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# ELECTROMAGNETIC INSTABILITIES IN HIGH BETA TOKAMAKS

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# THE IMPORTANCE OF PARALLEL MAGNETIC FIELD FLUCTUATIONS FOR ELECTROMAGNETIC INSTABILITIES IN STEP

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**Abstract**

The importance of parallel magnetic field perturbations in gyrokinetic simulations of electromagnetic instabilities and turbulence at mid-radius in the burning plasma phase of the conceptual high-$\beta$, reactor-scale, tight-aspect-ratio tokamak STEP is discussed. Previous studies have revealed the presence of unstable *hybrid* kinetic ballooning modes (KBMs) and subdominant microtearing modes (MTMs) at binormal scales approaching the ion-Larmor radius. Crucially, it was found that the *hybrid* kinetic ballooning mode requires the inclusion of parallel magnetic field perturbations for instability. Here, the extent to which the inclusion of parallel magnetic field perturbations can be relaxed is explored through gyrokinetic simulations. In particular, the frequently used MHD-approximation (setting the $\nabla B$ drift frequency equal to the curvature drift frequency) is discussed and simulations explore whether this approximation is useful for modelling STEP plasmas. If it were valid for STEP, the MHD-approximation would facilitate higher fidelity analysis using present day tools and models. It is shown that the one implementation of the MHD-approximation can reproduce some of the linear properties of the full STEP gyrokinetic system, but nonlinear simulations using the MHD-approximation result in a very different transport states. Unstable modes with very long binormal wavelengths (which are stable when the MHD-approximation is used) are identified as being responsible for this difference.

## 1. INTRODUCTION

Understanding and predicting turbulent transport in next-generation spherical tokamaks (STs) is critical for the optimisation of their performance. The UK STEP programme aims to generate net electric power $P_{\mathrm{el}} > 100\mathrm{MW}$ from fusion [1], by developing a compact prototype power plant, STEP, based on the ST concept. The first phase of this ambitious programme is to provide a conceptual design of a STEP prototype plant and a reference plasma equilibrium.

Modelling turbulent transport and optimising the plasma scenario in devices such as STEP is a complex challenge due to the higher $\beta$ accessible to STs, where $\beta = 2\mu_0 p/B^2$ is the ratio of the total plasma pressure $p$ to the magnetic field energy $B^2/(2\mu_0)$, with $B$ the magnetic field strength and $\mu_0$ the permeability of free space. High-$\beta$ equilibria can be unstable to microinstabilities which are electromagnetic in character, such as kinetic ballooning modes (KBMs) and microtearing modes (MTMs). These electromagnetic modes, less well understood than their electrostatic counterparts such as ion-temperature-gradient (ITG) driven modes and trapped-electron driven modes

(TEMs), are often not captured in the most advanced reduced core transport models and are thus difficult to model in STEP. Fortunately, all microinstabilities share broad characteristics which are well described by local linearised gyrokinetics provided that $k_\perp \rho_s \sim 1$ and $\rho_{\star s} \ll 1$, where $\rho_{\star s}$ is the ratio of the species thermal gyroradius $\rho_s$ to a typical equilibrium length scale $a$ and $k_\perp$ is the perpendicular wavelength of the instability. Thus, we can exploit high fidelity gyrokinetic (GK) simulations to try to assess microinstability and predict the turbulent transport that might be expected in designs proposed for conceptual reactors like STEP.

The first gyrokinetic analyses of the preferred flat-top operating point in STEP have revealed the presence of unstable *hybrid*-KBMs and subdominant MTMs at binormal scales approaching the ion-Larmor radius [2, 3]. In [3] it was shown that nonlinear gyrokinetic simulations suggest the existence of a transport steady state for a local equilibrium at mid-radius in the STEP-EC-HD flat-top operating point if equilibrium flow shear and/or $\beta'$ (the radial derivative of $\beta$) stabilisation are sufficient. However, it was left unclear as to whether a viable route to accessing such a burning flat top can be found, due in part to the nature of the turbulence at lower $\beta'$ en route to the flat top. At lower $\beta'$, and in the absence of developed equilibrium flow shear, *hybrid*-KBMs were found to drive very large turbulent transport in all channels. The local gyrokinetic simulations performed in [3] found that the *hybrid*-KBM can drive heat fluxes that exceed the available heating power by orders of magnitude. This state of large transport is characterised by turbulent eddies that are highly extended radially meaning that turbulence driven by these *hybrid*-KBMs may not be well described by the local gyrokinetic model.

