i^\theta \lesssim 4 \times 10^4$ ms^{-1}. Since $B_\varphi \simeq B_\theta$, this result combined with Eq. (11) suggests that $\nu_\varphi^* \lesssim 0.075\nu_\theta^*$. Now $\nu_\theta^* \simeq 3 \times 10^2$ s^{-1} if this is assumed to be of the order of the ion-ion collision frequency; this implies a value of ν_φ^* that is of the same order as that estimated on the basis of the energy confinement time.

It is straightforward to generalise the calculation of σ^Ψ to include the contributions of electrons and N bulk ion species. Assuming, as in the above discussion, that poloidal friction forces are unimportant, we obtain

$$\sigma^\Psi \quad = \quad \frac{1}{B_\theta^2}\left\{ m_e n_e \nu_{\varphi e}^* + \sum_{j=1}^{N} m_j n_j \nu_{\varphi j}^* \right\}, \tag{15}$$

where m_j, n_j and $\nu_{\varphi j}^*$ denote the mass, number density and toroidal momentum relaxation rate of the j-th ion species. This expression reinforces the point that electrons make a negligible contribution to the return current. On the other hand, the return current associated with massive plasma impurities such as carbon could be significant, particularly if the $\nu_{\varphi j}^*$ are determined by $\mathbf{E} \times \mathbf{B}$ turbulence and hence independent of m_j. Note that a general requirement for the return current to be carried by ions of any species is that there must be a continuous supply of those ions at the edge. Effectively, every fast ion lost promptly must be replaced by a cold ion from the edge.

III. Insulating plasma scenario

In this section we again consider the early phase of counter injection, immediately after the beam is switched on: specifically the first 100 μs or so, when collisional interactions between fast ions and bulk plasma particles can be neglected. This enables us to consider the fast ion dynamics somewhat more easily. We assume that the magnetic fields are essentially unaffected during this phase, so that we may treat the fast ions as moving in specified, azimuthally-symmetric toroidal and poloidal fields. The fast ion pressure is initially negligible and does not disturb the equilibrium magnetic configuration. However, as we shall shortly demonstrate, the fast ions are capable, through their losses, of re-organizing the plasma electric field. As this essentially radial field starts to build up, the bulk plasma particles must drift toroidally and poloidally under its influence. They can have no direct interaction with the injected fast ions. The motion of bulk ions in response to the radial electric field could give rise to poloidal and toroidal friction forces arising from turbulence and/or collisional dissipation [cf. Eq. (4)]: these forces can be balanced by a Lorentz force associated with an inward-directed radial bulk ion current. As shown in the previous section, this radial current can in turn balance the outward fast ion loss current. When dissipation is absent, however, there is nothing to prevent the electric field from rising to a sufficiently high level that all the fast ions are electrostatically confined. When this has occurred, no further fast ion loss is possible on timescales shorter than the plasma confinement timescale, in which collisional effects or turbulence can produce ambipolar transport.

Denoting the fast ion distribution function by $F_{\rm f}(\mathbf{r},\mathbf{v},t)$, neglecting collisions between particle species but taking account of the beam source and the charge-exchange sink outside the last closed flux surface, we can write down a Vlasov equation for $F_{\rm f}$:

$$\frac{\partial F_{\rm f}}{\partial t}+\mathbf{v}\cdot\frac{\partial F_{\rm f}}{\partial \mathbf{r}}+\frac{e}{m_i}\left[\mathbf{E}+\mathbf{v}\times\mathbf{B}\right].\frac{\partial F_{\rm f}}{\partial \mathbf{v}} \;=\; S_{\rm f}(\mathbf{r},\mathbf{v},t)-\gamma_{\rm f}(\mathbf{r})F_{\rm f}. \qquad (16)$$

The source $S_{\rm f}$ can be taken to be a delta function in position space, localised around $R = 1.3$, $Z = 0$ and a delta function in kinetic energy, around 40 keV. The velocity components at birth are determined by the beam geometry. The sink $\gamma_{\rm f}$ is effectively a charge-exchange loss rate of the fast ions which can be taken to be finite only outside the last closed flux surface, $\Psi = \Psi_{\rm edge}$. Since $\mathbf{B}$ is assumed to be known and $\mathbf{E} = -\nabla\Phi$, this equation can, in principle, be solved by the method of characteristics if Φ is known. The fast ion current density $\mathbf{j}_{\rm f}$ is then given by

$$\mathbf{j}_{\rm f} \;=\; e\int F_{\rm f}\mathbf{v}d\mathbf{v}, \qquad (17)$$

where the integration is performed over the whole of velocity space. Since collisions play no role in the scenario we are considering, the sole effect of the fast ions on bulk plasma particles is an ambipolar cross-field drift due to $\mathbf{E}$ and possibly a parallel electron current due to potential variations within a flux surface. However, we will show later that such variations will always be shorted out by parallel electron flows, so that Φ can always be well-approximated by a flux function. Since $\mathbf{E}\times\mathbf{B}$ drifts do not generate currents, $\mathbf{j}_i + \mathbf{j}_e$ must, in the absence of parallel electron currents, remain divergence-free, and in that case the law of conservation of charge [Eq. (1)] becomes

$$\frac{\partial \rho}{\partial t}+\nabla\cdot\mathbf{j}_{\rm f} \;=\; 0. \qquad (18)$$

Equation (18) combined with Eq. (3) gives us an evolution equation for the electrostatic potential Φ:

$$\epsilon_0 \nabla^2 \frac{\partial \Phi}{\partial t} \quad = \quad \nabla \cdot \mathbf{j}_{\mathrm{f}}. \tag{19}$$

Equations (16)-(19) must be solved simultaneously as an initial value problem, with $F_{\mathrm{f}} = 0$ at $t = 0$. It is reasonable to take Φ also to be zero initially, although this is not essential.

The general character of the solution can be inferred without solving the equations in detail. Initially, when the beam is turned on in a quasi-neutral ohmic target plasma, $\rho \simeq 0$ and $\Phi \simeq 0$. Particles injected by the source into regions of phase space corresponding to confined orbits remain in the plasma, slowly increasing the fast ion number density. These replace the slow ions which charge-exchanged with the beam to create them; they do not alter ρ. On the other hand, fast ions that are born in the prompt loss region of phase space will move beyond Ψ_{edge} during the course of their orbits; eventually they will charge-exchange with neutrals outside the last closed flux surface. They will thereby produce a $\mathbf{j}_{\mathrm{f}}$ with a finite divergence, since they transport positive charge out of the plasma. According to Eq. (19), this must lead to a changing electric field which will generally point inwards since the plasma is negatively charged due to the prompt loss of positively charged fast ions. As the field rises, however, the loss region of phase space must become progressively smaller, until the net fast ion current out of the system falls to zero.

