

UKAEA-STEP-PR(25)25

D. Kennedy, C. M. Roach, M. Giacomini, P. Ivanov, T.  
Adkins, F. Sheffield-Heit, T. Gorler A. Bokshi, D.  
Dickinson, H. Dudding, B. Patel

# **On the importance of parallel and perpendicular magnetic field fluctuations for electromagnetic instabilities in STEP**

Enquiries about copyright and reproduction should in the first instance be addressed to the UKAEA Publications Officer, Culham Science Centre, Building K1/O/83 Abingdon, Oxfordshire, OX14 3DB, UK. The United Kingdom Atomic Energy Authority is the copyright holder.

The contents of this document and all other UKAEA Preprints, Reports and Conference Papers are available to view online free at [scientific-publications.ukaea.uk/](https://scientific-publications.ukaea.uk/)

# **On the importance of parallel and perpendicular magnetic field fluctuations for electromagnetic instabilities in STEP**

D. Kennedy, C. M. Roach, M. Giacomini, P. Ivanov, T. Adkins, F. Sheffield-Heit, T. Gorler A. Bokshi, D. Dickinson, H. Dudding, B. Patel



# On the importance of parallel magnetic-field fluctuations for electromagnetic instabilities in STEP

D. Kennedy<sup>1</sup>, C. M. Roach<sup>1</sup>, M. Giacomini<sup>2,3</sup>, P. Ivanov<sup>4</sup>, T. Adkins<sup>5</sup>, F. Sheffield<sup>6</sup>, T. Görler<sup>6</sup>, A. Bokshi<sup>3</sup>, D. Dickinson<sup>3</sup>, H. G. Dudding<sup>1</sup>, and B. S. Patel<sup>1</sup>

<sup>1</sup>Culham Centre for Fusion Energy, Abingdon OX14 3DB, United Kingdom

<sup>2</sup>York Plasma Institute, University of York, York, YO10 5DD, United Kingdom

<sup>3</sup>Dipartimento di Fisica “G. Galilei”, Università degli Studi di Padova, Padova, Italy

<sup>4</sup>Rudolf Peirels Centre for Theoretical Physics, University of Oxford, OX1 3PU, UK

<sup>5</sup>Department of Physics, University of Otago, Dunedin, 9016, New Zealand

<sup>6</sup>Max Planck Institute for Plasma Physics, Boltzmannstr. 2, Garching, 85748, Germany

E-mail: daniel.kennedy@ukaea.uk

**Abstract.** This paper discusses the importance of parallel perturbations of the magnetic-field in gyrokinetic simulations of electromagnetic instabilities and turbulence at mid-radius in the burning plasma phase of the conceptual high- $\beta$ , reactor-scale, tight-aspect-ratio tokamak STEP. Previous studies have revealed the presence of unstable *hybrid* kinetic ballooning modes (hKBMs) and subdominant microtearing modes (MTMs) at binormal scales approaching the ion Larmor radius. It was found that the hKBM requires the inclusion of parallel magnetic-field perturbations to be linearly unstable. Here, the extent to which the inclusion of fluctuations in the parallel magnetic-field can be relaxed is explored through gyrokinetic simulations. In particular, the frequently used MHD approximation (dropping  $\delta B_{\parallel}$  and setting the  $\nabla B$  drift frequency equal to the curvature drift frequency) is discussed and simulations explore whether this approximation is useful for modelling STEP plasmas. It is shown that the MHD approximation can reproduce some of the linear properties of the full STEP gyrokinetic system, but nonlinear simulations using the MHD approximation result in very different transport states. It is demonstrated that it is difficult to improve the agreement even in conditions that appear to be more compatible with the assumptions that underlie the MHD approximation. Furthermore, it is shown that the sensitivity of STEP to  $\delta B_{\parallel}$  fluctuations is primarily because the plasma sits close to marginality and it is shown that in slightly more strongly driven conditions the hKBM is unstable without  $\delta B_{\parallel}$ . Crucially, it is demonstrated that the state of large transport typically predicted by local electromagnetic gyrokinetic simulations of STEP plasmas is not solely due to  $\delta B_{\parallel}$  physics.

## 1. Introduction

The performance of magnetic-confinement-fusion devices such as spherical tokamaks (STs) is often limited by the presence of turbulent fluctuations that typically dominate the transport losses of heat, particles, and momentum. Therefore, understanding and

predicting turbulent transport in next-generation STs is crucial for the optimisation of their performance. The UK STEP programme aims to generate net electric power  $P_{\text{el}} > 100$  MW from fusion [1, 2], by developing a compact prototype power plant, STEP, based on the ST concept. The first phase of this ambitious programme is to provide a conceptual design of a STEP prototype plant and reference plasma equilibria; in this context, a set of preferred STEP plasma scenarios are being designed [3], using simplified reduced models to model the core transport.

Reduced core-plasma transport models are essential for the integrated modelling of tokamak plasma scenarios, since first principles approaches are generally too computationally expensive to evaluate over confinement or resistive discharge timescales. Conventional-aspect-ratio devices tend to be dominated by electrostatic microturbulence driven by instabilities, such as modes driven by the ion-temperature gradient (ITG), modes driven by the electron-temperature gradient (ETG), and trapped-electron-driven modes (TEMs), and several reduced-complexity models have been developed to model electrostatic microturbulence (see e.g. [4, 5]). Such reduced models have been quite successful at predicting turbulent transport in conventional-aspect-ratio present-day devices at low-to-modest  $\beta$  [where  $\beta = 2\mu_0 p/B^2$  is the ratio of the total plasma pressure  $p$  to the magnetic-field energy density  $B^2/(2\mu_0)$ , with  $B$  the magnetic-field strength and  $\mu_0$  the permeability of free space], where the turbulence is predominantly electrostatic: see e.g. [6]. However, in STs, the confinement and turbulent transport, reviewed in [7], are both quantitatively and qualitatively different to that in conventional-aspect-ratio devices. ST geometry has favourable stability properties that allow access to higher- $\beta$  regimes where the turbulence is more electromagnetic in character and the dominant unstable modes including kinetic ballooning modes (KBM) and microtearing modes (MTMs). These electromagnetic modes, less well understood than their electrostatic counterparts, are not well captured in the most advanced reduced core transport models; transport from electromagnetic turbulence has thus not been reliably captured in scenario modelling for STEP.

Fortunately, most microinstabilities share broad characteristics which are well described by local $\ddagger$  linearised  $\delta f$  gyrokinetics (GK) provided that  $k_{\perp}\rho_s \sim 1$  and  $\rho_{*s} \equiv \rho_s/a \ll 1$  where  $\rho_s$  is the gyroradius of species  $s$ ,  $a$  is a typical equilibrium length scale, and  $k_{\perp}$  is the perpendicular wavenumber of the instability. High-fidelity GK simulations must be exploited: (i) to assess electromagnetic microinstabilities and their associated turbulence in the conceptual power-plant designs developed for STEP; and (ii) to improve the physics basis of reduced models used in scenario design.

In this paper, we concern ourselves with electromagnetic instabilities in STEP, in particular the hKBM [8] that GK simulations indicate to dominate turbulent transport [9] in the conceptual designs of STEP. The inclusion of compressional magnetic fluctuations  $\delta B_{\parallel}$  was found to be essential in [8] for the hKBM to be unstable in STEP

$\ddagger$  All simulations in this paper are local simulations; i.e. simulations are performed in a domain of perpendicular size that is infinitesimal in comparison with the length scale over which the equilibrium varies. Plasma equilibrium gradients are taken to be constant across the simulation domain.

(see also discussion in Section 4). However,  $\delta B_{\parallel}$  physics is often simply missing from many higher-fidelity (e.g. global, full- $f$ ) GK codes and modelling tools. Instead, in GK theory and simulations of microinstabilities, it is common practice to neglect  $\delta B_{\parallel}$ , and to compensate for this through an approximation by modifying the magnetic-drift velocity. In this work, we will refer to this compensation as the MHD approximation $\S$  (see e.g. [10–12]). The primary focus of this paper is to explore sensitivity of hKBM stability to the inclusion of  $\delta B_{\parallel}$  and the appropriateness of the MHD approximation in STEP plasmas.

The paper is organised as follows: Section 2 briefly reviews the results of [8] and [9] and outlines the motivation for using the MHD approximation in STEP plasmas. Following a brief review of electromagnetic  $\delta f$  gyrokinetics, Section 3 introduces how the MHD approximation fits into the GK framework and its implementation in GK codes. Section 4 introduces the STEP equilibria and associated plasma parameters (with more details available in [8, 9]) and presents linear GK simulations (using the GK code GENE [13]) to assess the validity of the MHD approximation in STEP. It is explored whether the performance of the MHD approximation can be improved in conditions where the approximation’s underlying assumptions are better satisfied. In Section 5 we demonstrate that the the MHD approximation is typically not appropriate for modelling electromagnetic turbulence in STEP plasmas and explain why this is the case. We again discuss cases where the MHD approximation can more faithfully model high- $\beta$  turbulence. Section 6 explains that  $\delta B_{\parallel}$  is only required for instability in STEP because the local equilibrium is close to marginal stability (owing to large values of  $\beta'$ ). We demonstrate that it is in fact possible to model electromagnetic turbulence driven by the hKBM in the absence of  $\delta B_{\parallel}$  in conditions further from marginal stability without making an approximation in the magnetic drifts. The final summary and conclusions are presented in Section 7. Additional auxiliary material is provided in Appendices, including a brief derivation of the MHD approximation from GK in Appendix A, a discussion of the why the MHD 1 approximation is more physically motivated than the MHD-2 approximation in Appendix B, a useful expression for the magnetic-drift velocity given in Appendix C, and turbulent snapshots in Appendix D from different simulations cited at various points in the main text.

## 2. The importance of parallel magnetic-field perturbations

Recent GK analyses for mid-radius of the preferred flat-top operating point, STEP-EC-HD [3], revealed the presence of unstable *hybrid* KBMs $\P$  (hKBM) and subdominant MTMs at binormal scales approaching the ion Larmor radius [8, 9] (note that similar

$\S$  In Section 3.2, it will be convenient to describe two separate ‘MHD approximations’, based on two different prescriptions of the magnetic-drift velocity. We will refer to these two different approximations as ‘MHD-1’ and ‘MHD-2’.

$\P$  In this work,  $\delta B_{\parallel}$  fluctuations are set to zero in **all** simulations using the MHD approximation.

$\P$  The term ‘*hybrid* KBM’ was introduced in [8] to describe the dominant instabilities seen in STEP plasmas; typically these modes share attributes associated with KBM, ITG, and TEM.

MTMs were reported for an earlier proposed variant of STEP [14]). Nonlinear local GK simulations for the same local equilibrium in [9] find that the hKBM turbulence can drive heat fluxes that exceed the available heating power by orders of magnitude in absence of equilibrium flow shear, and that this large transport state is characterised by highly radially extended turbulent eddies. Further simulations in [9] indicate that a transport steady state can be reached if equilibrium flow shear and/or  $\beta'$  stabilisation are sufficient at the flat-top to regulate the turbulence. At lower  $\beta'$  and/or in the absence of equilibrium flow shear, however, hKBMs drive very large turbulent transport in all channels that may challenge access through lower  $\beta'$  states to the burning flat top.

A natural extension to nonlinear simulations in [9] is to attempt higher-fidelity **global** gyrokinetic simulations for conditions where local GK predicts large turbulent fluxes.  $\delta B_{\parallel}$ , indicated by local GK to be essential for the hKBM to be unstable in STEP-EC-HD [8], is unfortunately neglected in most global GK codes. To progress towards a global description of hKBM-driven turbulence, we must both (i) implement  $\delta B_{\parallel}$  in a global-capable code;<sup>+</sup> and (ii) strive to better understand the sensitive dependence of the hybrid mode on  $\delta B_{\parallel}$ . This paper focuses on the latter.

### 2.1. The MHD approximation for the magnetic-drift velocity

As remarked in Section 1, in GK theory and simulations of microinstabilities it is common to neglect the (typically destabilising [17, 18])<sup>\*</sup> parallel magnetic perturbation  $\delta B_{\parallel}$  at low  $\beta$ , and to compensate for this by artificially adding a correction term proportional to  $\nabla p$  to the magnetic drift velocity (see e.g. [10, 12, 19] and references therein). This idea of modifying the magnetic drifts, hereinafter referred to as the MHD approximation, is recommended in many GK codes on the neglect of  $\delta B_{\parallel}$ . The merits, or lack thereof, of the MHD approximation have been tested in different GK simulations by various authors (see e.g. [17], [12]). Whilst proper treatment of  $\delta B_{\parallel}$  physics is often crucial [17], the cancellation of  $\delta B_{\parallel}$  and the  $\nabla p$  term in the  $\nabla B$  drift has been demonstrated to be a good approximation when calculating the growth rates of KBMs at low  $\beta$  and long wavelengths [12]. Here we explore whether this approximation may be appropriate for use in STEP plasmas<sup>‡</sup>.

To reiterate, the sensitivity of hKBM stability to  $\delta B_{\parallel}$  poses difficulties for exploiting existing codes for STEP. If the MHD approximation were valid for STEP, it would be useful for two key reasons:

<sup>+</sup> Exciting progress is being made in this direction (see e.g. [15]), and there are several parallel efforts working towards implementing  $\delta B_{\parallel}$  globally in both Eulerian codes such as **GENE** [13] and Particle-In-Cell codes such as **ORB5** [16].