In this paper, we concern ourselves with these *hybrid*-KBMs and their sensitivity to the inclusion of parallel magnetic field perturbations $\delta B_\parallel$. In [2] (see also discussion in Section 2 of this paper), it was found that including $\delta B_\parallel$ was essential for capturing this instability. However, $\delta B_\parallel$ physics is often simply missing from many gyrokinetic codes and modelling tools. Instead, in gyrokinetic theory and simulations of microinstabilities, it is common practice to neglect the parallel magnetic perturbation $\delta B_\parallel$ and to compensate for this by modifying the magnetic drift velocity (see Section 3.1). In this work, we will refer to this compensation as the MHD-approximation (see e.g. [4, 5, 6]). The remainder of this work is structured as follows. We begin in Section 2 by introducing the STEP equilibria and the associated plasma parameters (the details can be found elsewhere [2, 3]). We also demonstrate the sensitivity of the *hybrid*-KBM to $\delta B_\parallel$ and discuss the motivation for using the MHD-approximation. In Section 3, we give a very brief review of electromagnetic $\delta f$ gyrokinetics and discuss how the MHD-approximation fits into this framework and how the approximation is implemented in gyrokinetic codes. In Section 4, we perform linear and nonlinear gyrokinetic simulations with and without the MHD-approximation and explore to what extent the approximation is a suitable model for STEP. Finally, we present our conclusions in Section 5.

## 2. THE HYBRID-KBM AND PARALLEL MAGNETIC FIELD PERTURBATIONS

Important to this current work, and to understanding the gyrokinetic simulations in [2, 3], is the *hybrid* nature of the KBM identified in STEP. In particular, we are interested in the properties of this mode, identified in Section 4 of [2], which distinguish it from other KBMs described in the literature (see e.g. [7] and references therein). Of most interest to us is that the mode requires access to $\delta B_\parallel$ drive to be unstable. The precise meaning of this statement, and the importance of $\delta B_\parallel$, can be seen from numerical results using the $\delta f$ gyrokinetic code GENE [8].

### 2.1. LINEAR GYROKINETIC SIMULATIONS OF THE HYBRID KBM

In this work, we focus on a single equilibrium flux-surface taken from close to mid-radius ($q = 3.5, \Psi_n = 0.49$) in a conceptual STEP flat-top operating point plasma, which we refer to as STEP-EC-HD-v5 (hereinafter STEP-EC-HD). A Miller parameterisation [9] was used to model the local plasma equilibrium, and the shaping parameters were fitted to the chosen surface using **Pyrokinetics** [10], a Python library developed to facilitate pre- and post-processing of gyrokinetic analysis performed using a range of different GK codes. Table 1 provides the values of various local equilibrium quantities at the flux surface examined in this paper, including magnetic shear, $\hat{s}$; safety factor, $q$; normalised minor radius, $\rho/a$; elongation and its radial derivative, $\kappa$ and $\kappa'$; triangularity and its radial derivative, $\delta$ and $\delta'$; the radial derivative of the Shafranov shift, $\Delta'$; and the normalised inverse density and temperature gradient scale lengths of species $s$, $a/L_{ns}$ and $a/L_{Ts}$ respectively. Included also is the binormal wavenumber $k_y^{n=1}\rho_D$ corresponding to the toroidal mode number $n = 1$. Our simulations evolve three species, electrons, deuterium, and tritium and neglect entirely any impact of impurities or fast particles. The interested reader is referred to [2, 3] for more details on the equilibrium and on the setup of the computational grids, which are identical to those used in the aforementioned works.

Previous linear analysis shows that the *hybrid*-KBM is the dominant ion-scale instability on this surface, with a subdominant MTM also found to be unstable on a subset of these binormal scales (see [2], Figure 19 and Figure