In this scenario the electric field adjusts on the timescale of a few bounce times until all the injected fast ions are confined due to the combined effects of the magnetic field (assumed unaltered by injection) and the self-consistent electric potential. After this time the electric field and the fast ion number density will evolve in a relatively quasi-stationary manner. The longer time behavior will be dominated by the balance of fast ion collisions with the bulk ions and electrons, resulting in fast ion slowing down, momentum/energy equilibration with the bulk plasma, and ambipolar particle transport. The bulk electrons and ions will of course $\mathbf{E} \times \mathbf{B}$ drift in the electric field resulting from the loss of fast ions. These ions carry away mechanical angular momentum and energy as they are lost. The change of mechanical momentum is taken up by the electric field, which is changing during the prompt loss phase. Once the field is sufficiently established to prevent any further loss of fast ions, all the mechanical momentum of the beam goes into the confined fast ions, just as it would in the case of co-injection. Thus, the initial loss of fast ions is the price one pays to set up a radial electric field that confines all the fast ions subsequently injected. This field will also make the bulk plasma drift toroidally. The poloidal drift of the bulk ions is dissipated by neoclassical ion viscosity. Ultimately, the toroidal drift angular momentum must also be transported radially for the system to reach a long-term stationary state, assuming that the total current, line-averaged density and fast ion source are kept fixed. The toroidal spin up is very fast in the counter-injected case due to the prompt losses, which do not occur in the co-injected case.

The spin up of the plasma due to fast ion loss in the return current model is also fast. However, the total spin up in that case is due to both the prompt effect described above and also to the slowing-down momentum delivered to the plasma by the confined fast ions. In the absence of return currents, the electric fields generated must be sufficient to confine the fast ions. They are necessarily larger than the electric fields produced in the return current scenario, and the plasma rotation is consequently more rapid. It is also possible that the return current scenario could be applicable initially but not at later times: as the electric field increases the plasma gets hotter [cf. Eq. (14)] and consequently less collisional, and turbulence could be suppressed by the rotation. If the resultant viscosities are not large enough to produce sufficient friction forces in the bulk ions, the field may build up until it completely confines the fast ions. Which scenario is relevant depends on the precise plasma conditions in any specified device.

A complete solution of the problem formulated above would require a particle-in-cell (PIC) code or a self-consistent Vlasov solver, taking account of the initial and boundary conditions on Eqs. (16) and (19). The effect of the evolving electric potential on fast particle orbits must, of course, be taken into account in any such solution. At the end of this section we will show that the electric potential associated with fast ion loss in both the return current and insulating scenarios is likely to be be well-approximated by a flux function (i.e. that the radial component of the electric field E_Ψ is much larger than the poloidal or toroidal components), as in the limit of ideal magnetohydrodynamics (MHD) for a toroidally symmetric system [7]. The integral form of Gauss' law then takes the form

$$\int_\Psi \Phi'(\Psi)|\nabla\Psi| 2\pi R d\ell \quad = \quad -\int\int_{V(\Psi)} \frac{\rho}{\epsilon_0}\frac{2\pi R d\ell}{|\nabla\Psi|} d\Psi, \tag{20}$$

where the prime denotes differentiation with respect to Ψ, $d\ell$ is an element of arc along a flux surface in the (R, Z) plane, the integral on the left hand side is over the flux surface Ψ and the integral on the right hand side is over the volume enclosed by that surface. Since $\Phi' \equiv \Omega$ is assumed to be a flux function, it can be taken out of the integral on the left hand side. For the case of a predominantly toroidal flow Ω can be identified as the toroidal angular velocity of the plasma [7]. The time derivative of Eq. (20) and the charge conservation equation [Eq. (18)] then give an evolution equation for Ω:

$$\epsilon_0 \frac{\partial \Omega(\Psi)}{\partial t} \quad = \quad \frac{\int_\Psi \mathbf{j}_\mathrm{f} \cdot \mathbf{e}_\Psi R d\ell}{\int_\Psi |\nabla\Psi| R d\ell}, \tag{21}$$

This is evidently the integrated form of Eq. (19). The fast ion motion can now be determined by solving simultaneously Eqs. (16) and (21). As the (inward) radial electric field rises, $\mathbf{j}_{\rm f}$ will fall until steady conditions are reached and there can be no further prompt losses. The entire plasma must now drift in the resultant electric field. In the absence of collisions, this will lead to both toroidal and poloidal drifts.

We noted previously that if the electric field required by radial force balance in the steady-state return current scenario [Eq. (9)] is sufficient to confine electrostatically the fast ions, the need for return currents does not in fact arise: in this case the insulating scenario must apply. Electrostatic confinement of fast ions will occur if the electric field is of the order of $\varepsilon_{\rm NBI}/ea$; assuming that the radial resistivity is determined by the toroidal momentum relaxation rate, as in Eq. (12), we infer the following condition for the applicability of the steady-state return current model:

$$j_{\rm f}^{\Psi}\frac{B_\theta^2}{m_i n_i \nu_\varphi^*} \quad \lesssim \quad \frac{\varepsilon_{\rm NBI}}{ea}. \tag{22}$$

Equivalently, if a high proportion of the fast ions are lost, so that $j_{\rm f}^{\Psi} \simeq eP_{\rm NBI}/(S\varepsilon_{\rm NBI})$, the toroidal momentum relaxation time $\tau_\varphi^* = 1/\nu_\varphi^*$ must satisfy the inequality

$$\tau_\varphi^* \quad \lesssim \quad \frac{m_i n_i S \varepsilon_{\rm NBI}^2}{ae^2 B_\theta^2 P_{\rm NBI}}. \tag{23}$$

If this inequality is not satisfied, the insulating scenario must apply. If, as in the case of counter-injected MAST discharges, the fast ions are mostly born closer to the plasma edge than the core, the value of a used in Eq. (23) should be less than the actual minor radius of the plasma.

Finally in this section, it is instructive to consider the response of electrons to the presence of an electrostatic potential associated with fast ion loss. Neglecting dissipation effects and inertia, the electron momentum balance equation is

$$0 \quad = \quad -\nabla n_e T_e + e n_e \nabla\Phi - e n_e \mathbf{v}_e \times \mathbf{B}, \tag{24}$$

where $\mathbf{v}_e$, T_e are the electron fluid velocity and temperature. Making the reasonable assumption that T_e is a flux function, we find that the component of this equation parallel to the magnetic field yields the familiar adiabatic relation

$$n_e \quad = \quad N(\Psi)\exp(\frac{e\Phi}{T_e}), \tag{25}$$

where N is a flux function. This result is, of course, valid for both the return current and insulating scenarios. It is important to note that n_e fails to be a flux function if and only if Φ is not a flux function; any measured variation of n_e on a flux surface thus implies, strictly speaking, a violation of ideal MHD [7]. Thomson scattering measurements close to the midplane of MAST during counter-current NBI suggest that n_e increases from inboard to outboard on a flux surface by at most a factor of two or so [4]. The variation of Φ implied by Eq. (25) in such cases is of the order of T_e/e or less. We can write, quite generally, $\Phi = \bar{\Phi}(\Psi) + \tilde{\Phi}$ where $\tilde{\Phi}$ has zero flux surface average. Assuming that $|e\tilde{\Phi}/T_e| \ll 1$, we infer from Eq. (25) that

$$n_e \;=\; \tilde{N}(\Psi)\left[1 + \frac{e\tilde{\Phi}}{T_e}\right], \qquad (26)$$

where $\tilde{N}(\Psi) = N(\Psi)\exp(e\bar{\Phi}/T_e)$. This equation illustrates the fact that the radial component of the electric field can be large enough to confine all of the fast ions while the poloidal component remains small enough to ensure that there is not a very large inboard-outboard asymmetry in the electron density. The actual density asymmetry that would occur in the insulating scenario can only be determined experimentally or by solving the initial value Vlasov-Maxwell problem described by Eqs. (16) and (19). In general, however, the very high mobility of electrons parallel to the magnetic field is likely to ensure in all cases that $|e\tilde{\Phi}| \lesssim T_e$ and hence that $|\tilde{\Phi}| \ll |\bar{\Phi}|$. This justifies our assumption in Eqs. (20) and (21) that Φ can be well-approximated by a flux function.