<sup>\*</sup> It was demonstrated in [18] that neglecting  $\delta B_{\parallel}$  typically stabilises MHD-like modes because this omits the role of  $\delta B_{\parallel}$  in the “magnetic compression” term in  $\delta W$  (the potential energy of the plasma). If  $\delta B_{\parallel}$  is ignored then  $\delta W$  is minimised incorrectly which has a stabilising effect on pressure-driven modes.

<sup>‡</sup> Section 3.3 of [9] noted that the modes with longest perpendicular wavelength are typically expected to drive the most transport in STEP plasmas.



- *Global gyrokinetic codes often neglect  $\delta B_{\parallel}$ .*

Testing high turbulent fluxes predicted, but potentially not well resolved, by local GK in some local equilibria is of high priority for STEP design. In the absence of moderate equilibrium flow shear, progress requires global gyrokinetic simulations including profile variation that should better resolve the box-scale streamers observed in local simulations [9]. However, most global-capable codes simply do not include  $\delta B_{\parallel}$  at present. The MHD approximation has been used in a global code to study electromagnetic turbulence in NSTX plasmas [20].

- *Integrated modelling tools often neglect  $\delta B_{\parallel}$*

Quasilinear turbulence models such as TGLF [4] provide fast transport predictions that are essential for the integrated modelling codes used in scenario design. These models perform a fast computation of the linear-mode properties, and feed these into saturation rules developed to parameterise the turbulent fluxes. As such, these reduced models rely on both the fidelity of the saturation rules, and the accuracy of the linear physics calculation, which requires capturing  $\delta B_{\parallel}$  effects in STEP.

Sections 4–6 of this paper are devoted to: (i) exploring the degree to which the MHD approximation is appropriate for use in STEP plasmas; and (ii) seeking to understand conditions where the requirement to include  $\delta B_{\parallel}$  fluctuations can be relaxed.

### 3. Linear electromagnetic gyrokinetics and the MHD approximation

We are interested in plasmas that are well described by the GK framework (see e.g., [21, 22]): i.e. we are concerned with fluctuations  $\sim e^{i(\mathbf{k}\cdot\mathbf{r}-\omega t)}$ , having characteristic frequency  $\omega$  and wavenumbers  $k_{\parallel}$  and  $k_{\perp}$  parallel and perpendicular to the equilibrium magnetic field direction  $\mathbf{b}_0 = \mathbf{B}_0/B_0$ , that satisfy the standard GK ordering  $\omega/\Omega_s \sim \nu_{ss'}/\Omega_s \sim k_{\parallel}/k_{\perp} \sim q_s \delta\phi/T_{0s} \sim \delta B_{\parallel}/B_0 \sim \delta \mathbf{B}_{\perp}/B_0 \sim \rho_{\star,s} \ll 1$ , where  $\Omega_s = q_s B_0/m_s$  is the cyclotron frequency of species  $s$  with charge  $q_s$ , equilibrium density and temperature  $n_{0s}$  and  $T_{0s}$ , respectively, mass  $m_s$  and thermal speed  $v_{\text{ths}} = \sqrt{2T_{0s}/m_s}$ ,  $\nu_{ss'}$  is the typical collision frequency. The size of electrostatic perturbations is set by the perturbed electrostatic potential  $\delta\phi$ . Electromagnetic perturbations enter GK through  $\delta B_{\parallel}$  and  $\delta B_{\perp}$ , the fluctuations of the magnetic-field parallel and perpendicular to the equilibrium magnetic-field direction. Electromagnetic effects are most conveniently described in local GK by writing the fluctuating magnetic field  $\delta\mathbf{B} = \nabla \times (\delta\mathbf{A}_{\perp} + \delta A_{\parallel} \mathbf{b}) \simeq \nabla \times \delta\mathbf{A}_{\perp} + \nabla \delta A_{\parallel} \times \mathbf{b}$ , then relating its parallel component to the fluctuating vector potential  $\mathbf{A}$  by  $\mathbf{b} \cdot \delta\mathbf{B} \equiv \delta B_{\parallel} \simeq \mathbf{b} \cdot \nabla \times \delta\mathbf{A}_{\perp}$ .

It is convenient to write the GK guiding-centre distribution function in the form  $f_s = F_{0s}(1 - q_s \delta\phi/T_{0s}) + g_s = F_{0s} + \delta f_s$ ,  $\delta f_s = -(q_s \delta\phi/T_{0s})F_{0s} + g_s$ . Here, the GK distribution function consists of a Maxwellian piece  $F_{0s}$  and an order- $\rho_{\star,s}$ -small perturbation  $\delta f_s$ , with  $g_s$  being the non-adiabatic part of  $\delta f_s$ . Under the above ordering,

the collisionless, linear, GK equation is given in Fourier space by

$$\begin{aligned}
 (\omega - \omega_{ds} - k_{\parallel}v_{\parallel})g_s &= \frac{q_s F_{0s}}{T_{0s}}(\omega - \omega_{*s}^T) \\
 &\times J_0(b_s) \left[ \delta\phi - v_{\parallel}\delta A_{\parallel} + \frac{m_s v_{\perp}^2}{q_s B} \frac{J_1(b_s)}{b_s J_0(b_s)} \delta B_{\parallel} \right]
 \end{aligned} \tag{1}$$

where:  $b_s = k_{\perp}v_{\perp}/\Omega_s$ ;  $J_0$  and  $J_1$  are Bessel functions of the first kind that arise from gyroaveraging; the drive frequency  $\omega_{*s}^T = \omega_{*s}[1 + \eta_s(v^2/v_{\text{ths}}^2 - 3/2)]$  involves  $\eta_s = d \ln T_{0s}/d \ln n_{0s}$  and the diamagnetic frequency  $\omega_{*s} = (k_{\perp}T_{0s}/q_s B_0)d \ln n_{0s}/dr$  is defined in terms of the radial coordinate  $r$ . The magnetic drift frequency is given by

$$\omega_{ds} = \frac{1}{\Omega_s} \left( \omega_{\kappa} v_{\parallel}^2 + \omega_{\nabla B} \frac{v_{\perp}^2}{2} \right) \tag{2}$$

where the  $\nabla B$  drift frequency  $\propto \omega_{\nabla B} = \mathbf{k} \cdot \mathbf{b}_0 \times \nabla B_0/B_0$ , and the curvature drift frequency  $\propto \omega_{\kappa} = \mathbf{k} \cdot \mathbf{b}_0 \times (\mathbf{b}_0 \cdot \nabla \mathbf{b}_0)$ .<sup>††</sup> The equilibrium magnetic field can be written as  $\mathbf{B}_0 = \nabla\psi \times \nabla\alpha$ , where  $\psi$  is poloidal flux, and  $\alpha = \xi - q(\psi)\theta - \nu(\psi, \theta)$  is the binormal coordinate, with  $\xi$  the toroidal angle,  $\theta$  the poloidal angle, and  $\nu(\psi, \theta)$  a periodic function of  $\theta$  that depends on flux-surface shaping. Perpendicular gradients can be expressed in terms of binormal and radial wavenumbers as  $\nabla_{\perp} = k_{\alpha}\nabla_{\alpha} + k_{\psi}\nabla_{\psi}$ .

The fluctuating field quantities appearing in the GK equation are determined through the field equations. The perturbed electrostatic potential  $\delta\phi$  is determined through quasineutrality

$$\sum_s \frac{n_{0s} q_s^2}{T_{0s}} \delta\phi = \sum_s q_s \int d^3\mathbf{v} J_0(b_s) g_s. \tag{3}$$

The parallel magnetic vector potential  $\delta A_{\parallel}$  is determined by parallel Ampère's law

$$k_{\perp}^2 \delta A_{\parallel} = \mu_0 \sum_s q_s \int d^3\mathbf{v} v_{\parallel} J_0(b_s) g_s. \tag{4}$$

The magnetic fluctuation  $\delta B_{\parallel}$  is determined by perpendicular Ampère's law, leading to:

$$\frac{\delta B_{\parallel}}{B_0} = -\frac{\mu_0}{B_0^2} \sum_s \int d\mathbf{v}^3 g_s m_s v_{\perp}^2 \frac{J_1(b_s)}{b_s} \tag{5}$$

The MHD approximation is obtained in the long wavelength limit, where (5) reduces to:

$$\lim_{b_s \rightarrow 0} \frac{\delta B_{\parallel}}{B_0} = -\frac{\mu_0}{2B_0^2} \sum_s \int d\mathbf{v}^3 g_s m_s v_{\perp}^2 = -\frac{\mu_0}{B_0^2} \delta P_{\perp} \tag{6}$$

involving the perturbed gyrokinetic perpendicular pressure that is defined in this limit as:

$$\delta P_{\perp} = \sum_s \int d\mathbf{v}^3 \frac{g_s m_s v_{\perp}^2}{2}. \tag{7}$$

<sup>††</sup>Note that  $\omega_{\kappa}$  and  $\omega_{\nabla B}$  [appearing in (2)] are **not** themselves frequencies. These quantities are related to the curvature and  $\nabla B$  drifts,  $\Omega_{\kappa s}$  and  $\Omega_{\nabla B}$  viz  $\Omega_{\kappa s} = [(v_{\text{ths}}^2)/(2\Omega_s)]\omega_{\kappa}$  and  $\Omega_{\nabla B} = [(v_{\text{ths}}^2)/(\Omega_s)]\omega_{\kappa}$ .

Equation 6 implies that perpendicular force balance is satisfied to leading order in gyrokinetics, which is essential to exclude fast compressional Alfvén modes with frequencies  $\sim \Omega/\sqrt{\beta}$  that lie outside of the gyrokinetic approximation (as discussed in Appendix A of [23]).

### 3.1. The MHD approximation in GK

The historical precedent for the MHD approximation in GK comes from the fact that, in the one fluid MHD limit ( $k_y \rho_s \rightarrow 0$ ), the compressional magnetic fluctuations cancel the contribution of the  $\nabla B$  drift in a low- $\beta$  expansion in the absence of magnetic curvature [10]. Our first goal in this paper is to understand how this cancellation manifests itself in the GK framework.

Linear GK force balance in 6 requires finite  $\delta B_{\parallel}$  to sustain a perpendicular pressure perturbation. It is therefore only appropriate to drop  $\delta B_{\parallel}$  if  $\delta P_{\perp}$  can also be neglected. In Appendix A we calculate  $\delta P_{\perp}$  from the perturbed gyrokinetic distribution function, exploiting an exact relationship (given in Appendix C) between curvature and  $\nabla B$  drifts that follows straightforwardly from equilibrium force balance:

$$\omega_{\nabla B} = \omega_{\kappa} - (\mathbf{k} \cdot \mathbf{b}_0) \frac{\nabla p}{\mu_0 B_0^2}, \quad (8)$$

$\omega_{\kappa}$  and  $\omega_{\nabla B}$  differ by a term proportional to the pressure gradient (see derivation of C.4 in Appendix C). It is demonstrated in Appendix A that  $\delta P_{\perp}$  vanishes in the long-wavelength, low- $\beta$  limit with  $k_{\parallel} v_{\text{ths}} \ll \omega_{ds} \ll \omega$ , if:

$$\omega_{\kappa} - \omega_{\nabla B} = 0. \quad (9)$$

Enforcing equation 9 can therefore be used to implement the MHD approximation and drop  $\delta B_{\parallel}$  from GK in the appropriate limits.

Two approaches have been adopted to enforce equation 9 in gyrokinetics. Denoting the physical values (i.e. those set by the actual equilibrium magnetic geometry) of  $\omega_{\kappa}$  and  $\omega_{\nabla B}$  by  $\omega_{\kappa}^{\text{act}}$  and  $\omega_{\nabla B}^{\text{act}}$ , we can satisfy 9 by either:

- (MHD-1) setting the  $\nabla B$  drift equal to the curvature drift

$$(\omega_{\kappa}, \omega_{\nabla B}) = (\omega_{\kappa}^{\text{act}}, \omega_{\kappa}^{\text{act}})$$

- (MHD-2) setting the curvature drift equal to the  $\nabla B$  drift

$$(\omega_{\kappa}, \omega_{\nabla B}) = (\omega_{\nabla B}^{\text{act}}, \omega_{\nabla B}^{\text{act}}).$$

Both approaches have been implemented in GK codes, but Appendix B explains why MHD-1 is better motivated physically for the study of a particular equilibrium and is consistent with an argument presented in the Appendix of [24].

### 3.2. Implementation of the MHD approximation in gyrokinetic codes

It is helpful to connect the MHD approximation to the magnetic-drift velocity  $\mathbf{v}_d$  rather than the magnetic-drift frequency  $\omega_{ds} = \mathbf{k}_\perp \cdot \mathbf{v}_{ds}$ . The exact magnetic drift velocity is given by

$$\mathbf{v}_{ds} = \mathbf{b}_0 \times \left( \frac{v_\parallel^2}{\Omega_s} (\mathbf{b}_0 \cdot \nabla) \mathbf{b}_0 + \frac{v_\perp^2}{2\Omega_s} \nabla \ln B_0 \right) \quad (10)$$

where it is shown in Appendix C that:

$$\nabla_\perp \ln B_0 = (\mathbf{b}_0 \cdot \nabla) \mathbf{b}_0 - \frac{\mu_0 \nabla p}{B_0^2}. \quad (11)$$

We can now understand both possible MHD approximations in terms of adding or subtracting corrections to this definition.