TABLE 1. LOCAL PARAMETERS AT THE STEP MID-RADIUS FLUX SURFACE CONSIDERED IN THIS PAPER ($q = 3.5$

| Parameter | Value | Parameter | Value |
|---|---|---|---|
| $\Psi_n$ | 0.49 | $\rho/a$ | 0.64 |
| $q$ | 3.5 | $\hat{s}$ | 1.2 |
| $\beta$ | 0.09 | $\beta'$ | -0.48 |
| $\kappa$ | 2.56 | $\kappa'$ | 0.06 |
| $\delta$ | 0.29 | $\delta'$ | 0.46 |
| $k_y^{n=1}\rho_D$ | 0.0047 | $\Delta'$ | -0.40 |
| $a/L_{n_e}$ | 1.03 | $a/L_{n_e}$ | 1.03 |
| $a/L_{n_D}$ | 1.06 | $a/L_{T_D}$ | 1.82 |
| $a/L_{n_T}$ | 0.99 | $a/L_{T_e}$ | 1.58 |

20). No unstable microinstabilities are observed at the electron Larmor radius scale. The dominant *hybrid*-KBM and the subdominant MTM can both be recovered physically; that is, one can recover the subdominant mode by either forcing the parity of the perturbed distribution function in an initial value calculation or by using an eigenvalue solver to return the unstable linear spectrum. However, importantly for our work, it was also shown that it is possible to recover the subdominant mode simply by artificially suppressing $\delta B_{\|}$ (thus stabilising the *hybrid*-KBM).

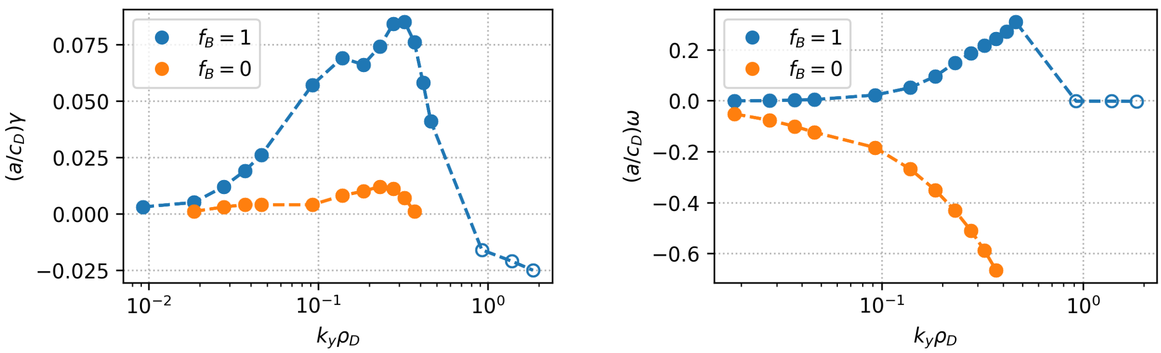


*FIG. 1. Growth rate (left) and mode frequency (right) as functions of the binormal wavenumber from linear simulations of the dominant instability in STEP-EC-HD on a mid-radius flux surface. Simulations are shown both with, $f_B = 1$ (blue), and without, $f_B = 0$ (orange), $\delta B_{\|}$. The two simulations are otherwise identical. This figure is adapted from [2].*

Fig 1 shows the linear growth rate and frequency (normalised to the deuterium sound speed, $c_D = \sqrt{T_e/m_d}$, divided by the minor radius of the last closed flux surface) as functions of the binormal wavenumber $k_y\rho_D = n\rho_{*D} d\rho/d\Psi_n$. Simulations are shown both including, $f_B = 1$ (blue), and neglecting, $f_B = 0$ (orange), $\delta B_{\|}$ fluctuations. The simulations are otherwise identical. Importantly, we see from Fig 1 that if $\delta B_{\|}$ is artificially excluded from calculations (this is routinely assumed in a number codes and modelling tools) then we recover the previously subdominant MTM (note the change in frequency) as the fastest growing unstable mode in the system. Succinctly, the *hybrid*-KBM is linearly stable on this surface (along with many others in the STEP flat top) without $\delta B_{\|}$.

## 2.2. THE MHD-APPROXIMATION FOR THE MAGNETIC DRIFT FREQUENCY

In gyrokinetic theory and simulations of microinstabilities, it is common to neglect the (typically destabilising) parallel magnetic perturbation $\delta B_{\|}$ at low $\beta$ and to erase the typically stabilising pressure gradient contribution to the $\nabla B$ drift by either removing it entirely or by correcting the $\nabla B$ drift by a term proportional to $\nabla p$ so that it is equal in magnitude to the curvature drift and points in the same direction as the curvature drift. Indeed, this latter treatment of the gyrokinetic drifts, hereinafter referred to as the MHD-approximation[1], is the recommended setting

[1]Later, in Section 3.1, it will be convenient to describe two separate "MHD-approximations", based on the two different prescriptions of the magnetic drift velocity described above, which we will refer to as "MHD-1" and "MHD-2".

in many gyrokinetic codes when parallel magnetic perturbations are neglected. The merits, or lack thereof, of the MHD-approximation have been discussed in different gyrokinetic simulations by various authors [11]. Whilst proper treatment of $\delta B_{\|}$ physics is often stressed, it should be noted that the cancellation of $\delta B_{\|}$ and $\nabla p$ is usually very good when calculating the growth rates of KBMs (and becomes better at longer wavelengths).