IV. Distribution of fast ion current density

There is an important aspect of the fast ion loss that is not addressed by considering Φ to be a flux function, as in Sec. II. If every fast ion were to charge exchange with a neutral immediately upon crossing the last closed flux surface, we would expect the fast ion losses to be be concentrated in a small area of the surface close to the upper mirror point for the injected particles. In this event $j_{\rm f}^{\Psi}$ would vary strongly with poloidal angle. For the fast ion current to be cancelled everywhere by return currents, the toroidal flows and/or viscosities would then have to be much larger in the region where most of the losses occur. There is also the possibility of strong toroidal asymmetry in the losses, due to the fact that the fast ion source is itself strongly localised in toroidal angle φ (cf. Fig. 1). If the majority of losses were to occur promptly, i.e. before the completion of a single trapped particle bounce orbit, we would thus expect $j_{\rm f}^{\Psi}$ to be localised in both φ and poloidal angle θ. However, as discussed in Sec. I, fast ions leaving the MAST plasma are not in general lost immediately; they can either be neutralised due to charge exchange with neutral atoms or they can re-enter the plasma. Typically, fast ions cross the last closed flux surface and re-enter the plasma several times before being neutralised, with the result that the losses are much less localised than would otherwise be the case.

To quantify $j_{\rm f}^{\Psi}$ as a function of toroidal and poloidal angle in a counter-injected MAST discharge, we have used the full orbit test particle code CUEBIT [8] and a simple model equilibrium. We assume that the fast ions are born within a cylindrical region of the MAST plasma, with the cylinder axis lying in the midplane; the tangency major radius of this axis is taken to be 0.7m. The beam ion birth profile is assumed to peak at a point on the axis of the beam cylinder lying approximately 1.1m from the tokamak symmetry axis and the radius of the cylinder is taken to be 0.2m (cf. Fig. 1). Within the beam cylinder and the last closed flux surface of the plasma, the beam ion birth probability is assumed to decay exponentially with distance from the position of maximum probability, the e-folding distance being 0.1m. Beam ions are assumed to be born with energies of 40keV and velocity vectors directed along the axis of the beam cylinder.

To model the equilibrium magnetic field we use a Solov'ev-type solution of the Grad-Shafranov equation in a form given by Freidberg [9]:

$$\Psi = \Psi_0 \left\{ \frac{\gamma}{8} \left[\left(R^2 - R_0^2 \right)^2 - R_b^4 \right] + \frac{1-\gamma}{2} R^2 Z^2 \right\}. \tag{27}$$

Here, R_0, R_b, γ and Ψ_0 are constants that can be chosen to give MAST-like values of the plasma major and minor radii, elongation and plasma current; the plasma boundary is defined by $\Psi = \Psi_{\rm edge} = 0$. We compute the collisionless trajectories of beam ions in this equilibrium, with initial positions and velocities chosen according to the probability distribution indicated above. The majority of beam ions are born into trapped orbits, and, as discussed in Sec. I, most of these ions cross the plasma boundary on the outboard leg of their bounce orbit. For modelling purposes, we assume that for any beam ion instantaneously lying outside the plasma there is a fixed probability of neutralisation per time step. To estimate this probability, we assume that the temporal variation of the fast ion population $N_{\rm f}$ in this region can be represented by an equation of the form [cf. Eq. (16)]

$$\frac{dN_{\rm f}}{dt} = -\sigma_{\rm cx} n_n v_{\rm f} N_{\rm f}, \tag{28}$$

where n_n is the neutral particle density, $v_{\rm f}$ is the fast ion speed and $\sigma_{\rm cx}$ is the charge exchange cross-section. Strictly speaking, the value used for $v_{\rm f}$ should take into account the presence of radial electric fields, if any, but for simplicity we use a charge exchange time $\tau_{\rm cx} = 1/(\sigma_{\rm cx} n_n v_{\rm f})$ based on $v_{\rm f} = 2 \times 10^6 {\rm ms}^{-1}$, the appropriate value for a deuteron whose kinetic energy is equal to the assumed total beam injection energy (40keV), whether a radial electric field is present or not. A typical value for the neutral particle density in the MAST vacuum vessel is $5 \times 10^{17}\mathrm{m}^{-3}$ (see Ref [10]), and $\sigma_{\mathrm{cx}} \simeq 6 \times 10^{-20}\mathrm{m}^2$ for a 40keV deuteron (see Fig 5.3.1 in Ref [2]). These figures imply a charge exchange time of about 15μs. The neutralisation probability in a single time-step Δt is then taken to be $\Delta t/\tau_{\mathrm{cx}}$. Beam ions in MAST can also be removed permanently from the plasma by striking in-vessel components [4]: we do not include this process in our simulations. If, as is generally the case in counter-injected MAST discharges, the plasma is rotating toroidally with a sonic Mach number of order unity, the associated radial electric field can have a significant effect on beam ion orbits. We assume that the entire plasma is rotating toroidally as a single rigid body, i.e. with a constant angular velocity Ω. The ideal MHD form of Ohm's law then implies that $\Phi = \Omega\Psi$ (cf. Sec. III): from this we can compute the radial electric field

$$\mathbf{E} = -\nabla\Phi = -\frac{d\Phi}{d\Psi}\nabla\Psi = -\Omega\nabla\Psi. \tag{29}$$

Toroidal rotation rates as high as $4 \times 10^5\mathrm{s}^{-1}$ have been measured in counter-injected MAST shots [4]: we use this figure to estimate the radial electric field.