*3.2.1. The MHD-1 approximation* The historical precedent for setting the  $\nabla B$  drift equal to the curvature drift comes from a result of [11] showing that, in the one fluid MHD limit ( $k_y \rho_D \rightarrow 0$ ), there is a cancellation of terms such that  $\delta B_\parallel$  can be dropped as long as the  $\nabla B$  drift is corrected by a term proportional to  $\nabla p$  so that it is equal in magnitude to the curvature drift and points in the same direction (hereinafter referred to as ‘‘MHD-1’’):

$$\mathbf{v}_{ds} = \mathbf{b}_0 \times \frac{\mathbf{b}_0 \cdot \nabla \mathbf{b}_0}{\Omega_s} \left( v_\parallel^2 + \frac{v_\perp^2}{2} \right) \quad (12)$$

In MHD-1 a term proportional to the pressure gradient has been added to the right hand side of Equation 10.

*3.2.2. The MHD-2 approximation* In MHD-2 the drift velocity is written as:

$$\mathbf{v}_{ds} = \mathbf{b}_0 \times \frac{\nabla B_0}{B_0 \Omega_s} \left( v_\parallel^2 + \frac{v_\perp^2}{2} \right) \quad (13)$$

As discussed in Appendix C, this effectively corresponds to changing the equilibrium magnetic field curvature by setting  $\mathbf{b}_0 \cdot \nabla \mathbf{b}_0 = \nabla B_0 / B_0$ , and again removing the pressure gradient contribution to the  $\nabla B$  drift in this different local equilibrium. MHD-2 allows  $B_\parallel$  to be dropped from the GK equation (1), but modifies the local equilibrium.

While both MHD-1 and MHD-2 are implemented in GENE code, MHD-1 is the recommended setting as it is more physical and does not change the local equilibrium.

## 4. Linear gyrokinetic simulations using the MHD approximation

The gyrokinetic analysis in this paper focuses primarily on a single equilibrium flux-surface taken from close to mid-radius ( $q = 3.5, \Psi_n = 0.49$ ) in the STEP reference scenario STEP-EC-HD † (where EC stands for Electron Cyclotron heating and current

† SimDB UUID: 2bb77572-d832-11ec-b2e3-679f5f37cafe, Alias: smars/jetto/step/88888/apr2922/seq-1

drive, and HD stands for High Density), which is designed to deliver a fusion power  $P_{\text{fus}} = 1.8$  GW. We note that this is precisely the same flux surface as examined in [9], a choice that has been made intentionally to allow benchmarking between our results against those reported in [9]. It should also be noted that whereas [9] focused on accurate investigation of the turbulent transport in STEP; the focus of this paper is instead on a more conceptual study of the hKBM.

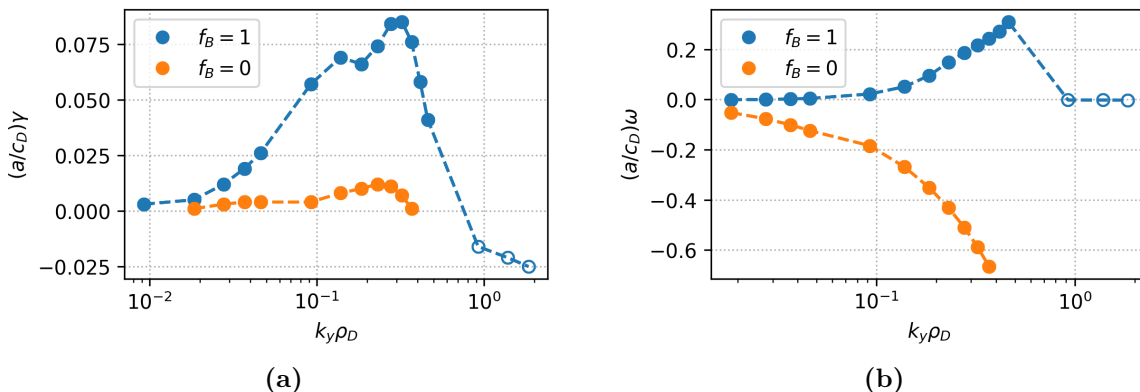
A Miller parameterisation [25] was used to model the local magnetic equilibrium geometry, and the shaping parameters were fitted to the chosen surface using `pyrokinetics` [26,27], a Python library developed to facilitate pre- and post-processing of gyrokinetic analysis performed using a range of different GK codes. `pyrokinetics` also contains an ideal-ballooning solver which has been used throughout this work to ensure that all equilibria remain MHD-stable when the local equilibrium parameters are varied. Table 1 provides the values of various local equilibrium quantities at the flux surface examined in this paper, including magnetic shear  $\hat{s}$ ; safety factor  $q$ ; normalised minor radius  $\rho/a$ ; elongation  $\kappa$  and its radial derivative  $\kappa'$ ; triangularity  $\delta$  and its radial derivative  $\delta'$ ; the radial derivative of the Shafranov shift  $\Delta'$ ; and the normalised inverse density and temperature gradient scale lengths of species  $s$ ,  $a/L_{n_s}$  and  $a/L_{T_s}$ , respectively. Our simulations evolve three species: electrons, deuterium, and tritium and neglect entirely impurities and fast particles. The interested reader is referred to [8,9] for more details on the equilibrium and on the setup of the computational grids, which are identical to those used in the aforementioned works.

Parameter	Value	Parameter	Value
$\Psi_n$	0.49	$\rho/a$	0.64
$q$	3.5	$\hat{s}$	1.2
$\beta$	0.09	$\beta'$	-0.48
$\kappa$	2.56	$\kappa'$	0.06
$\delta$	0.29	$\delta'$	0.46
$k_y^{n=1} \rho_D$	0.0047	$\Delta'$	-0.40
$a/L_{n_e}$	1.03	$a/L_{T_e}$	1.58
$a/L_{n_D}$	1.06	$a/L_{T_D}$	1.82
$a/L_{n_T}$	0.99	$a/L_{T_T}$	1.82

**Table 1:** Local equilibrium quantities at the STEP mid-radius flux surface considered in this paper.

Previous linear analysis shows that the hKBM is the dominant ion-scale instability on this surface, with a subdominant MTM also found to be unstable on a subset of these binormal scales (see Fig 19 and Fig 20 of [8]). No unstable microinstabilities are observed at the electron Larmor radius scale. The dominant hKBM and the subdominant MTM can both be recovered physically; that is, one can recover the subdominant mode by either by exploiting the up-down symmetry in the local equilibrium and forcing the parity of the perturbed distribution function in an initial-value calculation, or by using

an eigenvalue solver to return the unstable linear spectrum. However, importantly for our work, it was also shown that for this equilibrium it is possible to recover the subdominant mode simply by artificially suppressing  $\delta B_{\parallel}$  (thus stabilising the hKBM). This latter point is exemplified in Fig 1.

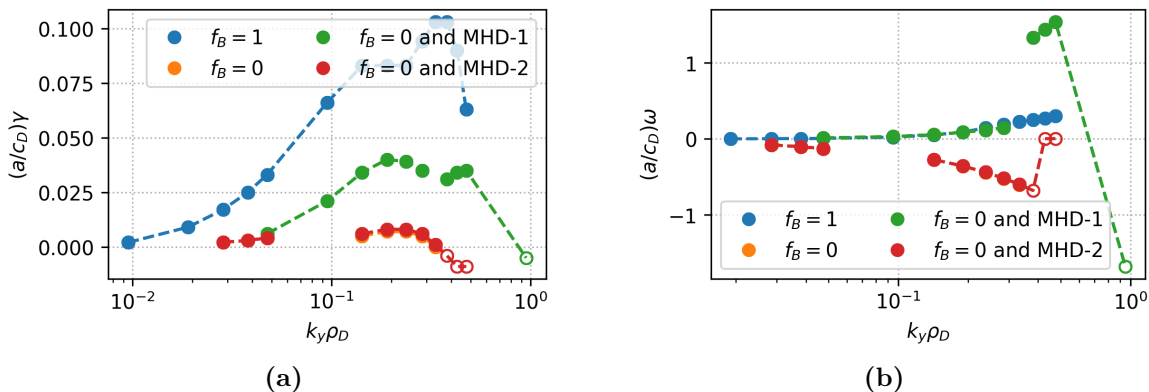


**Figure 1:** Growth rate (a) and mode frequency (b) as functions of the binormal wavenumber from linear simulations of the dominant instability in STEP-EC-HD on a mid-radius flux surface. Simulations are shown both with,  $f_B = 1$  (blue), and without,  $f_B = 0$  (orange),  $\delta B_{\parallel}$ . The two simulations are otherwise identical and the exact magnetic-drift velocity is used in each simulation. This figure is adapted from [8].

Fig 1 shows the linear growth rate (a) and frequency (b) (normalised to the deuterium sound speed,  $c_D = \sqrt{T_e/m_D}$ , divided by the minor radius of the last closed flux surface) as functions of the binormal wavenumber  $k_y\rho_D = n\rho_{*D}d\rho/d\Psi_n$ . Table 1 also includes the binormal wavenumber  $k_y^{n=1}\rho_D$  corresponding to the toroidal mode number  $n = 1$ . Simulations are shown both including,  $f_B = 1$  (blue), and neglecting,  $f_B = 0$  (orange),  $\delta B_{\parallel}$  fluctuations (the two simulations are otherwise identical) and no approximations are made concerning the magnetic drift velocity. Importantly, we see from Fig 1 that if  $\delta B_{\parallel}$  is artificially excluded from calculations (as routinely assumed in a number codes and modelling tools) then we recover the previously subdominant MTM (note the change in frequency) as the fastest growing unstable mode in the system. Succinctly, the hKBM is linearly stable on this surface (along with many other surfaces in the STEP equilibrium) without  $\delta B_{\parallel}$ .

#### 4.1. Linear gyrokinetic simulations using the MHD approximation in STEP

The MHD approximation introduced in Section 3.1 may allow us to study the hKBM in the absence of  $\delta B_{\parallel}$  fluctuations. In Section 3.1, we introduced two possible approximations: (i) MHD-1 ( $\mathbf{v}_d$  given by equation 12); and (ii) MHD-2 ( $\mathbf{v}_d$  given by equation 13), that were designed to compensate for the effect of excluding  $\delta B_{\parallel}$ . We now seek to explore whether either of these approximations are appropriate for modelling STEP plasmas.



**Figure 2:** Growth rate (a) and mode frequency (b) as functions of the binormal wavenumber from linear simulations of the dominant instability in STEP-EC-HD on a mid-radius flux surface. Simulations are shown both with,  $f_B = 1$  (blue), and without,  $f_B = 0$  (orange, green, red),  $\delta B_{\parallel}$ . For the simulations without  $\delta B_{\parallel}$ , different treatments of the drift velocity are also shown: (i) the full drift velocity [ $\mathbf{v}_{ds}$  given by equation 10 (orange)]; (ii) MHD-1 [ $\mathbf{v}_{ds}$  given by equation 12 (green)]; and MHD-2 [ $\mathbf{v}_{ds}$  given by equation 13 (red)]. Note that the MHD-2 curve (red) lies on top of the curve that excludes  $\delta B_{\parallel}$  but uses the exact form of the magnetic-drift velocity (orange).

Fig 2 shows the linear growth rate (a) and frequency (b) as functions of the binormal wavenumber for simulations both including,  $f_B = 1$  (blue), and excluding,  $f_B = 0$  (orange, green, red)  $\delta B_{\parallel}$  fluctuations. The simulations without  $\delta B_{\parallel}$  fluctuations employ different prescriptions of the magnetic drift velocity: (i) the full drift velocity [ $\mathbf{v}_d$  given by equation 10 (orange)]; (ii) MHD-1 [ $\mathbf{v}_d$  given by equation 12 (green)]; and MHD-2 [ $\mathbf{v}_d$  given by equation 13 (red)]. Using the MHD-2 approximation (Fig 2, red) we see that the hKBM is linearly stable and we once again recover the previously subdominant MTM (note the change in frequency) as the fastest growing unstable mode in the system.‡ However, using the MHD-1 approximation [setting the  $\nabla B$  drift parallel to the curvature drift (Fig 2, green)] does find the hKBM but with a strongly reduced growth rate. These results indicate that the MHD-1 approximation is the more appropriate treatment of the magnetic drift when  $\delta B_{\parallel}$  is neglected in STEP plasmas, as we might expect from the fact that MHD-1 does not change the local equilibrium. However, we note that even though this approximation recovers the hKBM, accurately capturing this mode clearly requires a proper treatment of  $\delta B_{\parallel}$  and exact magnetic drifts (particularly at low  $k_y \rho_D$  where MHD-1 suggests hKBMs are stable at wavenumbers where full physics finds the mode to be unstable).

‡ In fact, the simulation using the MHD-2 approximation (red) returns the exact same linear spectrum as the simulation that excludes  $\delta B_{\parallel}$  but uses the exact form of the magnetic-drift velocity (orange). This arises because (i) the hKBM is more stable at reduced curvature in the MHD-2 local equilibrium, and (ii) MTMs are typically only very weakly sensitive to  $\nabla p$  (see Fig 15b of [14]).