### 2.3. MOTIVATION FOR USING THE MHD-APPROXIMATION IN STEP

The extreme sensitivity of the *hybrid*-KBM mode to $\delta B_{\|}$ poses difficulties for exploiting existing codes in modelling for STEP. If valid, use of the MHD-approximation would be very attractive, for two key reasons.

*Global gyrokinetic codes often neglect* $\delta B_{\|}$. Testing the potential failure of the local gyrokinetic approximation is of high priority for the design of STEP and progress towards this goal necessitates the use of global gyrokinetic simulations which include the effects of profile variation and provide a promising avenue for avoiding the box-scale streamers observed in local simulations [3]. However, most global-capable codes simply do not include $\delta B_{\|}$ at present.

*Integrated modelling tools often neglect* $\delta B_{\|}$. Quasilinear turbulence models such as TGLF [12] form an integral part of modern turbulence prediction and integrated modelling efforts, due to the rapidity with which they can estimate turbulent transport. This is achieved by combining the properties of the linearly unstable modes with a saturation rule, which prescribes the shape of the saturated potential spectrum of the turbulence against binormal wavenumber. As such, these reduced models rely on both a saturation rule and accurate calculation of the linear physics for which capturing $\delta B_{\|}$ effects appears to be essential.

The remainder of the paper is devoted to exploring whether the MHD-approximation is appropriate for use in STEP plasmas.

## 3. LINEAR ELECTROMAGNETIC GYROKINETICS AND THE MHD-APPROXIMATION

We are interested in plasmas that are well described by the gyrokinetic framework (see e.g., [13]): that is, we are concerned with fluctuations, having characteristic frequency $\omega$ and wavenumbers $k_{\|}$ and $k_{\perp}$ parallel and perpendicular to the equilibrium magnetic field direction $\mathbf{b}_0 = \mathbf{B}_0/B_0$, that satisfy the standard gyrokinetic ordering $\omega/\Omega_s \sim \nu_{ss'}/\Omega_s \sim k_{\|}/k_{\perp} \sim q_s\phi/T_{0s} \sim \delta B_{\|}/B_0 \sim \delta\mathbf{B}_{\perp}/B_0 \sim \rho/a \equiv \rho_{\star,s} \ll 1$, where $\Omega_s = q_s B_0/m_s$ is the cyclotron frequency of species $s$ with charge $q_s$, equilibrium density and temperature $n_{0s}$ and $T_{0s}$, respectively, mass $m_s$ and thermal speed $v_{\mathrm{th}s} = \sqrt{2T_s/m_s}$, $\nu_{ss'}$ is the typical collision frequency, $\rho_s$ is the thermal Larmor radius, and $a$ is a typical equilibrium length scale. The size of electrostatic perturbations is set by the perturbed electrostatic potential $\phi$ and electromagnetic perturbations enter gyrokinetics through $\delta B_{\|}$ and $\delta B_{\perp}$, the fluctuations of the magnetic field parallel and perpendicular to the equilibrium direction. Electromagnetic effects are most conveniently described in local gyrokinetics by writing the fluctuating magnetic field $\delta\mathbf{B} = \nabla\times(\delta\mathbf{A}_{\perp} + \delta A_{\|}\mathbf{b}) \simeq \nabla\times\delta\mathbf{A}_{\perp} + \nabla\delta A_{\|}\times\mathbf{b}$, then relating its parallel component to the fluctuating vector potential $\mathbf{A}$ by $\mathbf{b}\cdot\delta\mathbf{B} \equiv \delta B_{\|} \simeq \mathbf{b}\cdot\nabla\times\delta\mathbf{A}_{\perp}$.