Since we are using cylindrical coordinates to define the equilibrium [Eq. (27)], and the flux surface cross-section in the (R, Z) plane is non-circular, there is no unique definition of poloidal angle θ. For the present discussion it is sufficient to use the simple definition

$$\tan\theta = \frac{Z}{R - R_0}, \tag{30}$$

where R_0, as in Eq. (27), is the major radius of the magnetic axis. We are specifically concerned here with calculating j_{f}^{Ψ}. Suppose we compute the trajectories of N_{tot} beam ions: if dN of these ions cross an area dA of the plasma surface, and are lost before they re-enter the plasma, the associated current is given by

$$j_{\mathrm{f}}^{\Psi} = e\frac{P_{\mathrm{NBI}}}{\varepsilon_{\mathrm{NBI}}}\frac{1}{N_{\mathrm{tot}}}\frac{dN}{dA}. \tag{31}$$

Using CUEBIT it is straightforward to compute dN for toroidal and poloidal angles in the range $\varphi \rightarrow \varphi + d\varphi$ and $\theta \rightarrow \theta + d\theta$: to obtain j_{f}^{Ψ} we then require an expression for dA. Such an expression can be obtained by introducing a minor radial distance variable

$$\rho = \left[(R - R_0)^2 + Z^2\right]^{1/2}, \tag{32}$$

and considering an element of arc $d\ell$ of the last closed flux surface in the (R, Z) plane:

$$d\ell = d\theta \left[\rho^2 + \left(\frac{d\rho}{d\theta}\right)^2\right]^{1/2}. \tag{33}$$

On any flux surface we have

$$d\Psi = \frac{\partial\Psi}{\partial\rho} d\rho + \frac{\partial\Psi}{\partial\theta} d\theta = 0, \tag{34}$$

where

$$\frac{\partial\Psi}{\partial\rho} = \frac{\partial\Psi}{\partial R}\frac{\partial R}{\partial\rho} + \frac{\partial\Psi}{\partial Z}\frac{\partial Z}{\partial\rho} = R\left(B_Z \cos\theta - B_R \sin\theta\right), \tag{35}$$

and

$$\frac{\partial\Psi}{\partial\theta} = \frac{\partial\Psi}{\partial R}\frac{\partial R}{\partial\theta} + \frac{\partial\Psi}{\partial Z}\frac{\partial Z}{\partial\theta} = -R\rho\left(B_Z \sin\theta + B_R \cos\theta\right). \tag{36}$$

Using Eqs. (34), (35) and (36) in Eq. (33) we obtain

$$dA = R d\varphi d\ell = \frac{R\rho B_\theta d\varphi d\theta}{|B_Z \cos\theta - B_R \sin\theta|}, \tag{37}$$

where $B_\theta = (B_R^2 + B_Z^2)^{1/2}$. This is a convenient formula, since the code is used to compute the magnetic field components for each point on the trajectory of the particle, including the point at which it crosses the plasma boundary.

Figures 2, 3 and 4 show the computed fast particle current as a function of toroidal and poloidal angle with $\Omega = 0$, 4×10^5 rad s^{-1} and 10^6 rad s^{-1} respectively. For comparison, Figs. 5, 6 and 7 show the corresponding results when the last closed flux surface is treated as a perfectly absorbing boundary: this would be a reasonable assumption for devices such as the National Spherical Torus Experiment (NSTX) [11], in which the plasma is enclosed within a tightly-fitting vacuum vessel. The first point to note from these plots is that strong variations of j_f^Ψ with both φ and θ occur in all of the scenarios considered. Infra-red images of the MAST vessel obtained during counter NBI do indeed show a high degree of asymmetry, both poloidally and toroidally, in the heat load due to escaping fast ions [12]. However, since parallel electric fields are rapidly shorted out by electrons, we would not expect to observe a similar asymmetry in the electric potential. The degree of toroidal asymmetry in the computed fast ion current is found to be greatly reduced when the assumed charge exchange lifetime is increased by a factor of ten; this is due to the smearing effect of differential fast ion toroidal precession. Fast ion losses never occur in the vicinity of the inner midplane ($|\theta| > 110°$), since all of the ions on loss orbits are deeply trapped. Figures 2 and 3 indicate that there are regions of the plasma boundary where the fast ion current density is significantly higher than 2.5 $\mathrm{Am^{-2}}$, the figure estimated in Sec. II for the current that could be balanced by a return current under MAST conditions. The maximum current is reduced somewhat (to about 7 $\mathrm{Am^{-2}}$) if the observed rotation rate of the plasma is taken into account (Fig. 3). Any remaining discrepancy between this maximum current and the current estimated using Eq. (13) is probably not significant, in view of the approximations used to derive the latter and uncertainties in the values of plasma parameters such as ν^*_φ. If the plasma is rotating at 10^6 rad $\mathrm{s^{-1}}$ (this corresponds to a sonic Mach number of about 3 in MAST), the radial electric field is sufficient to confine around 99% of the fast ions, and the maximum fast ion current density is very small, about 0.3 $\mathrm{Am^{-2}}$ (Fig. 4).

When the last closed flux surface is treated as a perfectly absorbing boundary (Figs. 5-7) j_f^Ψ is even more localised than it is in the case of delayed loss, rising to about 300 $\mathrm{Am^{-2}}$ in the vicinity of the upper mirror point when the plasma is non-rotating (Fig. 5). The main peak in Fig. 7, and also the secondary peaks in Figs. 5 and 6, are produced by ions that are born within one Larmor radius of the last closed flux surface. Figures 4 and 7 correspond essentially to the insulating plasma scenario discussed in Sec. III, in which there is a radial electric field that is sufficiently large to confine all (or nearly all) of the fast ions.

V. Conclusions and discussion

We have explored the electrodynamic consequences of energetic ion loss on tokamak plasmas in the light of results from recent MAST experiments with counter-current neutral beam injection. Such experiments are characterised by substantial losses of energetic beam ions and also rapid toroidal rotation. As noted by Helander and co-workers [5], there are two possible consequences of the presence of a substantial current associated with escaping energetic ions: either a compensating return current is set up, carried by bulk plasma particles, or the plasma behaves as an insulator, with the fast ion current balanced by a displacement current rather than a bulk plasma conduction current. Radial electric fields and hence toroidal flows occur in both cases, but higher fields are predicted in the insulating case. A fully rigorous analysis of this scenario will require the numerical solution of the Vlasov-Maxwell system of equations, Eqs. (16 - 19). Work on this problem is currently in progress, and will be reported in a future paper. We have shown that the return current scenario, which appears to be operative during the counter injection shots in MAST, is applicable if there is a sufficiently high level of momentum transport in the bulk ions: the radial electric fields corresponding to the highest rotation rates observed in MAST are insufficient to confine the beam ions, and hence the insulating scenario cannot be applicable. Test particle calculations for a model counter-injected MAST discharge show that the current arising from lost fast ions is invariably localised both poloidally and toroidally, with the degree of localisation increasing greatly if the plasma edge acts as a perfect absorber of fast ions.

Strictly speaking, the return current scenario described in Sec. II requires that the bulk ion toroidal momentum relaxation rate be sufficiently large locally that Eq. (22) is satisfied at every point on the plasma boundary. In the case of a machine such as MAST, it seems improbable that this would be the case if $j_{\rm f}^{\Psi}$ were to approach the maximum values in Figs. 5 and 6. These values are unlikely to be realised in MAST, however, because fast ions crossing the last closed flux surface have a high probability of re-entering the plasma before being neutralised. This is not in case in NSTX, nor indeed in the majority of other tokamaks. Nevertheless, even if the current due to lost fast ions is strongly localised in poloidal angle, the high mobility of electrons parallel to the magnetic field implies that the electric field arising from fast ion loss is always likely to be predominantly radial (cf. discussion at the end of Sec. III).