#### 4.2. Improving the fidelity of the MHD approximation

This section explores whether it is possible to study the hKBM in the absence of  $\delta B_{\parallel}$  by modifying the local equilibrium parameters such that the MHD approximation is better satisfied. There are three key assumptions in the derivation of the MHD approximation presented in Appendix A:

- (i) A long-wavelength assumption  $k_y \rho_D \ll 1$ .
- (ii) An MHD-like ordering of the frequencies  $k_{\parallel} v_{\text{ths}} \ll \omega_{ds} \ll \omega$ .
- (iii) A low- $\beta$  approximation  $\beta \ll 1$ .

These assumptions are explored in the following subsections through parameter scans. Unless it is explicitly stated otherwise, only the scan parameter is varied and other local parameters are held constant to isolate the effect of each parameter.

*4.2.1. Assumption (i)  $k_y \rho_D \ll 1$*  The long-wavelength assumption in STEP is well satisfied since we are primarily concerned with modes with  $k_y \rho_D \ll 1$ .

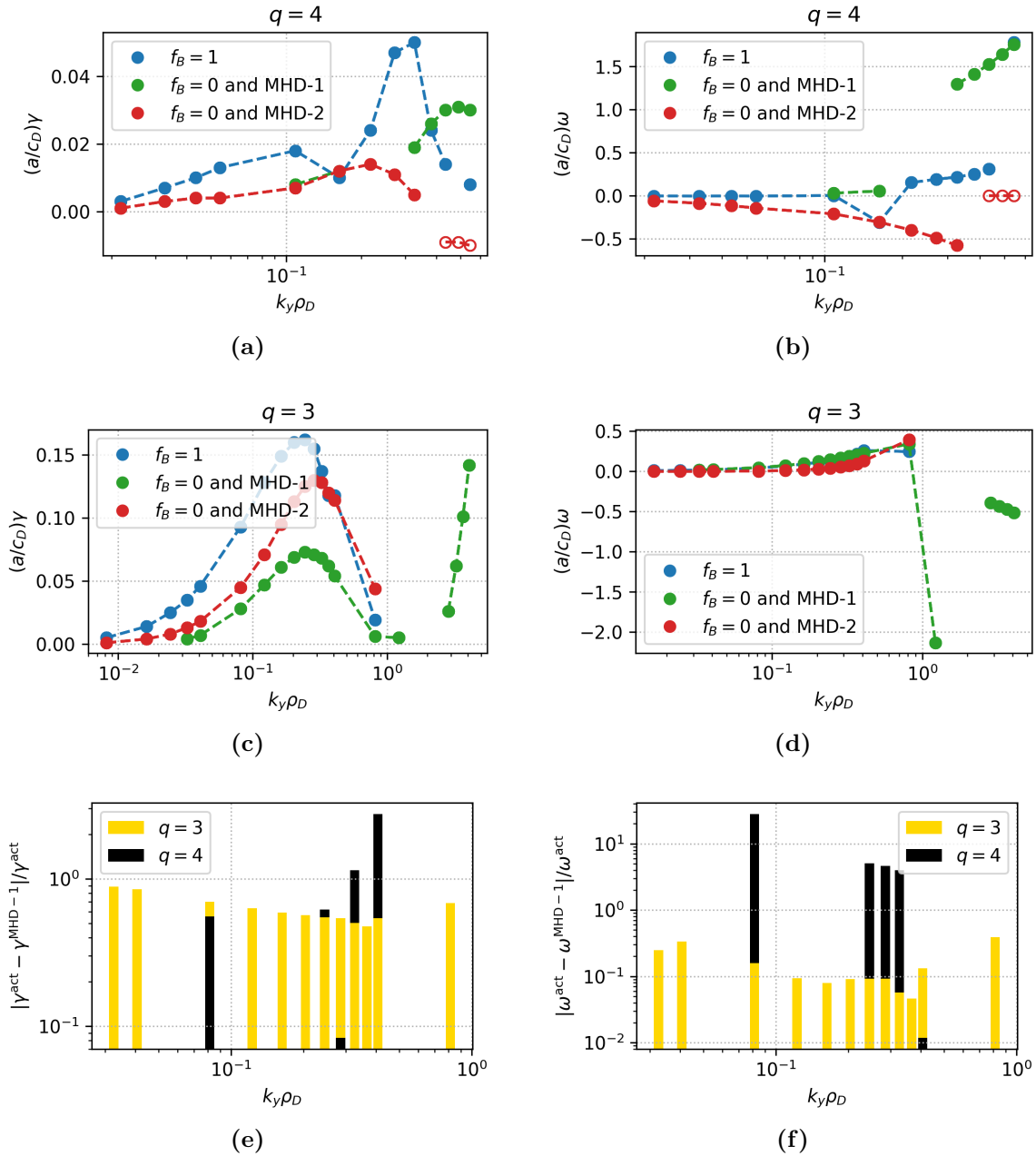
*4.2.2. Assumption (ii)  $k_{\parallel} v_{\text{ths}} \ll \omega_d \ll \omega$*  Basic estimates give rise to the scaling  $k_{\parallel} v_{\text{ths}} \propto v_{\text{ths}}/(qR)$ . Thus, we see that increasing (decreasing) the value of the safety factor  $q$  decreases (increases) the value of  $k_{\parallel}$ . It therefore appears straightforward at first glance to modify how well the left-hand-side of assumption (ii) is satisfied; we expect the MHD approximation to more (less) accurately capture the linear physics at larger (smaller) values of  $q$ .

Fig 3 shows the results of simulations using different values of  $q$  and both including,  $f_B = 1$  (blue), and excluding,  $f_B = 0$  (green, red),  $\delta B_{\parallel}$  fluctuations. Simulations without  $\delta B_{\parallel}$  fluctuations employ either the MHD-1 (green) or MHD-2 (red) approximation. Perhaps somewhat surprisingly the performance of the MHD approximation only improves very slightly if at all (see Fig 3e) with increasing  $q$ . The reason for this that the scaling  $k_{\parallel} v_{\text{ths}} \propto v_{\text{ths}}/(qR)$  is very poor in this case. Fig 4 shows that the parallel mode structure of the hKBM at is largely unchanged at different values of  $q$  and therefore no discernible improvement of the MHD approximation should be expected.

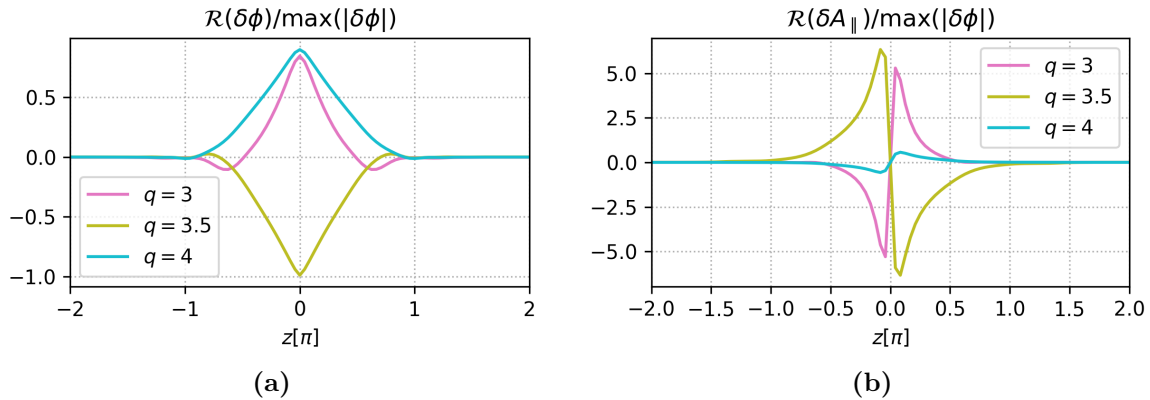
*4.2.3. Assumption (iii)  $\beta \ll 1$*  The derivation of the MHD approximation presented in Appendix A required the assumption that  $\beta \ll 1$ . We therefore might expect that decreasing the value of  $\beta$  will improve the performance of the MHD approximation. However, this cannot be the case since sending  $\beta \rightarrow 0$  is bound to make the MHD approximation fail as  $\delta B_{\parallel} = 0$  when  $\beta = 0$  regardless of any additional physics. In fact, the derivation of the MHD approximation given in Appendix A relies on taking  $k_y \rho_D \rightarrow 0$  at fixed  $\beta$  and much more care must be taken if we wished to exchange these limits.

Fig 5 shows the results of simulations using different values of  $\beta$  (keeping  $\beta'$  fixed) and both including,  $f_B = 1$  (blue), and excluding,  $f_B = 0$  (green, red),  $\delta B_{\parallel}$  fluctuations.





**Figure 3:** Growth rates (a, c) and mode frequencies (b, d) as functions of the binormal wavenumber from linear simulations of the dominant instability in STEP-EC-HD on a mid-radius flux surface. We attempt to vary the value of  $k_{\parallel}$  by varying the value of the safety factor  $q$  (all other local equilibrium quantities are the same as in Table 1). Results are shown for  $q = 4$  (a, b) and  $q = 3$  (c, d). Simulations are shown both with,  $f_B = 1$  (blue), and without,  $f_B = 0$  (green, and red),  $\delta B_{\parallel}$ . For the simulations without  $\delta B_{\parallel}$ , different treatments of the drift velocity are also shown: (i) MHD-1 (green); and (ii) MHD-2 (red). The absolute error made by the MHD-1 approximation for the growth rate and the mode frequency are shown in panels (e) and (f) respectively. The error is computed only for modes which are unstable and propagating in the ion direction.



**Figure 4:** Parallel mode structure of  $\Re(\delta\phi)$  (a) and  $\Re(\delta A_{\parallel})$  (b) at the peak of the linear spectrum from Fig 1 for different values of the safety factor  $q$ . Changing the value of  $q$  does not vastly alter the parallel structure of the mode.

Simulations without  $\delta B_{\parallel}$  fluctuations employ either the MHD-1 (green) or MHD-2 (red) approximation. As expected, we observe no noticeable improvement in the performance of the MHD approximation.

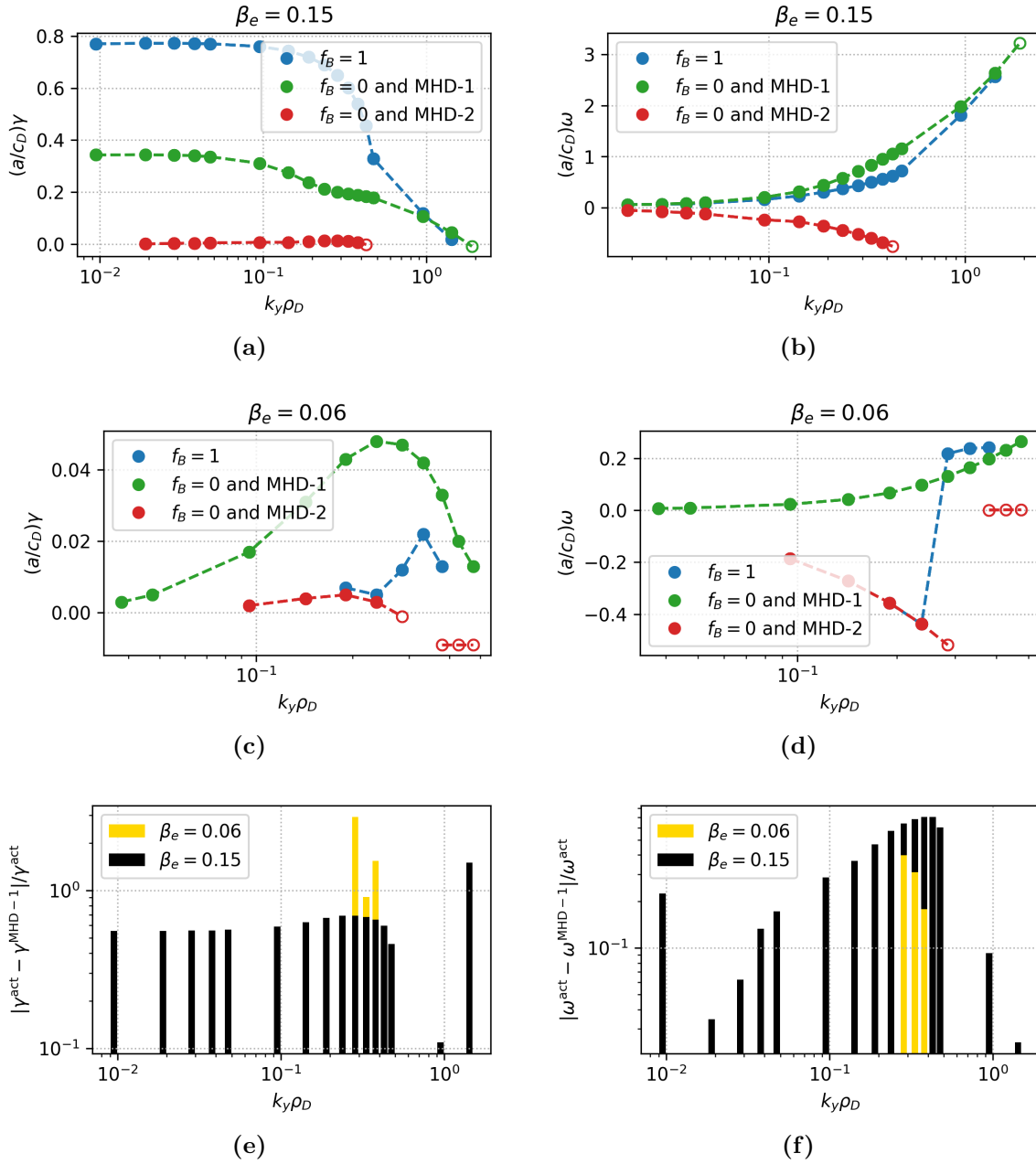
*4.2.4. Discussion* the results of Sections 4.2.2 and 4.2.3 highlight that it is very difficult to improve the performance of the MHD-approximation whilst only making small perturbations to the local equilibrium (i.e. only changing one parameter).