It is convenient to write the gyrokinetic distribution function in the form $f_s = F_{0s}\left(1 - q_s\phi/T_s\right) + g_s = F_{0s} + \delta f_s,\ \delta f_s = -(q_s\phi/T_s)F_{0s} + g_s$. Here, the gyrokinetic distribution function consists of a (Maxwellian) piece $F_{0s}$, and an order $\rho_{\star s}$ small perturbation $\delta f_s$, with $g_s$ being the non-adiabatic part of $\delta f_s$. Under the above ordering, the collisionless, linear, gyrokinetic equation is given in Fourier space by

$$(\omega - \omega_{ds} - k_{\|}v_{\|})g_s = \frac{q_s F_{0s}}{T_s}(\omega - \omega_{\star s}^T)J_0(b_s)\left[\delta\phi - v_{\|}\delta A_{\|} + \frac{m_s v_{\perp}^2}{q_s B}\frac{2J_1(b_s)}{b_s J_0(b_s)}\delta B_{\|}\right]. \quad (1)$$

where $b_s = k_{\perp}v_{\perp}/\Omega_s$, and $J_0$ and $J_1$ are the Bessel functions of the first kind which arise during the gyroaveraging due to finite-Larmor-radius effects. The magnetic drift frequency is given by

$$\omega_{ds} = \frac{1}{\Omega_s}\left(\omega_{\kappa}v_{\|}^2 + \omega_{\nabla B}\frac{v_{\perp}^2}{2}\right) \quad (2)$$

where we have also introduced the $\nabla B$ drift frequency coefficient $\omega_{\nabla B} = \mathbf{k}\cdot\mathbf{b}\times\nabla B/B$, and the curvature drift frequency coefficient $\omega_{\kappa} = \mathbf{k}\cdot\mathbf{b}\times(\mathbf{b}\cdot\nabla\mathbf{b})$. Other notation used in Equations 1 and 2 is the drive frequency $\omega_{\star s}^T = \omega_{\star s}\left[1 + \eta_s\left(v^2/v_{\mathrm{th}s}^2 - 3/2\right)\right]$, where the diamagnetic frequency $\omega_{\star} = (k_{\perp}T_s/q_s B)\mathrm{d}\ln n_s/\mathrm{d}r$, and we have defined $\eta_s = \mathrm{d}\ln T_s/\mathrm{d}n_s$, where $r$ is the radial coordinate, $\psi$ is the poloidal flux, $\alpha = \xi - q(\psi)\theta - \nu(\psi,\theta)$ the dimensionless binormal coordinate, $\xi$ the toroidal angle, $\nu(\psi,\theta)$ a periodic function of $\theta$ determined by flux-surface shaping, and $k_{\alpha}$ the dimensionless binormal wave number.

The fluctuating field quantities appearing in the gyrokinetic equation are determined through the field equations. The perturbed electrostatic potential $\phi$ is determined through quasineutrality

$$\sum_s \frac{n_s q_s^2}{T_s}\phi = \sum_s q_s \int \mathrm{d}^3\mathbf{v}\, J_0(b_s) g_s. \tag{3}$$

The parallel magnetic vector potential $A_\parallel$ is determined by parallel Ampère's law

$$k_\perp^2 A_\parallel = \mu_0 \sum_s q_s \int \mathrm{d}^3\mathbf{v}\, v_\parallel J_0(b_s) g_s. \tag{4}$$

The magnetic fluctuation $B_\parallel$ is determined by perpendicular Ampère's law

$$\delta B_\parallel = -\frac{\mu_0}{B}\sum_s \delta p_\perp,\ \delta p_\perp = \sum_s \int \mathrm{d}\mathbf{v}^3\ \frac{1}{2} m v_\perp^2 g_s. \tag{5}$$

### 3.1. THE MHD-APPROXIMATION IN GYROKINETICS

The concept currently under scrutiny is that the removal of $\delta B_\parallel$ can be compensated for by adding a pressure gradient contribution to the $\nabla B$ drift (setting the $\nabla B$ drift equal to the curvature drift). The magnetic drift velocity is given by

$$\mathbf{v}_d = \mathbf{v}_\kappa + \mathbf{v}_{\nabla B} = \frac{v_\parallel^2}{\Omega}(\nabla\times\mathbf{b}) + \frac{v_\perp^2}{2\Omega}\mathbf{b}\times\nabla\ln B. \tag{6}$$