If the full insulating scenario were to be realised in a MAST-like device with strong counter-current NBI, none of the beam energy or momentum would be lost, except during a short transient phase at the start of beam injection when the confining electric field is being established. The higher electric fields predicted in the insulating scenario are also more likely to suppress turbulent transport and hence lead to H-mode [6]. In MAST H-mode is easily achieved only when the upper and lower X-points on the plasma boundary lie on flux surfaces that are separated in the outer midplane by no more than one bulk ion Larmor radius: Meyer and co-workers [13] have found that this sensitivity of H-mode access to the magnetic configuration may be due to differences in ion orbit losses and the effects thereof on the radial electric field.

In general, the insulating plasma scenario is most likely to be applicable to high confinement regimes in tokamak plasmas containing a significant population of trapped energetic particles with large normalised Larmor radii. It is possible that the required conditions might be satisfied in future devices such as the proposed spherical tokamak components test facility (CTF) [14] and the proposed spherical tokamak power plant (STPP) [15]. Spherical tokamaks generally tend to lose energetic particles at a high rate, due to compact geometry and the use of relatively low toroidal fields, although such losses are reduced to some extent by orbit squeezing in the outboard midplane due to the poloidal field. Both CTF and STPP would have a substantial population of trapped fusion α-particles, some of whose orbits would cross the plasma boundary

[16]. Alternatively, it is possible that the insulating scenario, with its possible benefits for plasma confinement, could be realised in such devices through the strategic use of counter-current NBI. Losses of fast particles from a plasma can only be properly assessed if one takes account of collective electric field effects, which can manifest themselves either through displacement currents (leading to large confining electric fields) or fields consistent with significant radial transfer of toroidal momentum in the bulk ions (leading to return currents). In proposed burning plasma devices, including the International Tokamak Experimental Reactor (ITER) [17], approximately half the fusion α-particles will be born with velocity vectors in the counter-current direction. In advanced ITER scenarios, a significant fraction of these α-particles are lost, either promptly or as a result of toroidal field ripple [18]. Our analysis suggests that such losses must lead to large self-consistent electric fields and plasma rotation. It is straightforward to show that perfect electrostatic confinement of 3.5 MeV α-particles in an ITER-like device (i.e. the insulating scenario) would require the sonic Mach number of the toroidal flow to be of order unity. In the return current scenario the localization of the current density associated with escaping fast ions could pose design problems, because of the associated additional heat load on plasma-facing components. In this respect the insulating plasma scenario, with its associated rapid rotation, may prove to be more attractive in both confining the fast ions and spinning the burning plasma with no sources of external momentum input. It would be of interest to investigate further, experimentally and theoretically, the possible role of this scenario in existing and future devices.

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Figure Captions

FIG. 1. Calculated beam ion production contours for MAST discharge #8321; colors from black through yellow indicate increasing birth rate. The beam line is parallel to the Y axis. The curve passing through $X = 0$, $Y = -90$ cm corresponds to the magnetic axis, and the curve passing through $X = 0$, $Y = -140$ cm indicates the outer plasma boundary.

FIG. 2. Fast ion current versus toroidal and poloidal angle assuming a charge exchange lifetime outside the plasma of 15μs and no toroidal rotation. Essentially all of the fast ions are lost; the average time for these ions to be neutralised is 71μs.

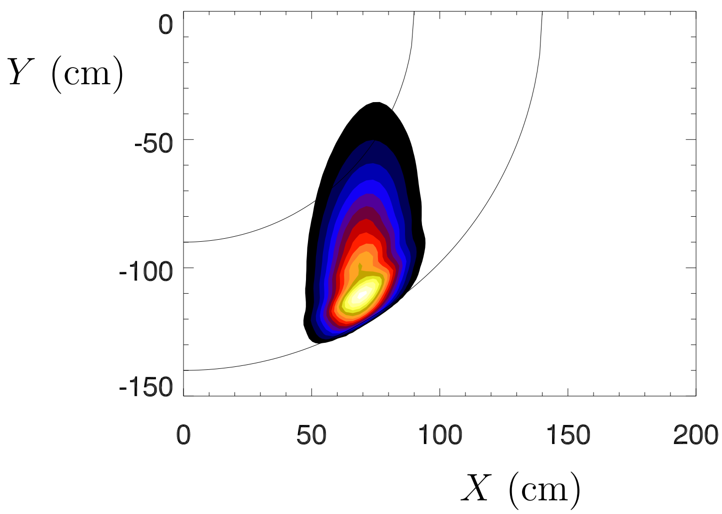
FIG. 3. Fast ion current versus toroidal and poloidal angle assuming a charge exchange lifetime outside the plasma of 15μs and rigid body toroidal rotation at 4×10^5 rad s^{-1}. Approximately 46% of the fast ions are lost; the average time for lost ions to be neutralised is 51μs.

FIG. 4. Fast ion current versus toroidal and poloidal angle assuming a charge exchange lifetime outside the plasma of 15μs and rigid body toroidal rotation at 10^6 rad s^{-1}. Around 1% of the fast ions are lost; the average time for lost ions to be neutralised is 25μs.

FIG. 5. Fast ion current versus toroidal and poloidal angle when the last closed flux surface is a perfectly absorbing boundary and there is no toroidal rotation. Essentially all of the fast ions are lost.

FIG. 6. Fast ion current versus toroidal and poloidal angle when the last closed flux surface is a perfectly absorbing boundary and the plasma is rotating toroidally as a rigid body at 4×10^5 rad s^{-1}. Approximately 96% of the fast ions are lost.

FIG. 7. Fast ion current versus toroidal and poloidal angle when the last closed flux surface is a perfectly absorbing boundary and the plasma is rotating toroidally as a rigid body at 10^6 rad s^{-1}. Fewer than 2% of the fast ions are lost.