## 5. Nonlinear GK simulations using the MHD-1 approximation

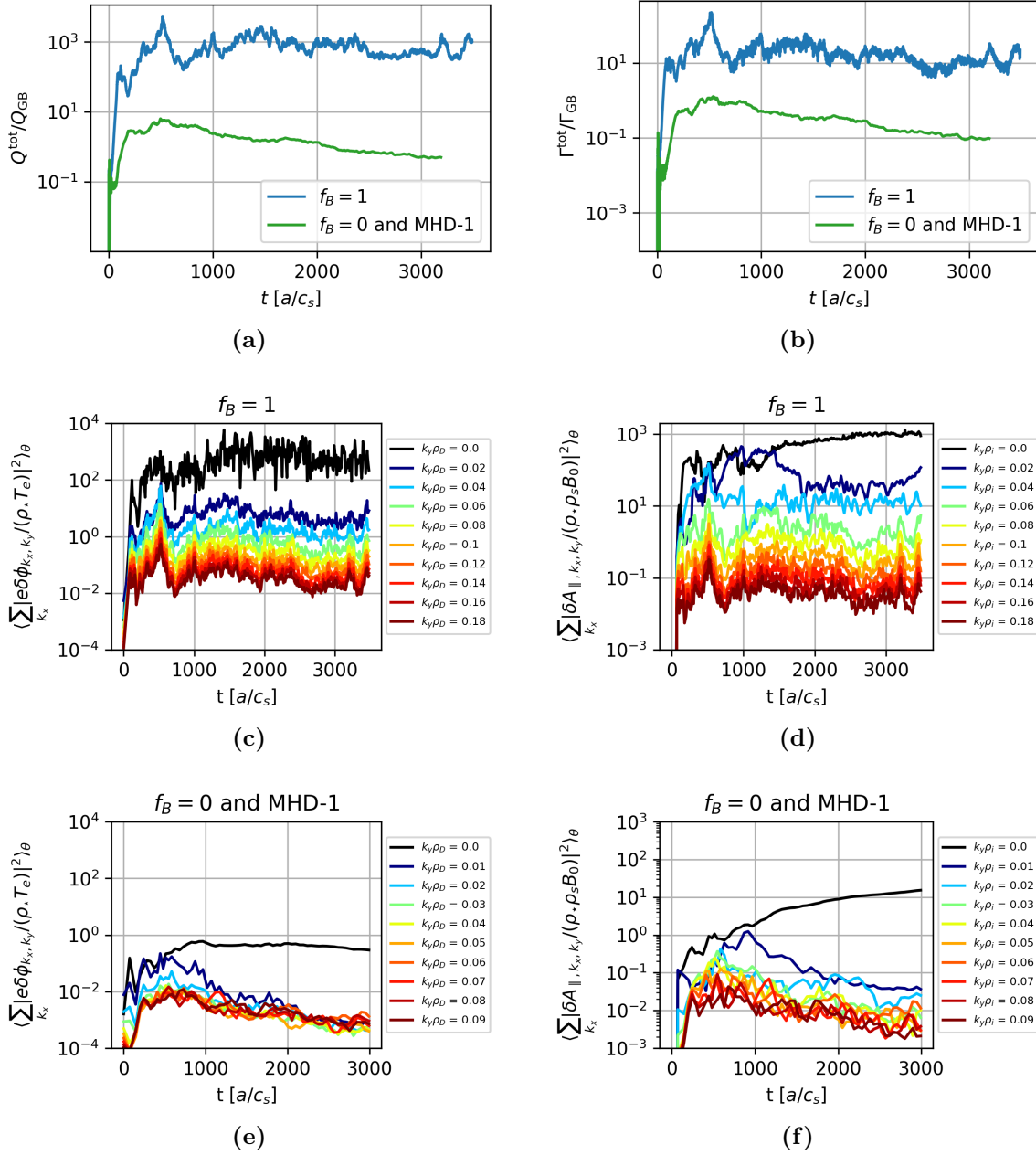
It was demonstrated in Section 4.1 that using the MHD-1 approximation (Appendix C demonstrates that this is the more physical of the two approximations) for STEP results in the fastest growing linear instability being the hKBM (albeit with a reduced growth rate). This suggests that it might be possible to simulate hKBM driven turbulence in STEP using the MHD-1 approximation in a global code, where the full inclusion of  $\delta B_{\parallel}$  is not yet routinely available. As a first step towards this goal, we first perform a *local* nonlinear simulation using the MHD-1 approximation; this simulation is otherwise identical to the hKBM simulation performed in [9] in the absence of equilibrium flow shear.

### 5.1. The MHD approximation does not accurately reproduce the large-transport state predicted by full-physics simulations

Fig 6 shows time traces of the total heat flux from two nonlinear GENE simulations. The first simulation (blue) includes  $\delta B_{\parallel}$  and uses (10) for the drift velocity (this simulation is equivalent to that shown in Fig 3a of [9] and will henceforth be referenced as the ‘STEP baseline’ simulation). The second simulation (green) shown in Fig 6 does not include



**Figure 5:** Growth rates (a, c) and mode frequencies (b, d) as functions of the binormal wavenumber from linear simulations of the dominant instability in STEP-EC-HD on a mid-radius flux surface. The value of  $\beta_e$  has been varied keeping  $\beta'$  fixed (all other local equilibrium quantities are the same as in Table 1). Results are shown for  $\beta_e = 0.15$  (a, b) and  $\beta_e = 0.06$  (c, d). Simulations are shown both with,  $f_B = 1$  (blue), and without,  $f_B = 0$  (green, red),  $\delta B_{\parallel}$ . For the simulations without  $\delta B_{\parallel}$ , different treatments of the drift velocity are also shown: (i) MHD-1 (green); and (ii) MHD-2 (red). The absolute error made by the MHD-1 approximation for the growth rate and the mode frequency are shown in panels (e) and (f) respectively. The error is computed only for modes which are unstable and propagating in the ion-direction.



**Figure 6:** Time traces of the total heat flux (a) and total particle flux (b) from *GENE* simulations. Simulations are shown both with,  $f_B = 1$  (blue), and without,  $f_B = 0$  (green),  $\delta B_{\parallel}$ . The case without  $\delta B_{\parallel}$  fluctuations uses the MHD-1 approximation with  $\mathbf{v}_a$  given by equation 12. Also shown is the time trace of the  $k_y$  spectrum of  $\delta\phi$  (c, e) and  $\delta A_{\parallel}$  (d, f) from the same simulations. Only the first decade of poloidal wavenumbers is shown.

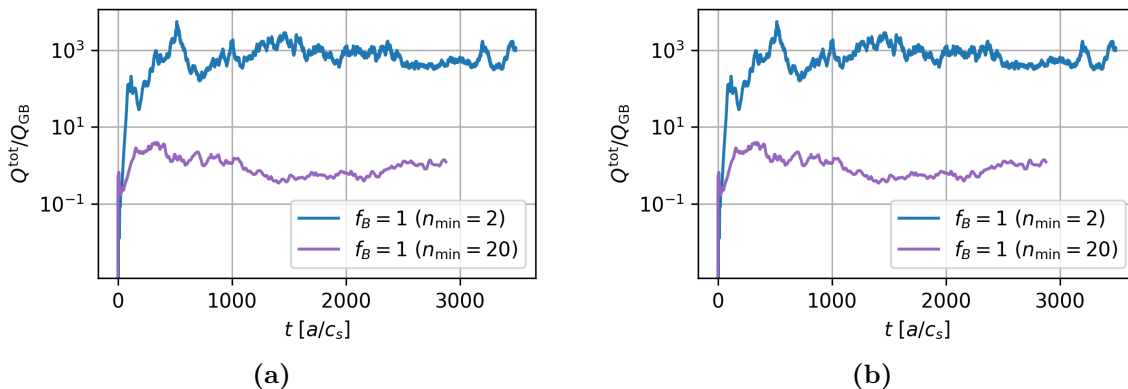
$\delta B_{\parallel}$  and instead uses the MHD-1 approximation. Although the linear physics is broadly similar (both linear spectra have hKBMs as the dominant unstable mode), the nonlinear physics produced by these two simulations is strikingly different. The simulation using the MHD-1 approximation appears to achieve a robustly steady§ saturated state at values of the heat flux around two orders of magnitude smaller than those reached by the simulation that includes full  $\delta B_{\parallel}$  physics||. Fig 6 also shows the time trace of the  $k_y$  spectrum of  $\delta\phi$  and  $\delta A_{\parallel}$  for the two simulations. In both cases, the zonal mode dominates over the non-zonal modes and it is likely that zonal flows and fields play a role in saturating the hKBM instability¶. Despite the difference between the two simulations, it is important to remark that the heat flux predicted using the MHD-1 approximation is still orders of magnitude larger than the heat flux driven by the subdominant MTM (see Fig 14 of [9]), showing the strong effect of the hKBM instability even without  $\delta B_{\parallel}$  when MHD-1 is used. Snapshots of the turbulence are shown in Appendix D, Fig D1. Note that the MHD-1 simulation without  $\delta B_{\parallel}$  Fig D1(d,e,f) does not show the same extended radial structures observed in simulations which include  $\delta B_{\parallel}$  Fig D1(a,b,c). The lower transport fluxes are associated with the absence of these structures.

The reason that we see saturation at much lower fluxes using the MHD-1 approximation in Fig 6 is revealed by closer inspection of Fig 2a. From this figure, we can see that long wavelength modes, which are unstable with  $\delta B_{\parallel}$ , are stable with MHD-1 (without  $\delta B_{\parallel}$ ), and it is precisely these wavenumbers at  $k_y \rho_D \ll 1$ , that are associated with the very large turbulent fluxes. To be explicit, it appears to be possible to find a lower-flux saturated state even when hKBMs are unstable across a wide range of the linear spectrum provided that they are not unstable up to very long wavelengths. Further evidence for this claim is demonstrated in Fig 7. This figure compares the total heat and particle fluxes from the baseline simulation (blue) to a second simulation (purple) that also includes the full-physics model but on a reduced grid of poloidal wavenumbers such that the only unstable modes evolved are those which are also unstable when the MHD-1 approximation is used. Removing the low- $k_y$  modes results in saturation at much smaller levels even when the exact same linear instabilities are present at intermediate  $k_y$ .

§ In this paper, we use the same definition of robustly steady as was introduced in [9]. The augmented Dickey-Fuller statistical test [28] is applied to the time trace of the total heat flux in the saturated or pseudo-saturated phase of each simulation (i.e. after the linear growth phase) to determine whether the saturation corresponds to a robust stationary state. The null hypothesis of the augmented Dickey-Fuller test is the presence of a unit root, while the alternative hypothesis is the stationarity of the time series. The null hypothesis is rejected when the **p-value** returned by the statistical test is below a threshold value that is taken to be **p= 0.1** in this paper. Unless explicitly stated otherwise, **all** simulations in this paper reach such a steady state.

|| In [9], simulations including  $\delta B_{\parallel}$  did not reach a state of robust saturation within the considered time interval. Recent computational advances, in particular the release of the GPU-enabled code **GENE-3.0** [29], have allowed us to run these simulations for much longer times over which saturation can be achieved.

¶ The exact saturation mechanism of the hKBM in the absence of equilibrium flow shear demands further study which will be the focus of future work.



**Figure 7:** Time traces of the total heat flux (a) and total particle flux (b) from *GENE* simulations. Both simulations include  $\delta B_{\parallel}$  and use the exact prescription of the magnetic-drift velocity. The value  $n_{\min}$  is the toroidal mode number corresponding to the value of  $k_{y,\min}$ . The simulation with the larger value of  $n_{\min}$  only evolves those unstable modes that are also unstable when the MHD-1 approximation is used. Removing the lowest  $k_y$  modes (purple) in this way results in saturation at values two orders of magnitude lower than the baseline (blue).

The results in Fig 6 appear to support the conclusions of [8,9] that  $\delta B_{\parallel}$  fluctuations play an essential role in hKBM driven turbulence in these STEP local equilibria and in particular in the transition to a large-transport state. However, we have already seen in Sections 4.2.2 and 4.2.3 (see also [8,9]) that it is possible to vary the local equilibrium in such a way as to change the hKBM threshold. In Section 6 we study whether it is possible to extend this result. Is it possible to drive the hKBM linearly unstable in the absence of  $\delta B_{\parallel}$  with exact drifts?

## 6. The importance of parallel magnetic perturbations to the hKBM

In [9] it was shown that the total heat and particle fluxes from nonlinear simulations vary with  $\beta$  when  $\beta'$  is varied consistently (see Fig. 11 of [9]). However, the heat and particle fluxes remained very large even at much smaller values of  $\beta$  than the STEP baseline. It was argued that reduced  $\beta'$  stabilisation of the hKBM plays a significant role in maintaining such large turbulent fluxes. In the STEP baseline case,  $\beta'$  stabilisation is very important since it pushes the hKBM towards marginality. The sensitive dependence of the hKBM on  $\beta'$  is made most obvious when  $\beta'$  is varied holding all other local equilibrium parameters fixed (albeit inconsistent).