Starting from the perpendicular force balance of the gyrokinetic equilibrium (see e.g. equation (128) of [13]) it is simple to show that the drift velocity is connected to the pressure gradient viz

$$\mathbf{v}_d = \mathbf{b}\times\left[\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\frac{\nabla B}{B} + v_\parallel^2 \frac{\mu_0}{B^2}\nabla p\right]\Big/\ \Omega_s, \tag{7}$$

or equivalently

$$\mathbf{v}_d = \mathbf{b}\times\left[\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\mathbf{b}\cdot\nabla\mathbf{b} v_\perp^2 \frac{\mu_0}{2B^2}\nabla p\right]\Big/\ \Omega_s. \tag{8}$$

The historical precedent for the MHD-approximation in gyrokinetics comes from a result of [5] showing that, in the one fluid MHD limit ($k_y\rho_D \to 0$), there is a cancellation of terms such that $\delta B_\parallel$ can be dropped as long as the $\nabla B$ drift is corrected by a term proportional to $\nabla p$ so that it is equal in magnitude to the curvature drift and points in the same direction (hereinafter referred to as "MHD-1"):

$$\mathbf{v}_{\kappa+\nabla B} = \mathbf{b}\times\left[\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\frac{\nabla B}{B} + v_\parallel^2 \frac{4\pi}{B^2}\nabla p + \frac{v_\perp^2}{2}\frac{4\pi}{B^2}\nabla p\right]\Big/\ \Omega_s. \tag{9}$$

We remark that this approximation, the recommended setting in GENE when $\delta B_\parallel$ is neglected, is different to the approximation implemented in [6]. Instead, [6] argues that the $\nabla p$ contribution to the magnetic drift should be erased so that the terms proportional $\nabla p$ are removed from equation 10 (hereinafter referred to as "MHD-2"):

$$\mathbf{v}_{\kappa+\nabla B} = \mathbf{b}\times\left[\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\frac{\nabla B}{B}\right]\Big/\ \Omega_s. \tag{10}$$

We have identified two different MHD-approximations (MHD-1 and MHD-2). In this work, we use GENE which is capable of running with either the MHD-1 approximation ($\mathbf{v}_d$ given by equation 9) or the MHD-2 approximation ($\mathbf{v}_d$ given by equation 10) used in [6].

### 3.2. PHYSICAL INSIGHT INTO THE MHD-APPROXIMATION

We can gain some physical insight into the approximations above from equation 5 (which follows from first order force balance) where it can be seen that $\delta B_\parallel$ physics is needed to sustain a perpendicular pressure perturbation. In order to drop $\delta B_\parallel$ in the gyrokinetic equation, for consistency $\delta p_\perp$ must also be negligible. The perturbed perpendicular pressure can be calculated by solving the linear gyrokinetic equation 1 in the limit $\omega_{d,s}, k_\parallel v_\parallel \ll \omega$ and substituting into equation 5 to give

$$\delta p_\perp = \sum_s \int \mathrm{d}\mathbf{v}^3\, \frac{m_s q_s v_\perp^2}{2T_s}\left[1 - \frac{\omega_{\star s}^T}{\omega} + \frac{\omega_{d,s}}{\omega}\right]\left[\phi - v_\parallel A_\parallel + \frac{m_s v_\perp^2}{q_s B}B_\parallel\right] \tag{11}$$

Using equation 5 and evaluating the terms on the right-hand-side of this equation gives the consistency condition

$$\frac{2}{\beta}(\omega_\kappa - \omega_{\nabla B}) = \omega_\kappa + 2\omega_{\nabla B}. \quad (12)$$

We now simply note that $\omega_{\nabla B} = \omega_\kappa$ is a solution to equation 12 in the limit $\beta \ll 1$. That is, we can think of either of the approximations (MHD-1 and MHD-2), both of which set $\omega_\kappa = \omega_{\nabla B}$ albeit by different prescriptions of the magnetic drift velocity, as simply being attempts to maintain a pressure that is consistent with the plasma equilibrium in the absence of $\delta B_{\|}$.

## 4. GYROKINETIC SIMULATIONS WITH THE MHD-APPROXIMATION

Having introduced two possible approximations: (i) MHD-1 ($\mathbf{v}_d$ given by equation 9); and (ii) MHD-2 ($\mathbf{v}_d$ given by equation 10), we now ask whether either of these approximations are appropriate for modelling STEP plasmas.