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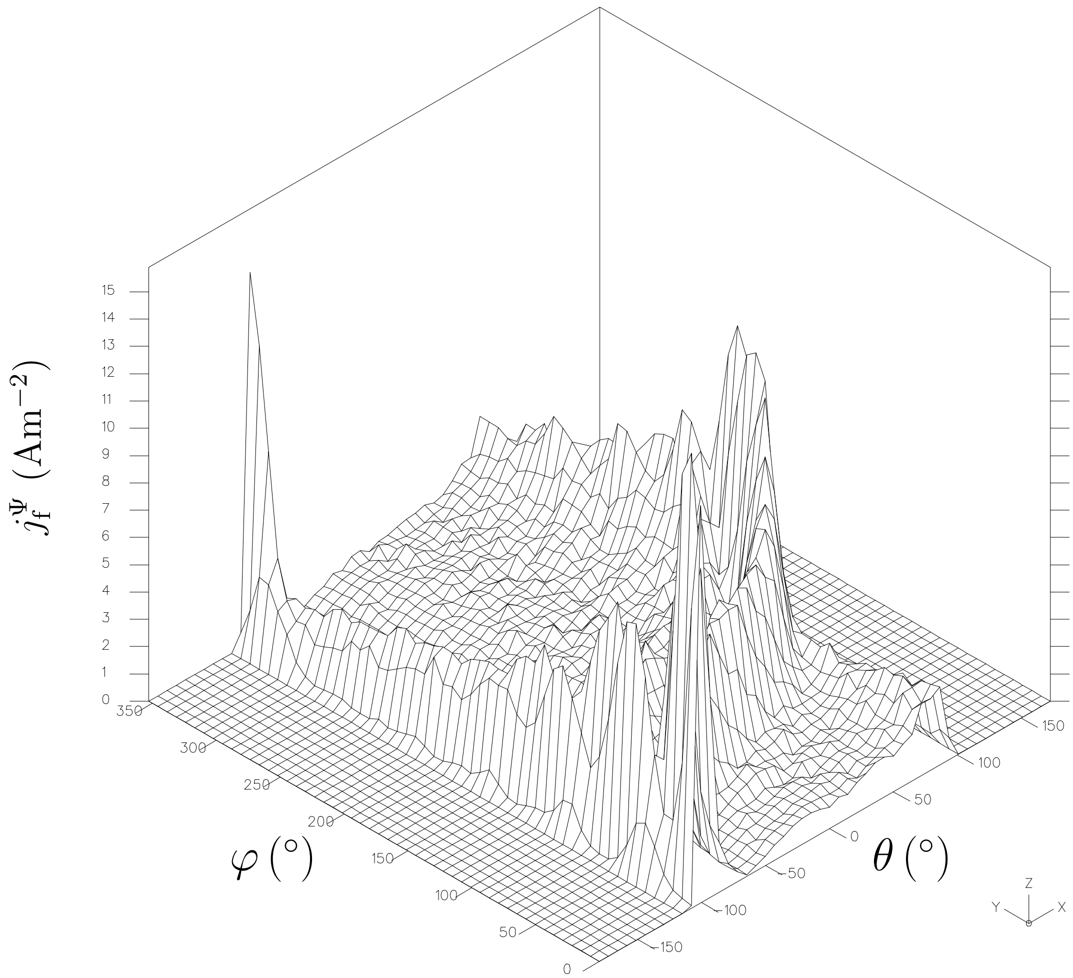
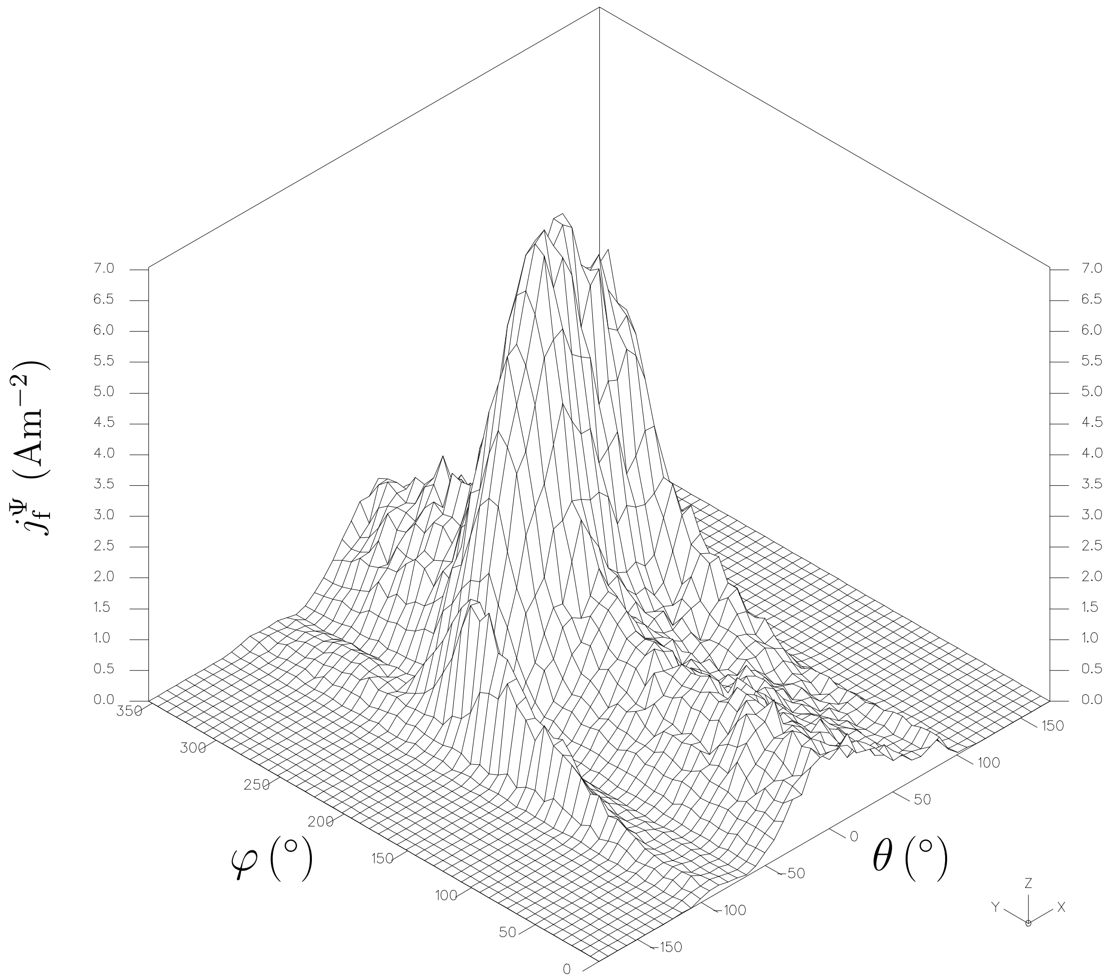


j_f^{Ψ} (Am^{-2})
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
$\varphi\,(^{\circ})$
350
300
250
200
150
100
50
0
$\theta\,(^{\circ})$
150
100
50
0
-50
-100
-150
Z
Y
X