Fig 8 shows the linear growth rate (a, c) and frequency (b, d) as functions of the binormal wavenumber for simulations with reduced  $\beta'$  with,  $f_B = 1$  (blue), and without,  $f_B = 0$  (orange, green, red),  $\delta B_{\parallel}$  fluctuations. In **all** cases the hKBM is much more unstable than the case with the nominal value of  $\beta'$  (see Fig. 2) due to the much decreased role of  $\beta'$  stabilisation. In each case, the hKBM is likely to drive much more

transport (note also that the mode does not become stable at  $n = 1$  and this is certainly a no-go zone for any actual device).

There are three key observations from Fig 8 that we wish to stress.

- In the limit  $\beta' \rightarrow 0$ , the MHD-1 and MHD-2 are identical, which is not surprising as  $\mathbf{b}_0 \cdot \nabla \mathbf{B}_0 \rightarrow \nabla B_0 / B_0$  as  $\nabla p \rightarrow 0$  (see Appendix C). This is clearly seen in Fig 8 where the difference between the MHD-1 and MHD-2 prediction reduces as  $|\beta'|$  decreases.
- We note that the MHD approximation does a much better job of capturing the linear growth rates of the hKBM at smaller values of  $|\beta'|$ . The limit  $|\beta'| \rightarrow 0$  is in effect a  $\omega_{\nabla B} - \omega_{\kappa} \rightarrow 0$  approximation but this does not say anything about  $\delta B_{\parallel}$ .
- The hKBM can be unstable up to  $n = 1$  in the absence of  $\delta B_{\parallel}$  with exact drifts.

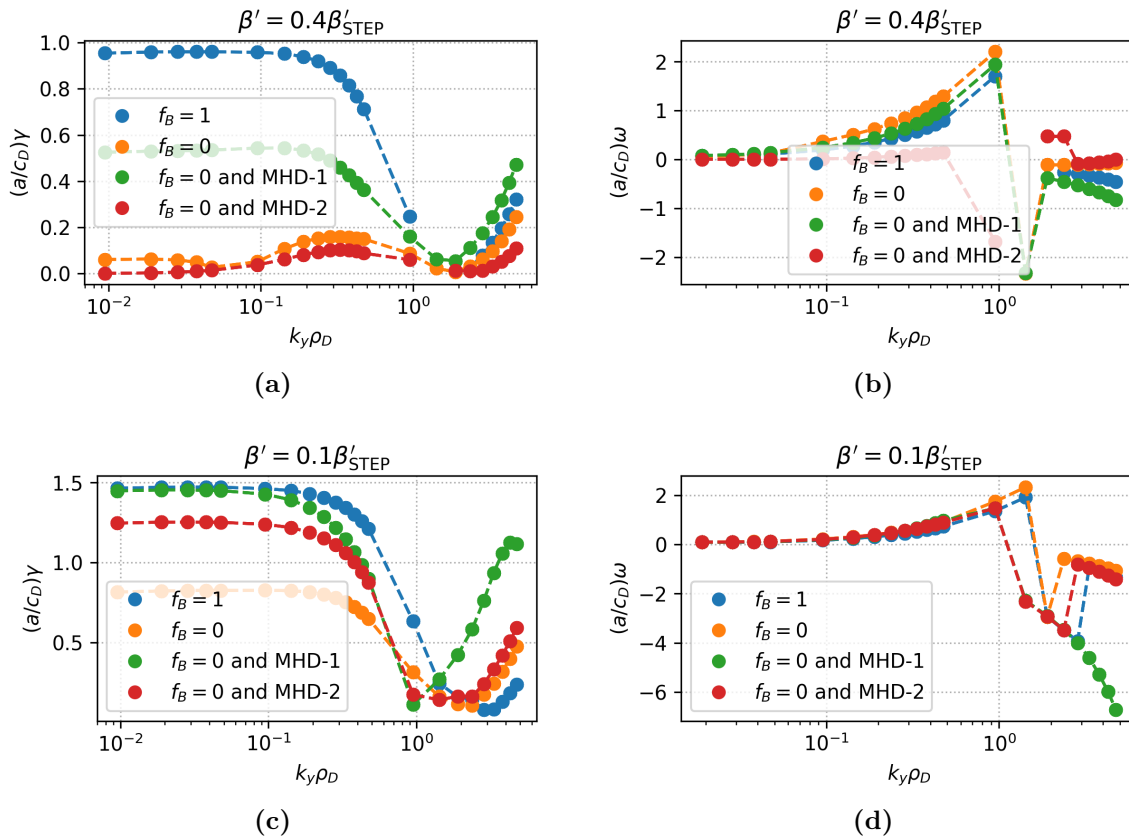
This final result suggests it should be possible to study hKBM-driven turbulence in more strongly driven regimes using global GK codes that do not include  $B_{\parallel}$ .

Fig 9 shows time traces of the total heat flux and particle flux from two nonlinear GENE simulations: (i) with the nominal value of  $\beta'$  that includes  $\delta B_{\parallel}$  fluctuations (blue); and (ii) excluding  $\delta B_{\parallel}$  fluctuations and using exact drifts (magenta) at a lower value of  $\beta' = 0.4\beta'_{\text{STEP}}$  (with all other parameters are identical). As expected from the linear spectrum (Fig 8), the nonlinear physics of this new case qualitatively mirrors the STEP baseline case even without  $\delta B_{\parallel}$  fluctuations. The heat fluxes rise to very large levels and no robustly steady saturated state is reached in this simulation on the timescale simulated (note that this is the only simulation in this paper that does not reach a robustly steady saturated state). Snapshots of the turbulent fields Fig D1(g,h,i) reveal the presence of box-scale radially elongated streamers.

The fact that at lower  $\beta'$  we obtain electromagnetic runaway fluxes without  $\delta B_{\parallel}$  is an important result; it reduces the number of ingredients needed to observe electromagnetic runaway fluxes in local gyrokinetic simulations. Put more simply, we observe qualitatively similar runaway fluxes in local simulations when there is electromagnetic instability at the longest-wavelengths in the system. We take great care to emphasise that the results in this paper suggests that there is an important distinction to be made between (A) runaway fluxes that eventually saturate at very large values such as those discussed in this paper, and (B) fluxes that will never saturate such as the runaways observed by [31] in cyclone-base-case [32] geometry without  $\delta B_{\parallel}$ . We find that results for the STEP equilibrium considered in this paper definitively fall into category (A). It is left unclear which category the results at lower  $\beta'$  fall into. Identification of these two categories motivates the need for a rigorous theory of electromagnetic turbulence saturation that can distinguish between the two types of behaviour.

## 7. Conclusions

In this paper, we have examined the importance of including parallel magnetic perturbations ( $\delta B_{\parallel}$ ) in gyrokinetic simulations of electromagnetic turbulence at mid-



**Figure 8:** Growth rates (a, c) and mode frequencies (b, d) as functions of the binormal wavenumber from linear simulations of the dominant instability in STEP-EC-HD on a mid-radius flux surface. Results are shown with reduced  $\beta' = 0.4\beta'_{STEP}$  (a, b) and  $\beta' = 0.1\beta'_{STEP}$  (c, d). All other parameters are kept fixed. Simulations are shown both with,  $f_B = 1$  (blue), and without,  $f_B = 0$  (orange, green, red),  $\delta B_{\parallel}$  fluctuations. For the simulations without  $\delta B_{\parallel}$ , different treatments of the drift velocity are also shown: (i) MHD-1 (green); and (ii) MHD-2 (red). In all cases, the hKBM is much more unstable than the reference case (see Fig 2) due to reduced  $\beta'$  stabilisation [30].

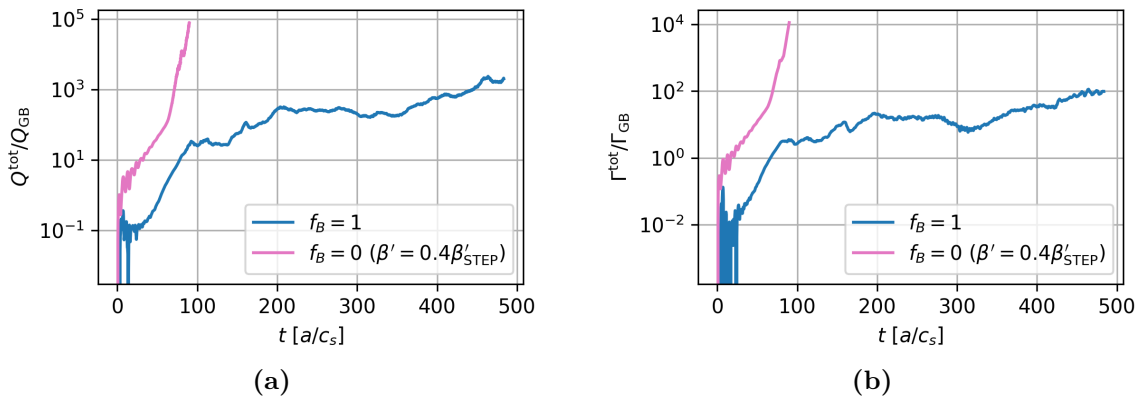
radius in the burning plasma phase of the conceptual high- $\beta$  reactor-scale, tight-aspect-ratio tokamak STEP. It has previously been found [8] that  $\delta B_{\parallel}$  is essential for the hKBM to be unstable in local equilibria taken from the STEP burning flat top.

### 7.1. The MHD approximation is typically not appropriate for use in high- $\beta_e$ reactor-scale plasmas

In Section 4, it was shown that inclusion of  $\delta B_{\parallel}$  and a rigorous treatment of the magnetic drift velocity is essential to capture the correct linear spectrum of the hKBM in the STEP flat-top. The MHD approximation<sup>+</sup> roughly captures the linear spectrum of the hKBM,

<sup>+</sup> The MHD approximation is most faithfully represented by variant MHD-1, discussed in the main text, which compensates for the neglect of parallel magnetic perturbations by setting  $\omega_{\nabla B} = \omega_{\kappa}$ .





**Figure 9:** Time traces of the total heat flux (a) and total particle flux (b) from *GENE* simulations. Simulations are shown both with,  $f_B = 1$  (blue), and without,  $f_B = 0$  (magenta),  $\delta B_{\parallel}$ . In the case without  $\delta B_{\parallel}$ ,  $\beta' = 0.4\beta'_{\text{STEP}}$ . No approximations are made concerning the magnetic-drift velocity.

but predicts lower growth rates in the STEP local equilibrium and is significantly more stable at long wavelengths.

Even though the MHD approximation (MHD-1) reproduces qualitatively similar linear physics, Section 5 reveals that the corresponding nonlinear simulations do not accurately track nonlinear simulations for STEP with full physics that were reported in [9]. Typically, simulations which attempt to resolve the hKBM with full physics (including  $\delta B_{\parallel}$ ) yield fluxes rising to very large values and it can be difficult to obtain a robustly steady saturation period in the absence of flow shear (i.e. this typically requires a large amount of computational resource). When the MHD-1 approximation is used, the turbulence saturates at a level around two orders of magnitude lower than that reached in a comparable time in the full physics simulations. The reason for this difference is that the longest wavelength (i.e., modes corresponding to  $n < 10$ ) are stable when the MHD approximation is used, but unstable with full physics. It was argued that these linearly unstable modes at very long-wavelength are responsible for the transition to very large fluxes (see Fig 7 and discussion).

In Section 4.2 it was found very difficult to improve the linear performance of the MHD-approximation whilst making only relatively minor changes to the local equilibrium.

*7.2.  $\delta B_{\parallel}$  is not essential for studying hKBM driven turbulence. However,  $\delta B_{\parallel}$  is essential for studying turbulence in the high  $\beta'$  conditions of the STEP flat-top.*

In Section 6,  $\beta'$  stabilisation [30] was identified as being responsible for the sensitivity of hKBM stability to  $\delta B_{\parallel}$  fluctuations, and it was noted that this plasma sits close to marginality. It was also shown, however, that the hybrid mode becomes unstable in the absence of  $\delta B_{\parallel}$  if  $\beta'$  is reduced to push the local equilibrium that are further from

marginality. In such local equilibria (further from marginal stability at long wavelength), electromagnetic runaway fluxes emerge in nonlinear simulations that neglect  $\delta B_{\parallel}$  (with or without the MHD approximation). This suggest that turbulence can be simulated in some high- $\beta$  ST plasmas using presently available tools and codes that neglect  $B_{\parallel}$ .

In the present design of the STEP flat-top, however, it is essential to include  $B_{\parallel}$  because the modes expected to dominate turbulence are both: (i) unstable only with  $\delta B_{\parallel}$  and; (ii) unstable up very long wavelengths. Global gyrokinetic codes currently being developed to include  $\delta B_{\parallel}$  rigorously will be important and timely for STEP.

Finally this work motivates the need for a rigorous theory of electromagnetic turbulence saturation, and an elucidation of the role of  $\delta A_{\parallel}$  in particular.