### 4.1. IS THE MHD-APPROXIMATION APPROPRIATE FOR CAPTURING LINEAR PHYSICS IN STEP?

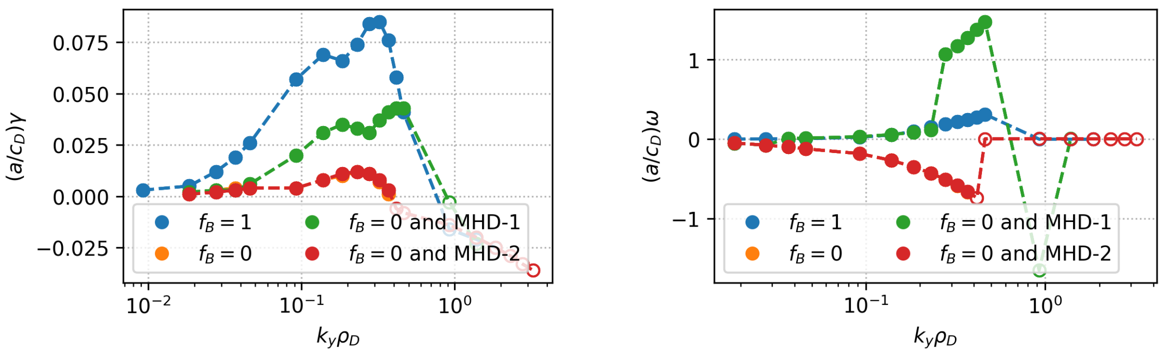


*FIG. 2. Growth rate (left) and mode frequency (right) as functions of the binormal wavenumber from linear simulations of the dominant instability in STEP-EC-HD on a mid-radius flux surface. Simulations are shown both with, $f_B = 1$ (blue), and without, $f_B = 0$ (orange, green, and red), $\delta B_{\|}$. For the simulations without $\delta B_{\|}$, different treatments of the drift velocity are shown: (i) the full drift velocity ($\mathbf{v}_d$ given by equation 8 (orange)); (ii) MHD-1 ($\mathbf{v}_d$ given by equation 9 (green)); and MHD-2 ($\mathbf{v}_d$ given by equation 10) used in [6].*

Fig 2 shows the linear growth rate (left) and frequency (right) as functions of the binormal wavenumber for simulations with ($f_B = 1$ (blue)) and without ($f_B = 0$ (orange)) $\delta B_{\|}$ and using different prescriptions of the magnetic drift velocity. If the pressure gradient is completely erased from the drift velocity (MHD-2, red) then we see that the *hybrid*-KBM is linearly stable and we once again recover the previously subdominant MTM (note the change in frequency) as the fastest growing unstable mode in the system (red). However, using the MHD-1 approximation (setting the $\nabla B$ drift parallel to the curvature drift (green)) **does** find the *hybrid*-KBM mode but with a strongly reduced growth rate. These results indicate that the MHD-1 approximation is the more appropriate treatment of the magnetic drift when $\delta B_{\|}$ is neglected in STEP plasmas. However, we note that even though this approximation recovers the *hybrid*-KBM, accurately capturing this mode clearly requires a proper treatment of $\delta B_{\|}$ and the full expression for the magnetic drifts (particularly at low $k_y\rho_D$).

### 4.2. IS THE MHD-APPROXIMATION APPROPRIATE FOR CAPTURING NONLINEAR PHYSICS IN STEP?

The MHD-1 approximation results in the fastest growing linear instability being the *hybrid*-KBM (albeit with a reduced growth rate). This result suggests that it might be possible to simulate *hybrid*-KBM driven turbulence [3] using this version of the MHD approximation in a global code. As a first step towards this goal, we first perform a *local* nonlinear simulation using the MHD-1 approximation.

Fig 3 shows time traces of the total heat flux from two nonlinear GENE simulations. The blue curve in Fig 3 includes $\delta B_{\|}$ and uses equation 8 for the drift velocity (this simulation is identical to that shown in Fig 3a of [3]). The orange curve in Fig 3 does not include $\delta B_{\|}$ and instead uses the MHD-1 approximation. The simulation using the MHD-1 approximation appears to achieve a robustly-steady saturated state at values of the heat flux