Figure 2: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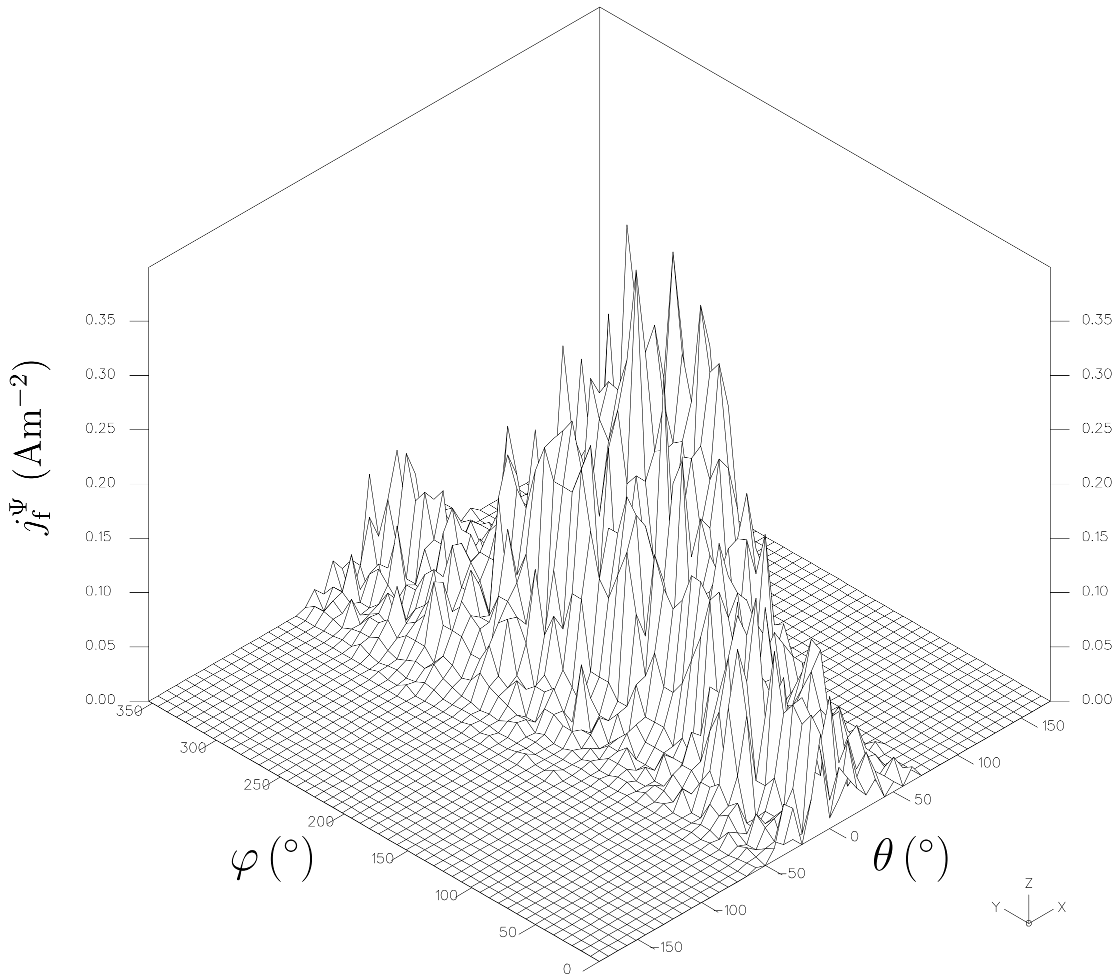
6.5
6.0
5.5
5.0
4.5
4.0
3.5
3.0
2.5
2.0
1.5
1.0
0.5
0.0
j_f^Ψ (Am^{-2})
350
300
250
200
150
100
50
0
$\varphi\,(^\circ)$
150
100
50
0
-50
-100
-150
$\theta\,(^\circ)$
Z
Y
X

Figure 3:



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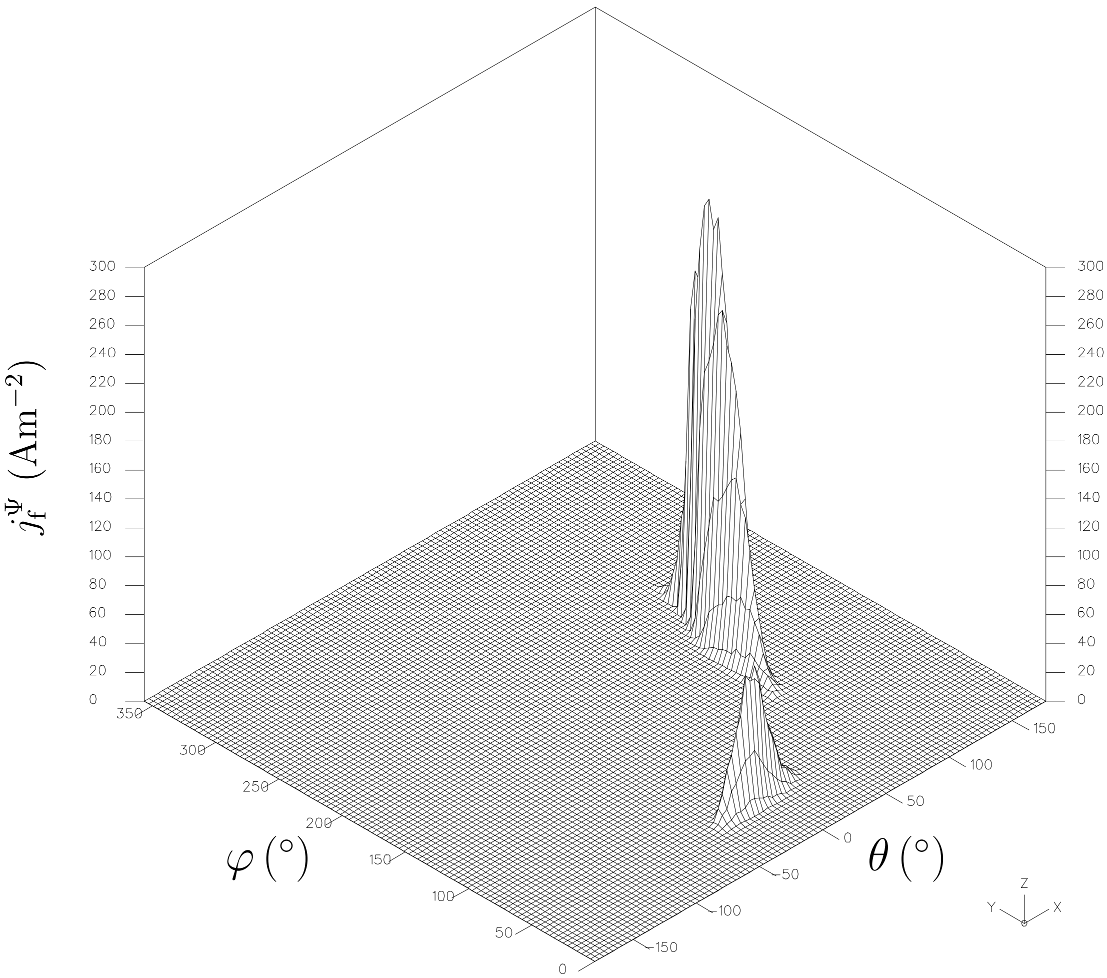
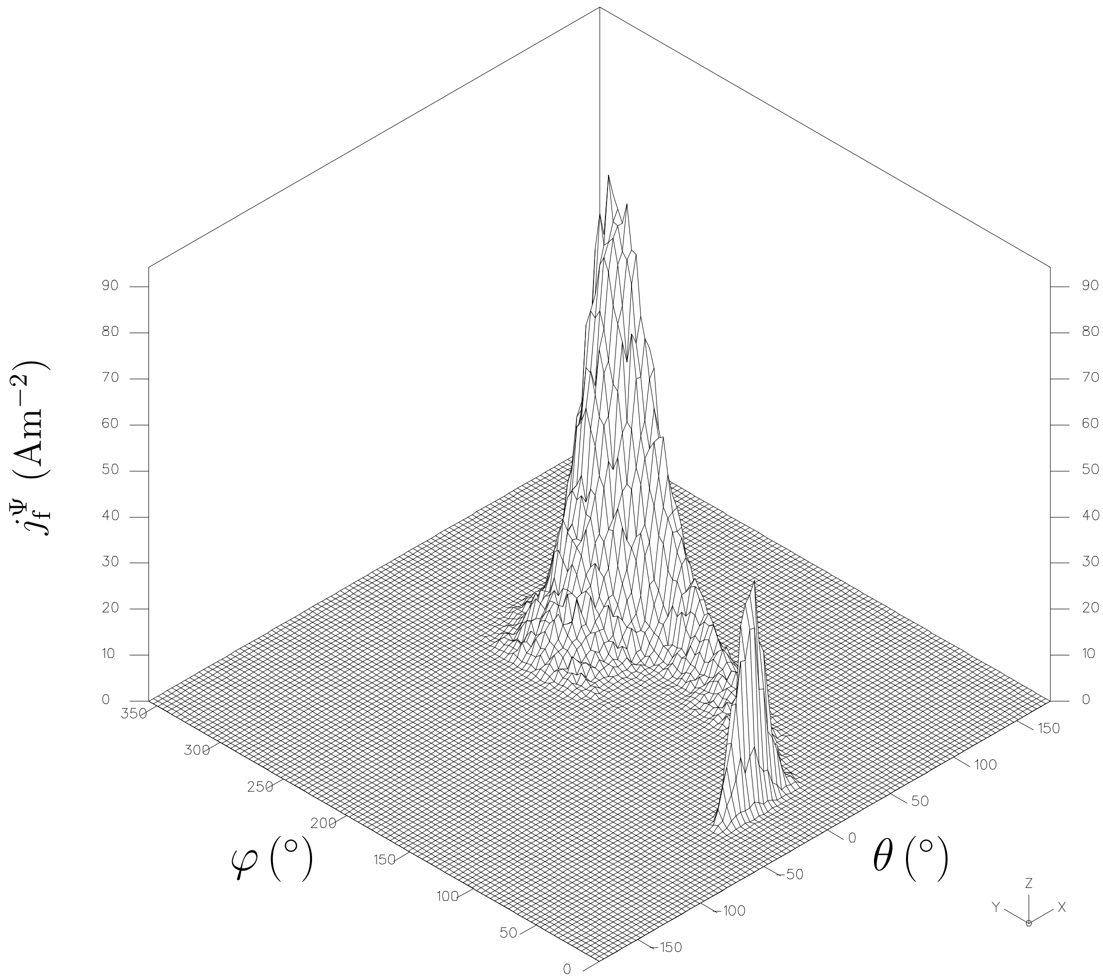
0.30
0.25
0.20
0.15
0.10
0.05
0.00
j_f^Ψ (Am^{-2})
350
300
250
200
150
100
50
0
$\varphi\,(^\circ)$
150
100
50
0
-50
-100
-150
$\theta\,(^\circ)$
Z
Y
X