## Acknowledgments

Simulations were performed using resources provided by the Cambridge Service for Data Driven Discovery (CSD3) operated by the University of Cambridge Research Computing Service ([www.csd3.cam.ac.uk](http://www.csd3.cam.ac.uk)), provided by Dell EMC and Intel using Tier-2 funding from the Engineering and Physical Sciences Research Council (capital grant EP/T022159/1), and DiRAC funding from the Science and Technology Facilities Council ([www.dirac.ac.uk](http://www.dirac.ac.uk)). The work of PI was supported by the Engineering and Physical Sciences Research Council (EPSRC) [EP/R034737/1]. The work of TA was supported in part by the Royal Society Te Apārangi Marsden grant MFP-UOO2221. The authors (CMR) would also like to acknowledge helpful discussions with R J Hastie and J W Connor.

To obtain further information on the data and models underlying this paper please contact [PublicationsManager@ukaea.uk](mailto:PublicationsManager@ukaea.uk).

## Appendix A. The MHD approximation in the GK framework

The perturbed perpendicular pressure  $\delta P_{\perp}$  appearing on the right hand side of the linear GK force balance equation 6 can be calculated by solving the linear gyrokinetic equation 1 in the limit  $k_{\parallel} v_{\text{ths}} \ll \omega_{ds} \ll \omega$  and substituting into equation 7 to give

$$\begin{aligned} \delta P_{\perp} = \sum_s \int d\mathbf{v}^3 \frac{m_s q_s v_{\perp}^2}{2T_{0s}} \left[ 1 - \frac{\omega_{*s}^T}{\omega} + \frac{\omega_{ds}}{\omega} \right] \\ \times \left[ \delta\phi - v_{\parallel} \delta A_{\parallel} + \frac{m_s v_{\perp}^2}{2q_s B_0} \delta B_{\parallel} \right] F_{0s}. \end{aligned} \quad (\text{A.1})$$

Replacing  $\delta B_{\parallel}$  with  $\delta P_{\perp}$  via (6), and dropping terms involving  $\delta A_{\parallel}$  that vanish because they are odd in  $v_{\parallel}$ , we can write:

$$\delta P_{\perp} = \sum_s \int d\mathbf{v}^3 F_{0s} \frac{m_s v_{\perp}^2 q_s}{2T_{0s}} \left[ 1 - \frac{\omega_{*s}^T}{\omega} + \frac{\omega_{ds}}{\omega} \right] \times \left[ \delta\phi - \frac{m_s v_{\perp}^2}{2q_s} \frac{\mu_0 \delta P_{\perp}}{B_0^2} \right]. \quad (\text{A.2})$$

That is, we obtain

$$\begin{aligned}
\delta P_{\perp} &= \sum_s \int d\mathbf{v}^3 F_{0s} \left\{ \frac{m_s v_{\perp}^2}{2T_{0s}} q_s \delta\phi \left[ 1 - \frac{\omega_{*s}^T}{\omega} + \frac{\omega_{ds}}{\omega} \right] - \frac{m_s^2 v_{\perp}^4}{4T_{0s}} \frac{\mu_0 \delta P_{\perp}}{B_0^2} \left[ 1 - \frac{\omega_{*s}^T}{\omega} + \frac{\omega_{ds}}{\omega} \right] \right\} \\
&\quad (A.3) \\
&= \underbrace{\delta\phi \sum_s \int d\mathbf{v}^3 \frac{q_s m_s v_{\perp}^2 F_{0s}}{2T_{0s}} \left[ 1 - \frac{\omega_{*s}^T}{\omega} + \frac{\omega_{ds}}{\omega} \right]}_{[I]} - \underbrace{\frac{\mu_0 \delta P_{\perp}}{B_0^2} \sum_s \int d\mathbf{v}^3 \frac{m_s^2 v_{\perp}^4 F_{0s}}{4T_{0s}} \left[ 1 - \frac{\omega_{*s}^T}{\omega} + \frac{\omega_{ds}}{\omega} \right]}_{[II]}
\end{aligned}$$

Performing the velocity space integrals and sums over species allows us to obtain:

$$[I] = \delta\phi \sum_s n_s q_s \left\{ 1 - \frac{\omega_{*s}}{\omega} (1 + \eta_s) + \frac{T_{0s}}{q_s B_0 \omega} (\omega_{\kappa} + 2\omega_{\nabla B}) \right\}. \quad (A.4)$$

The first term in (A.4) vanishes due to quasineutrality. On summing over species the second term reduces to:

$$\sum_s n_s q_s \omega_{*s} (1 + \eta_s) = \frac{(\mathbf{k} \times \mathbf{b}_0) \nabla P}{B_0} = \frac{B_0}{\mu_0} (\omega_{\kappa} - \omega_{\nabla B}), \quad (A.5)$$

where we have also used (C.4) relating  $\omega_{\kappa}$  and  $\omega_{\nabla B}$ , derived in Appendix C. Using these results in (A.4) we obtain:

$$[I] = \frac{B_0 \delta\phi}{\mu_0 \omega} \left( (\omega_{\nabla B} - \omega_{\kappa}) + \frac{\beta}{2} (\omega_{\kappa} + 2\omega_{\nabla B}) \right) \quad (A.6)$$

Evaluating similar velocity space integrals in [II] from the RHS of A.3, gives:

$$[II] = -\delta P_{\perp} \sum_s \beta_s \left[ 1 - \frac{\omega_{*s}}{\omega} (1 + 2\eta_s) + \frac{T_{0s}}{q_s B_0 \omega} (\omega_{\kappa} + 3\omega_{\nabla B}) \right] \quad (A.7)$$

Now substituting for [I] and [II] [from (A.6) and (A.7)] into (A.3), retaining only leading order terms in a small  $\beta$  expansion gives:

$$\delta P_{\perp} = \frac{B_0 \delta\phi}{\mu_0 \omega} (\omega_{\nabla B} - \omega_{\kappa}), \quad (A.8)$$

obtained in the limits  $k_{\parallel} v_{\text{th}s} \ll \omega_{ds} \ll \omega$  and  $\beta \ll 1$ . It follows from (A.8) that in these limits  $\delta P_{\perp}$  and therefore  $\delta B_{\parallel}$ , can only be neglected if:

$$\omega_{\nabla B} = \omega_{\kappa} \quad (A.9)$$

This is the MHD approximation that appears as (9) in the main text.

## Appendix B. Cancellation of Magnetic Drift and $\mathbf{B}_{\parallel}$ Terms in Appropriate Limits

Using the relationship between  $\nabla B$  and curvature drifts in C.4, the drift term on the left-hand side of the GK equation in 1 can be written in a form convenient for the MHD-1 approximation:

$$T_{ds} = -\frac{1}{\Omega_s} \left[ \omega_{\kappa} \left( v_{\parallel}^2 + \frac{v_{\perp}^2}{2} \right) - \frac{\mu_0 (\mathbf{k}_{\perp} \cdot \mathbf{b}) \nabla p v_{\perp}^2}{B_0^2} \frac{v_{\perp}^2}{2} \right] g_s \quad (B.1)$$

The final term is the pressure gradient contribution to the  $\nabla B$  drift:

$$T_{\nabla B}^{p'} = \frac{1}{q_s B_0} \frac{\mu_0 \mathbf{k}_\perp \cdot \mathbf{b}_0 \times \nabla p}{B_0^2} \frac{m_s v_\perp^2}{2} g_s = \omega_{\text{dia}} \frac{\mu_0 n_{0s} T_{0s}}{B_0^2} \frac{m_s v_\perp^2}{2 T_{0s}} g_s \quad (\text{B.2})$$

where  $\omega_{\text{dia}} = \mathbf{k}_\perp \cdot \mathbf{V}_{\text{dia}}$  and the diamagnetic velocity  $\mathbf{V}_{\text{dia}} = \mathbf{b}_0 \times \nabla p / (n_{0s} q_s B_0)$ .

Alternatively the magnetic drift term can be expressed as:

$$T_{ds} = -\frac{1}{\Omega_s} \left[ \omega_{\nabla B} \left( v_\parallel^2 + \frac{v_\perp^2}{2} \right) + \frac{\mu_0 (\mathbf{k}_\perp \cdot \mathbf{b}_0) \nabla p}{B_0^2} v_\parallel^2 \right] g_s. \quad (\text{B.3})$$

where the final term resembles a pressure gradient contribution to the curvature drift (though this decomposition is unphysical) and can be written as:

$$T_\kappa^{p'} = -\omega_{\text{dia}} \frac{\mu_0 n_{0s} T_{0s}}{B^2} \frac{m_s v_\parallel^2}{T_{0s}} g_s \quad (\text{B.4})$$

In the long wavelength high frequency limit ( $b_s \rightarrow 0$  and  $\omega \gg \omega_{*s}^T$ ) the compressional magnetic perturbation term on the RHS of the GKE reduces to:

$$T_{B_\parallel} = \omega \frac{\delta B_\parallel}{B_0} \frac{m_s v_\perp^2}{2 T_{0s}} F_{0s} = -\omega \frac{\mu_0 \delta P_\perp}{B_0^2} \frac{m_s v_\perp^2}{2 T_{0s}} F_{0s}, \quad (\text{B.5})$$

using perturbed perpendicular force balance in equation 6 to reach the second equality.

*Consideration of MHD-1:* In the long wavelength fluid limit,  $g_s$  is Maxwellian to leading order in  $\omega_{ds}/\omega$ , so  $T_{\nabla B}^{p'}$  and  $T_{B_\parallel}$  have approximately the same structure in velocity space. A cancellation between  $T_{\nabla B}^{p'}$  and  $T_{B_\parallel}$  will arise at leading order if:

$$\frac{q_s \delta \phi}{T_{0s}} = -\frac{\omega}{\omega_{\text{dia}} n_s T_{0s}} \frac{\delta P_\perp}{\omega_{\text{dia}} n_s T_{0s}}, \quad (\text{B.6})$$

This approximate cancellation between  $T_{\nabla B}^{p'}$  and  $T_{B_\parallel}$  allows the  $\nabla p$  contribution to the  $\nabla B$  drift and  $B_\parallel$  to be dropped from the GKE, and corresponds to MHD-1.

*Consideration of MHD-2:* On the other hand,  $T_\kappa^{\nabla p}$  and  $T_{B_\parallel}$  have manifestly different structure in velocity space, so exact cancellation is not possible in the GKE. While MHD-2 implements 9 to allow  $\delta B_\parallel$  to be dropped from the GKE, it only does this at the expense of *changing the local equilibrium and its magnetic curvature*,  $\mathbf{b}_0 \cdot \nabla \mathbf{b}_0$ . In effect MHD-2 *changes the local equilibrium field curvature*, and *again removes the pressure gradient contribution to the  $\nabla B$  drift*,  $T_{\nabla B}^{p'}$  defined in B.2, for this modified local equilibrium!

## Appendix C. A useful identity involving the magnetic-drift frequency

The magnetic-drift frequency is given by:

$$\omega_{ds} = \mathbf{k}_\perp \cdot \mathbf{b}_0 \times \left( \frac{v_\parallel^2}{\Omega_s} (\mathbf{b} \cdot \nabla) \mathbf{b} + \frac{v_\perp^2}{2\Omega_s} \nabla \ln B \right) = \frac{\omega_\kappa v_\parallel^2}{\Omega_s} + \frac{\omega_{\nabla B} v_\perp^2}{2\Omega_s} \quad (\text{C.1})$$

Using Ampère's law, we can write

$$\mu_0 \mathbf{J} \times \mathbf{B} = [\nabla \times \mathbf{B}] \times \mathbf{B}$$

$$\begin{aligned}
&= -\mathbf{B} \times [\nabla \times (B\mathbf{b})] \\
&= -\mathbf{B} \times [\nabla B \times \mathbf{b} + B(\nabla \times \mathbf{b})] \\
&= -\mathbf{B} \times [\nabla B \times \mathbf{b}] - \mathbf{B} \times B(\nabla \times \mathbf{b}) \\
&= -B\nabla B + (\mathbf{B} \cdot \nabla B)\mathbf{b} - B^2\mathbf{b} \times (\nabla \times \mathbf{b}) \\
\Rightarrow \mu_0\mathbf{J} \times \mathbf{B} &= -B\nabla_{\perp} B + B^2\mathbf{b} \cdot \nabla\mathbf{b} \tag{C.2}
\end{aligned}$$

where  $\mathbf{b}$  is a unit vector aligned to the magnetic field, so that  $\mathbf{b} \times (\nabla \times \mathbf{b}) = -\mathbf{b} \cdot \nabla\mathbf{b}$ . Now invoking equilibrium force balance  $\mathbf{J} \times \mathbf{B} = \nabla p$  [see equation (128) of [21]] in (C.2) gives:

$$\nabla_{\perp} \ln B = (\mathbf{b} \cdot \nabla)\mathbf{b} - \frac{\mu_0 \nabla p}{B^2}. \tag{C.3}$$

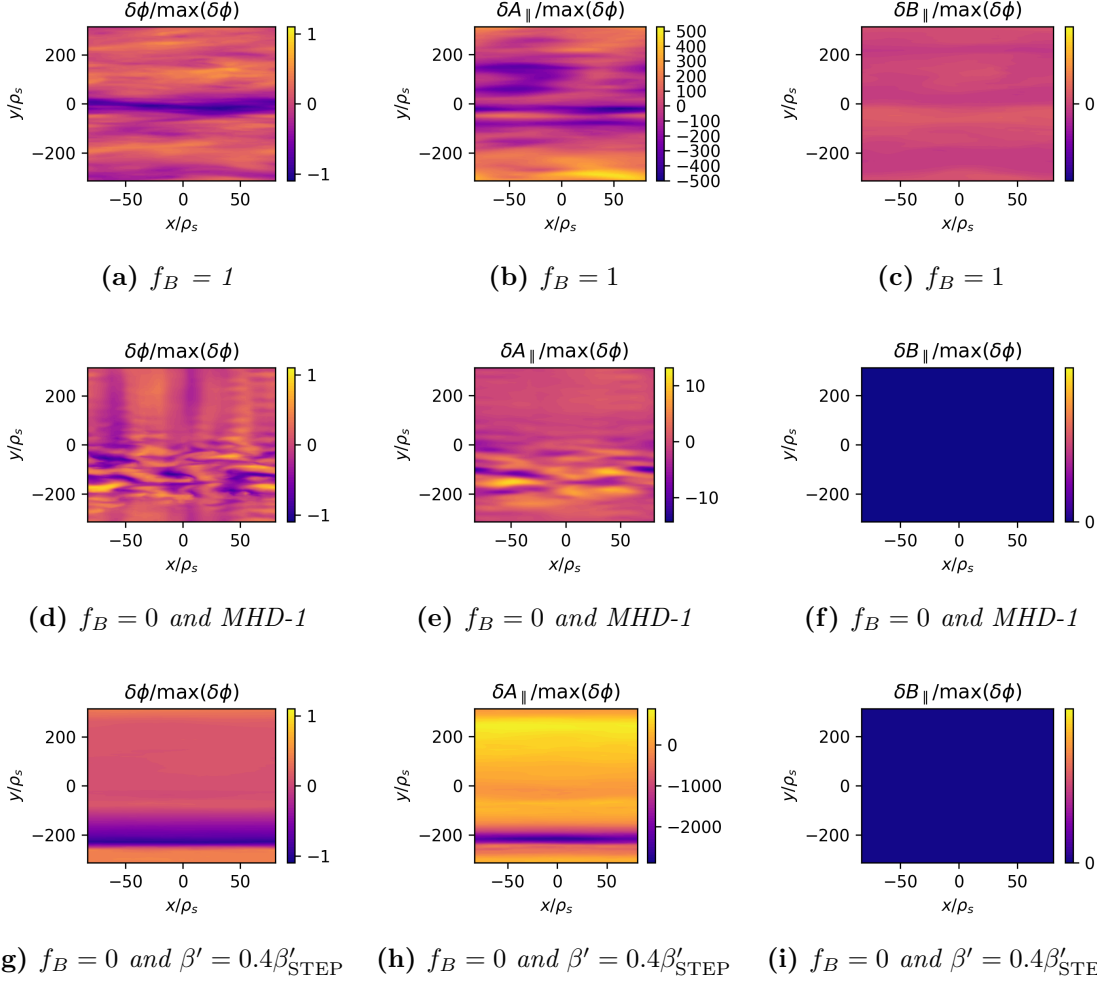
which usefully relates the gradient in magnetic-field strength to the curvature, and can be expressed as:

$$\omega_{\kappa} - \omega_{\nabla B} = \frac{\mu_0 \mathbf{k}_{\perp} \cdot \mathbf{b} \times \nabla p}{B^2} \tag{C.4}$$

Using (C.3) to substitute for  $\nabla \ln B$  in (C.1), the magnetic drift frequency can be re-expressed as:

$$\Omega_s \omega_{ds} = \mathbf{k}_{\perp} \cdot \mathbf{b} \times \left[ \left( v_{\parallel}^2 + \frac{v_{\perp}^2}{2} \right) (\mathbf{b} \cdot \nabla)\mathbf{b} - \frac{v_{\perp}^2 \mu_0 \nabla p}{2B^2} \right]. \tag{C.5}$$

## Appendix D. Snapshots of the turbulent fields



**Figure D1:** Snapshot contour plots at the outboard midplane ( $\theta = 0$ ) from the final time step of different nonlinear simulations discussed throughout the text: (a, d, g)  $e\delta\phi/(\rho_*T_e)$ , (b, e, h)  $\delta A_{\parallel}/(\rho_*\rho_s B_0)$ , and (c, f, i)  $\delta B_{\parallel}/(\rho_*T_e B_0)$ . Each field is normalised to the maximum value of  $e\delta\phi/(\rho_*T_e)$  at the outboard midplane.

## References

- [1] H Meyer for the STEP Plasma Control, Heating, Current Drive Team, and Contributors. The physics of the preferred plasma scenario for step. In *Proceedings of the 48th EPS Conference on Plasma Physics*, June 2022.
- [2] STEP - Spherical Tokamak for Energy Production. <https://step.ukaea.uk/>.
- [3] Tholerus, E and *et al.* Flat-top plasma operational space of the STEP power plant. in-preparation, 2024.
- [4] G. M. Staebler, J. E. Kinsey, and R. E. Waltz. Gyro-Landau fluid equations for trapped and passing particles. *Physics of Plasmas*, 12(10):102508, 10 2005.

- [5] C. Bourdelle, X. Garbet, F. Imbeaux, A. Casati, N. Dubuit, R. Guirlet, and T. Parisot. A new gyrokinetic quasilinear transport model applied to particle transport in tokamak plasmas. *Physics of Plasmas*, 14(11):112501, 11 2007.
- [6] J. Citrin, S. Maeyama, C. Angioni, N. Bonanomi, C. Bourdelle, F.J. Casson, E. Fable, T. Görler, P. Mantica, A. Mariani, M. Sertoli, G. Staebler, T. Watanabe, and JET Contributors. Integrated modelling and multiscale gyrokinetic validation study of etg turbulence in a jet hybrid h-mode scenario. *Nuclear Fusion*, 62(8):086025, jun 2022.
- [7] S M Kaye, J W Connor, and C M Roach. Thermal confinement and transport in spherical tokamaks: a review. *Plasma Physics and Controlled Fusion*, 63(12):123001, 2021.
- [8] D. Kennedy, M. Giacomini, F.J. Casson, D. Dickinson, W.A. Hornsby, B.S. Patel, and C.M. Roach. Electromagnetic gyrokinetic instabilities in step. *Nuclear Fusion*, 63(12):126061, 2023.
- [9] M. Giacomini, D. Kennedy, C.J. Ajay, F. Casson, D. Dickinson, B. S. Patel, and C. M. Roach. Electromagnetic gyrokinetic instabilities in the Spherical Tokamak for Energy Production (STEP) part II: turbulence and transport. *submitted to PPCF*, 2023.
- [10] H. L. Berk and R. R. Dominguez. Variational method for electromagnetic waves in a magnetoplasma. *Journal of Plasma Physics*, 18(1):31–48, 1977.
- [11] R. E. Waltz and R. L. Miller. Ion temperature gradient turbulence simulations and plasma flux surface shape. *Physics of Plasmas*, 6(11):4265–4271, 11 1999.
- [12] N. Joiner, Akira Hirose, and William Dorland. Parallel magnetic field perturbations in gyrokinetic simulations. *Physics of Plasmas*, 17:072104–072104, 07 2010.
- [13] F. Jenko, W. Dorland, M. Kotschenreuther, and B. N. Rogers. Electron temperature gradient driven turbulence. *Physics of Plasmas*, 7(5):1904–1910, 2000.
- [14] B. S. Patel, D. Dickinson, C. M. Roach, and H. Wilson. Linear gyrokinetic stability of a high  $\beta$  non-inductive spherical tokamak. *Nuclear Fusion*, 62(1):016009, 2021.
- [15] Alexey Mishchenko, Matthias Borchardt, Roman Hatzky, Ralf Kleiber, Axel Könies, Carolin Nührenberg, Pavlos Xanthopoulos, Gareth Roberg-Clark, and Gabriel G. Plunk. Global gyrokinetic simulations of electromagnetic turbulence in stellarator plasmas. *Journal of Plasma Physics*, 89(3):955890304, 2023.
- [16] E. Lanti, N. Ohana, N. Tronko, T. Hayward-Schneider, A. Bottino, B.F. McMillan, A. Mishchenko, A. Scheinberg, A. Biancalani, P. Angelino, S. Brunner, J. Dominski, P. Donnel, C. Gheller, R. Hatzky, A. Jocksch, S. Joliet, Z.X. Lu, J.P. Martin Collar, I. Novikau, E. Sonnendrücker, T. Vernay, and L. Villard. Orb5: A global electromagnetic gyrokinetic code using the pic approach in toroidal geometry. *Computer Physics Communications*, 251:107072, 2020.
- [17] K. Aleynikova, A. Zocco, P. Xanthopoulos, P. Helander, and C. Nührenberg. Kinetic ballooning modes in tokamaks and stellarators. *Journal of Plasma Physics*, 84(6):745840602, 2018.
- [18] J P Graves, D Zullino, D Brunetti, S Lanthaler, and C Wahlberg. Reduced models for parallel magnetic field fluctuations and their impact on pressure gradient driven mhd instabilities in axisymmetric toroidal plasmas. *Plasma Physics and Controlled Fusion*, 61(10):104003, 2019.
- [19] C. Bourdelle, W. Dorland, X. Garbet, G. W. Hammett, M. Kotschenreuther, G. Rewoldt, and E. J. Synakowski. Stabilizing impact of high gradient of  $\beta$  on microturbulence. *Physics of Plasmas*, 10(7):2881–2887, 2003.
- [20] A. Y. Sharma, M. D. J. Cole, T. Görler, Y. Chen, D. R. Hatch, W. Guttenfelder, R. Hager, B. J. Sturdevant, S. Ku, A. Mishchenko, and C. S. Chang. Global gyrokinetic study of shaping effects on electromagnetic modes at NSTX aspect ratio with ad hoc parallel magnetic perturbation effects. *Physics of Plasmas*, 29(11):112503, 11 2022.
- [21] I G Abel, G G Plunk, E Wang, M Barnes, S C Cowley, W Dorland, and A A Schekochihin. Multiscale gyrokinetics for rotating tokamak plasmas: fluctuations, transport and energy flows. *Reports on Progress in Physics*, 76(11):116201, oct 2013.
- [22] Peter J. Catto. Practical gyrokinetics. *Journal of Plasma Physics*, 85(3):925850301, 2019.
- [23] C. M. Roach, D. J. Applegate, J. W. Connor, S. C. Cowley, W. D. Dorland, R. J. Hastie, N. Joiner, S. Saarelma, A. A. Schekochihin, R. J. Akers, et al. Microstability physics as illuminated in the

- spherical tokamak. *Plasma Physics and Controlled Fusion*, 47(12B):B323, 2005.
- [24] A. Zocco, N. F. Loureiro, D. Dickinson, R. Numata, and C. M. Roach. Kinetic microtearing modes and reconnecting modes in strongly magnetised slab plasmas. *Plasma Physics and Controlled Fusion*, 57(6):065008, 2015.
- [25] R. L. Miller, M. S. Chu, J. M. Greene, Y. R. Lin-Liu, and R. E. Waltz. Noncircular, finite aspect ratio, local equilibrium model. *Physics of Plasmas*, 5(4):973–978, 1998.
- [26] B.S. Patel, L. Pattinson, P. Hill, M. Giacomini, D. Kennedy, D. Dickinson, H.G. Dudding, F.J. Casson, and A.C. Jayalekshmi. pyrokinetics, 10 2022.
- [27] B. Patel, P. Hill, L. Pattinson, M. Giacomini, A. Bokshi, D. Kennedy, H.G. Dudding, J.F. Parisi, T.F. Neiser, Ajay C. Jayalekshmi, D. Dickinson, and year = 2024 note = accepted-to appear J. Ruiz Ruiz, title = Pyrokinetics - A Python library to standardise gyrokinetic analysis.
- [28] D. A. Dickey and W. A. Fuller. Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American statistical association*, 74(366a):427–431, 1979.
- [29] K. Germaschewski, B. Allen, T. Dannert, M. Hrywniak, J. Donaghy, G. Merlo, S. Ethier, E. D’Azevedo, F. Jenko, and A. Bhattacharjee. Toward exascale whole-device modeling of fusion devices: Porting the GENE gyrokinetic microturbulence code to GPU. *Physics of Plasmas*, 28(6):062501, 06 2021.
- [30] C. Bourdelle, G.T. Hoang, X. Litaudon, C.M. Roach, and T. Tala. Impact of the  $\alpha$  parameter on the microstability of internal transport barriers. *Nuclear Fusion*, 45(2):110, 2005.
- [31] M. J. Pueschel, D. R. Hatch, T. Görler, W. M. Nevins, F. Jenko, P. W. Terry, and D. Told. Properties of high- $\beta$  microturbulence and the non-zonal transition. *Physics of Plasmas*, 20(10):102301, 2013.
- [32] A. M. Dimits, G. Bateman, M. A. Beer, B. I. Cohen, W. Dorland, G. W. Hammett, C. Kim, J. E. Kinsey, M. Kotschenreuther, A. H. Kritz, L. L. Lao, J. Mandrekas, W. M. Nevins, S. E. Parker, A. J. Redd, D. E. Shumaker, R. Sydora, and J. Weiland. Comparisons and physics basis of tokamak transport models and turbulence simulations. *Physics of Plasmas*, 7(3):969–983, March 2000.