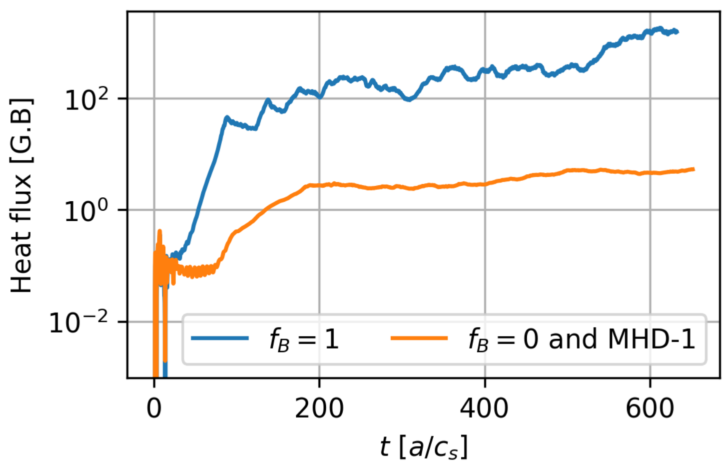


*FIG. 3. Time trace from GENE simulations of the total heat flux. Simulations are shown both with, $f_B = 1$ (blue), and without, $f_B = 0$ (orange), $\delta B_{||}$. The case without $\delta B_{||}$ fluctuations uses the MHD-1 approximation with $\mathbf{v}_d$ given by equation 9.*

around two orders of magnitude smaller than those reached by the simulation that includes $\delta B_{||}$. However, it is also important to remark that the heat flux predicted using the MHD-1 approximation is still orders of magnitude larger than the heat flux driven by the subdominant MTM (see Fig 14 of [3]) which indicates that this turbulence is indeed still being driven by the *hybrid*-KBM.

The reason that we see saturation at much lower fluxes using the MHD-1 approximation in Fig 3 is revealed by closer inspection of Fig 2a; from this figure we can see that there are long wavelength modes which are unstable with $\delta B_{||}$ but are stable without $\delta B_{||}$ (even when the MHD-1 approximation is used). It is these unstable modes at $k_y \rho_D \ll 1$, rather than any physics specific to the *hybrid*-KBM, that are responsible for the lack of saturation typically encountered in nonlinear simulations of electromagnetic turbulence. To be explicit, it appears to be possible to find a saturated state even when *hybrid*-KBMs are unstable **provided** that they are not unstable up to the very long wavelengths.

## 5. CONCLUSIONS

In this paper, we have examined the necessity of parallel magnetic perturbations ($\delta B_{||}$) in gyrokinetic simulations of electromagnetic turbulence at mid-radius in the burning plasma phase of the conceptual high$-\beta$ reactor-scale, tight-aspect-ratio tokamak STEP. It has previously been reported [2] that $\delta B_{||}$ is essential for the *hybrid*-KBM identified in STEP to be linearly unstable. It was shown that the MHD-2 approximation, standard in many gyrokinetic codes (see [6]), finds the *hybrid*-KBM to be completely stable.

We find that implementing the MHD-1 approximation (compensating for the neglect of parallel magnetic perturbations by correcting the $\nabla B$ drift by a term proportional to $\nabla p$ so that it is equal in magnitude to the curvature drift and points in the same direction) was able to roughly capture the linear spectrum of the *hybrid*-KBM. While the validity of the MHD-1 approximation for STEP plasmas may be questionable, it may still have some value in integrated modelling codes in that it at least partly captures the hybrid-KBM growth rate spectrum.

However, we find that nonlinear simulations using the MHD-1 approximation do not accurately capture the nonlinear physics expected in STEP [3]. Typically, simulations which attempt to resolve the *hybrid*-KBM with full physics (including $\delta B_{||}$) yields fluxes rising to very large values with no robustly-steady saturation period over the time simulated (any state of saturation is always lost if the simulation is run for a sufficiently long time). When the MHD-1 approximation is used, we find that the turbulence simulations saturate at a level around two orders of magnitude lower than that reached by the full physics simulations in comparable time. The reason for this difference is that there are modes in the system with very long wavelength (i.e., modes corresponding to $n < 10$) that are stable when the MHD-approximation is used but are unstable otherwise. It is likely that it is these unstable very long wavelength modes that are responsible for the lack of a saturated state, rather than anything intrinsically linked to the *hybrid*-KBM. Since the problematic modes at this surface are both: (i) unstable only with $\delta B_{||}$ and; (ii) unstable up to wavelengths where the local limit is questionable, this work highlights the importance and timeliness of global gyrokinetic codes with $\delta B_{||}$ solvers.

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