Figure 4:



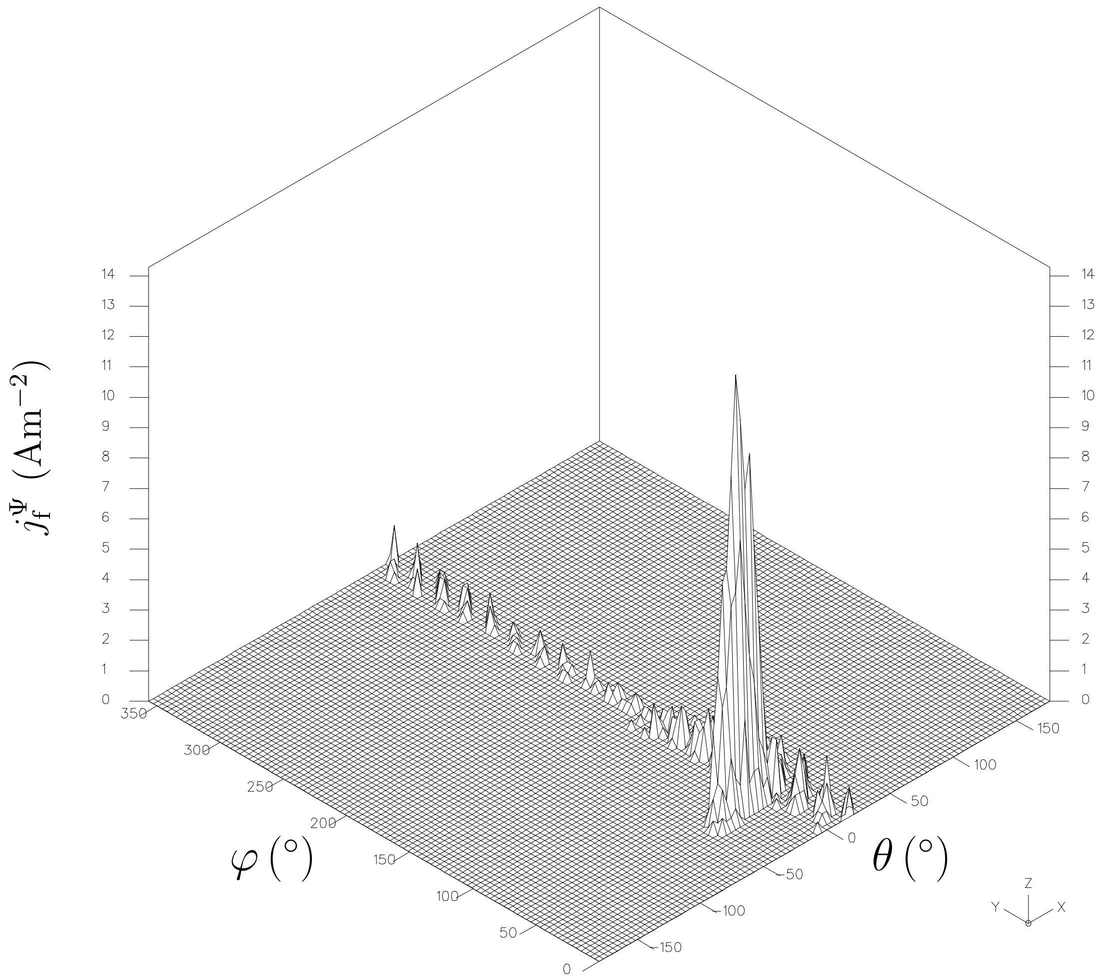
j_f^Ψ (Am^{-2})
0
20
40
60
80
100
120
140
160
180
200
220
240
260
280
300
φ (°)
0
50
100
150
200
250
300
350
θ (°)
-150
-100
-50
0
50
100
150
Z
Y
X

Figure 5:



j_f^Ψ (Am^{-2})
90
80
70
60
50
40
30
20
10
0
350
300
250
200
150
100
50
0
φ (°)
θ (°)
150
100
50
0
-50
-100
-150
Z
Y
X

Figure 6:



j_f^{Ψ} (Am^{-2})
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
φ (°)
350
300
250
200
150
100
50
0
θ (°)
150
100
50
0
-50
-100
-150
Z
Y
X

Figure 7: